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Trinity College Toronto

JOURNAL

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INSTITUTE OF ACTUARIES.

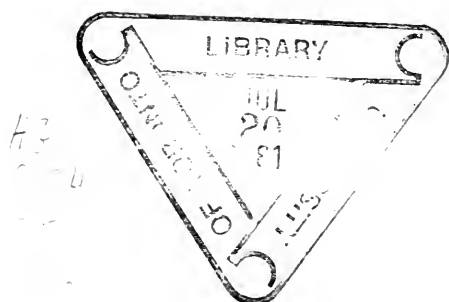
"I hold every man debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of wits to be a help and ornament thereunto."—BACON.

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JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The late William Matthew Makeham.

BY the death of Mr. William Matthew Makeham, so feelingly alluded to by the President of the Institute at the opening of the present session, the scientific literature of life contingencies has lost one of its most brilliant names. A reference to the pages of this *Journal* will convey some impression of the importance and variety of the subjects he dealt with from time to time, but the productions of such a master mind cannot be fitly judged by their length or number. In some of his shorter contributions the highest level of scientific ability and invention is reached, and in all—with scarcely a single exception—are exhibited remarkable powers of insight and analysis, combined with a lucid and luminous mode of exposition, rarely found in association save in intellects of the highest calibre. The value of Makeham's writings will endure for all time, and his will always be a foremost place among those early exponents of actuarial science who guided it into the channels along which it has steadily progressed.

Although Makeham's name is so closely associated with his formula of graduation, or "Law of Mortality", as it is indifferently termed, there are other eminent achievements of his which will immediately present themselves to the minds of readers of this *Journal*. The concentration of thought and mathematical

ingenuity, shown in his masterly contributions on Compound Survivorship Problems (*J.I.A.*, x, 241, and xii, 61), have called forth high praise, and led the way to greater simplification in the solution of these intricate and important matters. Another striking instance of Makeham's extraordinary ability to grapple with the most complicated problems, and reduce them to simple elements, is seen in his paper "On the Theory of Annuities-Certain" (*J.I.A.*, xiv, 189), than which nothing more elegant has, perhaps, found a place in the *Journal*. The easily applied formulas he there obtained for deriving the values of increasing annuities-certain are too well known to need further remark.

With equal success he dealt with the question of Extra Risks (*J.I.A.*, xiv, 159, 242, and xvii, 153), and it is interesting to reproduce the editorial note appended to the last of these papers, as evidence of his great practical foresight:

"* * * The above paper will, no doubt, prove practically useful to the managers of the companies which adopt the method of charging for impaired health therein discussed,* but it appears to us to be even more valuable to the student of the theory of life contingencies, as suggesting the course which future investigations into the mortality among under-average lives should follow. Mr. Makeham's investigations clearly demonstrate that the point to be ascertained is whether the increased mortality among such lives is more apparent immediately after the grant of the policy or in later years; and any future investigation into such mortality must be considered incomplete that does not give special attention to this point."—*ED. J.I.A.*

Other contributions of great merit, too numerous to mention, are scattered throughout the pages of this *Journal*. The "continuous method" now so often employed, and destined, we may confidently predict, to play an even more important part in actuarial analysis, received its impetus from his early papers. His many purely mathematical essays—the last of which appeared shortly before his death was announced—are valuable additions to the scientific equipment of a modern actuary. They have been justly admired, both in this country and in America, where Makeham's writings are held in the highest esteem. The amount of instruction conveyed in a short space is often astonishing, and, to appraise Makeham's worth as an author, we have only to imagine what actuarial knowledge would now be could his writings be expunged from the records and obliterated from the memory.

* That is, by way of contingent debt on the policy.

We have left till the last our reference to the papers dealing with the formula of graduation, or law of mortality, known as Makeham's hypothesis, to which a somewhat extended consideration will not be thought untimely. The first indication of his important modification of Gompertz's theoretical law of mortality was given to the world in January 1860 (*J.I.A.*, viii, 301). After premising that Gompertz's assumption could be defined "by stating that the logarithms of the probabilities of living over any given period proceed in geometrical progression", Makeham showed, by the aid of the Carlisle, Seventeen Offices', and Government Annuitants' Tables, that the facts did not conform to Gompertz's theory; and he proposed to make them obey the law assigned by the addition of a certain uniform quantity (x) to each term. The terms here referred to were the logarithms of ${}_np_x$; and, Gompertz's formula containing three constants, three values of the function were necessary for obtaining the values of such constants. Makeham operated upon l_{20} , l_{40} , and l_{60} , whence he passed to ${}_{20}p_{20}$, ${}_{20}p_{40}$, and ${}_{20}p_{60}$, and the logarithms of these quantities formed the terms which, by Gompertz's hypothesis, should be in geometrical progression. Denoting these terms by the letters (a), (b), and (c), the numerical value of (x) was easily found. Since $(a+x)$, $(b+x)$, and $(c+x)$ are in geometrical progression, we have

$$(a+x)(c+x) = (b+x)^2$$

whence

$$x = \frac{b^2 - ac}{a + c - 2b}.$$

By this means, the terms were brought into conformity with the hypothesis, and the law of mortality, as modified, was defined by stating that the "probabilities of living, *increased or diminished in a certain constant ratio*, form a series whose logarithms are "in geometrical progression."

From this simple device flowed all the merits and advantages which are associated with Makeham's formula. The close agreement of the hypothetical law of mortality with that shown in the Seventeen Offices' Experience was first pointed out, and the unique facilities afforded for the computation of annuities on any given number of lives were then demonstrated. The next stage in the development of the theory (*J.I.A.*, xiii, 325) was to deduce the well-known formula for the force of mortality:

$$\mu_x = A + Bq^x.$$

This formula, the author explains, "is derived from a modification

“ of Mr. Gompertz’s simple and highly ingenious theory, that the
“ power to oppose destruction loses equal proportions in equal
“ times. The modification which I propose to introduce consists
“ in the limitation of the theory to a *portion only* of the partial
“ forces of mortality, and the assumption that the remaining
“ forces operate (in the aggregate) with equal intensity at all
“ ages.” In the above formula, it is scarcely necessary to say, A is the sum of certain partial forces assumed to be of equal amount at all ages, and Bq^x the aggregate of several forces of a similar nature, but increasing obviously in constant relation to the age.

It is curious to look back, and observe how slow Makeham’s valuable and ingenious suggestions were to gain acceptance. No comment on his exposition of the new hypothesis is to be found in our *Journal* until, four years after publicity had been given to it, the late Peter Gray, in a generous and highly appreciative letter, written in November 1863 (*J.I.A.*, xi, 236), enforced the value and importance of the new investigations. We cannot refrain from thinking that the time of their publication was, by a freak of chance, unfavourable to that early discussion which the merit and originality of Makeham’s suggestions must be held to have deserved. The burning topic then, and for some time afterwards, was the claim—the unwarrantable claim, as we must now regard it—of Mr. T. R. Edmonds to have discovered an original and independent law of mortality; and the leading mathematical minds of that generation, De Morgan, Mr. Sprague, and Mr. Woolhouse, were engaged in proving that Edmonds’ “law of mortality” was, to all intents and purposes, identical with that which Gompertz had given to the world some years previously. If this conjecture appear far-fetched, it is not easy to comprehend the neglect of Makeham’s theory on any other ground.

After Gray’s letter, no mention was made of the subject until Makeham resumed his investigations in November 1865 (*J.I.A.*, xii, 305). The physiological interpretation of his theory was now given, and the hypothesis may be considered to have been fully enunciated. An incidental feature of this paper was the discovery that in the second enumeration of the population of Carlisle (in December 1787), as used by Milne in conjunction with the earlier enumeration (in January 1780) in the construction of the Carlisle Table, the ages of the members living were not derived by actual enumeration of the inhabitants age by age, but by distributing the later population, 8,677 in total, according

to the numbers living at each age in January 1780, 7,677 in total, by increasing each item in the proportion of 8,677 to 7,677. This removed one of the pillars on which the Carlisle Table rested, and by diminishing confidence in the original data, paved the way to its being superseded as a standard table for life assurance calculations by more exact experiences.

Makeham's modified law of mortality was next mentioned, cursorily, by Mr. M. N. Adler, in his "Memoir of the late Benjamin Gompertz" (*J.I.A.*, xiii, 14); and, passing by brief congratulatory references at the annual meetings of the Institute, we then find it examined and discussed, in July 1870, by Mr. Woolhouse (*J.I.A.*, xv, 103), who, after pointing out the great advantages to be derived from the formula, expressed the view that future investigations might reveal other important relations at present unknown. From this point it would be useless to trace the progress of Makeham's theory in the actuarial mind. It had at last received due recognition, and has since steadily increased its adherents and advocates throughout the scientific world.

Passing notice may be made of the later developments of Makeham's famous hypothesis. Mr. Emory McClintock, pursuing an elaborate investigation by Makeham, showed how the value of annuities on any number of lives, by any given constants of mortality, at any given rate of interest, may be found by means of the ordinary tables of the well-known gamma-function (*J.I.A.*, xviii, 242). In treating the formula as a means of adjusting tables of experience, Messrs. G. F. Hardy and G. King (*J.I.A.*, xxii, 191) introduced a most important modification by making the constants depend, not upon isolated values of $\log l_x$, but upon the summation of the values of $\log l_x$ in groups. This process (termed the Aggregate Method) gave a broader base to the resulting table, and it will no doubt be invariably adopted in future attempts to graduate tables by Makeham's formula.

Quite recently Makeham, returning to the subject of his early triumphs, has shown how Gompertz's law may be still further developed (*J.I.A.*, xxviii, 191). The successive developments may be exhibited as follows:

1. Gompertz's law $l_x = dg^{qx}$ ($\Delta^1 \log l_x$ in geom.¹ progression)
2. Makeham's first } $= dg^{qx} s^x$ (Δ^2 " " ")
- modification
3. Makeham's second } $= dg^{qx} s^x \omega^{x^2} (\Delta^3$ " " ")
- modification

By means of an elaborate analysis, Makeham then proves that the new term, ω^s , introduced into (3) would involve an increase in the rate of interest, a set of tables for equal ages at *successive rates of interest* being required for the computation of annuities on joint lives. The effect of this further extension of Gompertz's hypothesis is well summarized by Mr. Woolhouse (*J.I.A.*, xxviii, 481*): "The great value of Mr. Makeham's first development is the beautiful maintenance of the law of seniority under a somewhat different but equally convenient form to that belonging to Gompertz's law. When we come to Mr. Makeham's second development, there is found to exist a similar law of seniority, but it is unfortunately accompanied by an inconvenient imposition of a new rate of interest, having a determinate value depending on the relative set of lives which may enter any special calculation." At this time, it would appear that the second development, owing to its complexity, offers little scope or inducement for its practical application, though it is, of course, impossible to say to what use it may be put in the future.

Makeham's law is open to the great objection that, when employed to adjust mortality tables, it produces a curve of unbroken smoothness, and thus erases all irregularities in the series, which may be accidental or characteristic, according to the nature of the original observations. It thus becomes an unsuitable method of graduation under certain conditions, as when, for instance, the curve of mortality rapidly changes its form and progression. It would, therefore, be inapplicable to tables showing the influence of "selection" in the early years of insurance, where the mortality exhibits a totally different course for the first few years to that which it afterwards takes. But in the adjustment of an "average" table, required mainly as a basis for monetary values, the consistent regularity of the results ceases to be an objection, and, provided the graduated series reproduces the rough series with general fidelity, it becomes a positive advantage. Hence, Makeham's formula has been frequently employed in dealing with important tables of experience. It was adopted by the Thirty American Offices for their Male Life Table, by the Gotha Life Office in their Experience, and has been very skilfully applied to the II^M Mortality by Mr. G. King in the table appended to the Institute *Text-Book*, Part II. In many minor but important investigations it has also been used.

Experience in the use of Makeham's method of graduation

* On page 481, the 17th line from the bottom, s^2 is a misprint for s^x .

has brought to light one very interesting fact. The values of the constants d , g , and s , are different for different tables of observations, and even for different selected points (or groups) in the same table of observations. But a singular coincidence has been found to exist in the values of q , independently obtained from various bodies of facts. It is strange that, after his first investigation of the subject, Makeham himself should nowhere have given the constants derived from any of the numerous bodies of observations to which he applied his formula; but the curious agreement in the values of q was pointed out by him in 1867 (*J.I.A.*, xiii, 317) as significant and worthy of consideration. We may here collate the results of some of the principal applications of his method:

Makeham, Seventeen Offices' Table	$\log q =$.0409075
Woolhouse, H ^M 1st Curve0402225
" " 2nd "0395573
" " Mean Curve0400008
" Seventeen Offices' Mean Curve03956
Thirty American Offices041280
Gotha Life Office039625
King, H ^M (<i>Text-Book</i>) Aggregate Curve03965686

It was stated by Makeham that in the tables which he had calculated by means of his formula, an average value of q (or $\log q = .04$) could be used without materially affecting the results. Returning to this theme in one of his latest papers (*J.I.A.*, xxviii, 319), he proceeded to argue that the practically identical agreement in the value of $\log q$ could only result from the rate of deterioration of the vital force being the same for each individual. And he concluded, "Thus extended, Gompertz's law may be stated as follows:—The vital force or recuperative power of each individual "loses equal proportions in equal times; and the proportion of "vital force so lost by each is *universally the same*, being approximately represented by $\log q = .04$." Readers of these pages will not think we have spent too much time in bringing together the facts which bear upon and justify this memorable inference. Few such instances of the manifestation of a law, influencing and threading its way through vital statistics, are to be met with in the records of actuarial science; and it is a singular result, both full of suggestion and likely to prove of wide and useful application.

A final remark may be made regarding an interesting illustration of the significance of Makeham's formula for the *force of mortality*, $\mu_x = A + Bq^x$, given by Messrs. G. F. Hardy and H. J. Rothery in their paper on Mortality in the West Indies (*J.I.A.*, xxvii, 179). By comparing the values of the constants A and B for various tables of normal and extra mortality, the nature of the

incidence of the extra risk was shown in a simple and striking manner. Taking two examples only, the respective values of the constants are as follows:

	A	B
H ^M Table	·0061	·000093
West Indies (Barbados)	·0099	·000113

Comparing the values of A and B in the H^M and the West Indies observations, the authors pointed out that the effect of exposure to tropical climate upon the rate of mortality could be represented by a constant addition to the force of mortality (to allow for the increase in A) combined with an addition to the age (to correspond with the increase in B). And seeing that a constant addition to the force of mortality affects the annuity-values in the same way as an equivalent change in the rate of interest, Messrs. Hardy and Rothery found they were able to derive their special Barbados annuity-values at 4 per-cent interest from the H^M annuity-values by simply taking the latter values at $4\frac{1}{2}$ per-cent interest for ages three years older. In other words, a_x (Barbados 4 per-cent) is approximately equal to a_{x+3} (H^M $4\frac{1}{2}$ per-cent). In considering the influence of special mortality upon policy reserves, it was also clearly brought out that an increase in A while B remains constant diminishes the values of policies, while an increase in B while A remains constant increases them. In the course of the discussion, it was remarked that a further investigation of the relative values of A and B seemed to afford the means of determining the vexed question "How does an increased mortality affect policy-values?" There can be little doubt that further examinations of the constituent elements of the force of mortality, on the lines of Messrs. Hardy and Rothery's suggestive analysis, will throw light upon many of the problems to be met with in regard to the incidence of increased mortality.

This is not the place, nor is this the occasion, to dwell upon Makeham's personal characteristics—his unobtrusiveness, his love of the peace of domesticity, his sound literary judgment, and his many accomplishments outside the arena in which his skill was chiefly exhibited. But in closing this very imperfect account of the work he so effectively performed in the service of his profession, it is at once a pleasure and a duty to pay a tribute of admiration and gratitude to his memory for the zeal with which he devoted his energies to the advancement of actuarial science, and for the rare ability and inexhaustible resourcefulness which have opened up such fruitful fields of speculation to all earnest enquirers.

Formulas and Tables of Values for Life Interests and Reversions.

By H. ETHELSTON NIGHTINGALE, *Fellow of the Institute of Actuaries, and Actuarial Assistant to the Royal Exchange Assurance.*

[Read before the Institute, 21 December 1891.]

IT is with some reluctance that I venture to address the Institute on the subject of Reversionary Interests, a topic already exhaustively discussed by several members of great experience.

The chief motive which induces me to take this course is the desire to attract further attention to the advantages derived from the method of calculation originally adopted by Mr. Jellicoe (*J.I.A.*, ii, 159) and subsequently laid under contribution by Mr. Bunyon (*J.I.A.*, xviii, 32). This plan of operation, by means of which almost any formula may be derived from one general expression, was recommended by Mr. Bunyon for the following reasons:

- (1). The exceptional facility with which any required formula can be written down.
- (2). The great advantage secured by keeping the *component parts* of the formula clearly before the computer; thereby enabling him to modify the several parts without difficulty, in order to suit the case in question.

For instance, allowance may have to be made for loss of interest owing to delay in recovering the capital, and in re-investment of the same. A suitable premium may be necessary to cover a special assurance. Different rates of interest may be desired in possession and in reversion, convertible half-yearly or quarterly. In short, the actual conditions of a problem can be adhered to in the closest possible manner.

Mr. Bunyon considered Mr. Jellicoe's formulas as inexact, affirming that *complete* annuities should be used, on the ground that the Apportionment Act, 1870, rendered the income of a life tenant apportionable.

Proceeding on the lines indicated by Mr. Bunyon, I obtained several expressions different from those now in use. These new formulas appear to me more simple and exact than those generally adopted.

In order to increase the interest and utility of the paper, I

have appended a number of tables, showing the values of the various interests based on these formulas. Realizing that the sex of the life tenant considerably affected the results, I decided to calculate values for males and females separately. This step I felt to be essential, notwithstanding that the bulk of the tables was doubled in consequence.

I humbly trust that the result of my labour may prove of sufficient interest to warrant its intrusion on the space of the *Journal of the Institute*.

In building up a formula, whether for life interests or reversions, the following principles should be borne in mind:

- (1). That the total outlay be recovered in any event. An assurance should therefore be opened to cover every possible loss of capital not provided for by the reversion in question.
- (2). That interest on the total outlay be allowed for during the whole period intervening between the time when the loan or purchase is carried out, and the ultimate recovery of the capital.
- (3). That the total outlay be equal to the present value of interest thereon during the status in question, plus the present values of the outlay at the end of the status, and of the premiums required to cover any contingency by which the outlay may be lost.
- (4). The redemption money should be equal to the total outlay; except, in the case of life interests, where a portion of the premium should be deducted, as hereafter explained. (Special arrangements for redemption are, however, sometimes made for a fixed term of years).

Equity would require that the policy (if any), or the surrender-value thereof, should be handed over on redemption. Also, where the reversion is contingent and both the life tenant and reversioner are proved to be in good health, some allowance might be made for the unexpired joint-life annuity set up in the books, or assumed to have been purchased to provide premiums and interest during the remainder of the joint lives.

Let us start by assuming £1 as the total outlay. Then £1 will represent the amount to be recovered by payment of the sum assured, or by the falling in of the reversion; allowance having

been made for legacy duty, expense of realizing securities, possible depreciation of investments and any other *reversionary* charges likely to be incurred. Power to vary the securities must also be taken into consideration.

As regards *initial* expenses, they are divided into two classes:

- (1). Those representing a percentage on the sum assured, such as the first premium (when a policy has been effected) and the single premiums for whole-world licence and issue risk, if any.
- (2). Legal and other expenses usually adjusted upon the amount of loan or purchase-money.

Of the expenses under heading (1) the first premium only has been introduced into the formulas, this charge being universal whenever a policy is opened. On the other hand, should the remaining charges require to be taken into account, the total of such expenses should be substituted for the first premium.

The expenses under heading (2) have not been noticed in the formulas. They must be deducted from the results obtained, in order to find the so-called market value.

Now, taking the various interests in order, we have

I.—ABSOLUTE REVERSION TO £1 ON THE DEATH OF x .

(a). Here interest should be provided during the life of x .

The capital is returned by the falling in of the reversion, so that no assurance is necessary.

Hence, in accordance with principle (3) before mentioned, the total outlay, namely,

$$1 = \ddot{u}_x + R_x,$$

where R_x is the present value of £1 at the end of the status in question

$$\therefore R_x = 1 - \ddot{u}_x. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Specimen Values of Absolute Reversions—Interest 5 per-cent.

Age x	Carlisle A_x	TABLE I (see page 18)	
		Male	Female
40	·3145	·3473	·2897
60	·5267	·5336	·4895
80	·7612	·7707	·7542

(b). Power of re-purchase within a fixed term of years (generally five) is frequently desired by a borrower who believes the life tenant to be in bad health and wishes to secure any advantage that may accrue should the reversion determine at an early date.

Allowing for re-purchase within the first n years, on payment of R_x accumulated to the time of re-purchase at an agreed rate i' , we have

$$1 = i' a'_{\overline{n}|} + v'^n \cdot i a'_{x+n} + R_x$$

and

$$\begin{aligned} R_x &= 1 - i' \cdot a'_{\overline{n}|} - v'^n \cdot i a'_{x+n} \\ &= v'^n \cdot (1 - i a'_{x+n}) = v'^n \cdot R_{x+n} \quad \dots \quad (2) \end{aligned}$$

Example.—If the life tenant be aged 60, and a male, the value is 4672, as against 5336 given in the previous comparisons (n being taken = 5 and i' as 5 per-cent).

II.—CONTINGENT REVERSION TO £1 ON THE DEATH OF y , PROVIDED x BE THEN ALIVE.

(a). In this case the reversion returns the capital outlaid only if x survive y . An assurance is therefore required to cover the risk of x dying before y .

Interest must also be allowed for during the joint lives. Hence, the total outlay—

$$\text{namely,} \quad 1 = P^1_{x,y}(1 + a_{xy}) + i a_{xy} + R^1_{y,x}$$

$$\text{and} \quad R^1_{y,x} = 1 - P^1_{x,y}(1 + a_{xy}) - i a_{xy} \quad \dots \quad (3)$$

(Formulas (1) and (3) are given by Mr. Bunyon.)

Specimen Values of Contingent Reversions—Interest 5 per-cent.

Age of x	Age of y	$100 P^1_{x,y}$	Carlisle $A_{xy} - P^1_{x,y}(1 + a_{xy})$	TABLE II	
				Male Life Tenant	Female Life Tenant
20	50	1.271	.293	.321	.273
30	60	1.308	.442	.452	.406
40	70	1.546	.569	.579	.547
50	80	2.020	.678	.688	.668

(b). If re-purchase be allowed in the first n years as before, we have

$$1 = P + (P + i')a'_{\bar{n}} + v'^n \cdot (P \cdot a_{x+n, y+n} + i a_{x+n, y+n}) + R^1_{y, x}$$

and $R^1_{y, x} = 1 - P - (P + i')a'_{\bar{n}} - v'^n \cdot (P \cdot a_{x+n, y+n} + i a_{x+n, y+n})$. (4)

Formula (4) may be written thus—

$$R^1_{y, x} = 1 - P - (P + i')a'_{\bar{n}} - v'^n(P + i)a_{x+n, y+n} + v'^n \cdot P \cdot \frac{A_{x+n, y+n}}{2}$$

Whence, if $i' = i$,

$$R^1_{y, x} = 1 - P - (P + i)(a_{\bar{n}} + v^n \cdot a_{x+n, y+n}) + v^n \cdot P \cdot \frac{A_{x+n, y+n}}{2}$$

Therefore, if we substitute $'P$ for P , where $'P = P$ less the annual premium to assure $\frac{'P}{2}$ on the failure of the joint lives, we may write

$$R^1_{y, x} = 1 - 'P - ('P + i)(a_{\bar{n}} + v^n \cdot a_{x+n, y+n})$$

Now, without power of re-purchase, $R^1_{y, x} = 1 - 'P - ('P + i)a_{xy}$. Hence, the difference in the value, when re-purchase is allowed, and the full policy is set up in the first instance, is represented by the expression

$$('P + i)(a_{xy} - a_{\bar{n}} - v^n \cdot a_{x+n, y+n})$$

where $'P = P^1_{x, y} - P_{xy} \cdot \frac{'P}{2}$ and $'P = \frac{P^1_{x, y}}{1 + \frac{P_{xy}}{2}}$.

The last factor of the above expression has been tabulated for quinquennial ages (*vide* Table V).

If P be substituted for $'P$ in the formulas for $R^1_{y, x}$, only a slight loss of accuracy will be caused (the error being on the safe side), and the formulas will take a compact and practical form.

Formula (4) embodies the necessary requirement that R accumulated with the premiums paid for n years shall equal the then value of the reversion.

Thus, suppose the accumulated advance and premiums, namely,

$$(R^1_{y, x} + P)(1 + i')^n + P(s'_{\bar{n}} - 1) = 1 - P(1 + a_{x+n, y+n}) - i a_{x+n, y+n}$$

then

$$\begin{aligned} R^1_{y, x} + P + P \cdot \frac{1 - v'^n}{i} &= v'^n - v'^n \cdot (P \cdot a_{x+n, y+n} + i a_{x+n, y+n}) \\ &= 1 - i' a'_{\bar{n}} - v'^n \cdot (P \cdot a_{x+n, y+n} + i a_{x+n, y+n}) \end{aligned}$$

and

$$R^1_{y, x} = 1 - P - (P + i')a'_{\bar{n}} - v'^n \cdot (P \cdot a_{x+n, y+n} + i a_{x+n, y+n})$$

as before.

The formula given by Mr. R. P. Hardy (*J.I.A.*, xvii, 245) for a loan by way of accumulation, is

$$1 - (P + d)(1 + a_{\overline{n-1}|} + {}_{n-1}a_{xy}),$$

In this expression the premiums and interest payable after the first n years are discounted with interest and *mortality*. The formula thus involves the probability of x and y living over the first n years, and would not therefore appear to fulfil the requirement above mentioned.

(c). Next take the case where purchase is made of a reversionary life interest by way of reversionary charge (as a capital sum) and that such charge does not exceed two or three years' income of the life tenant.

Here, instead of effecting the usual whole-life policy, considerable reduction in the cost of assurance is obtained, as Mr. Sprague points out, by opening a contingent policy payable in case the reversioner die before the life tenant *or within t years after*.

We have then to consider whether the premium for such a policy should be payable:

- (1) During the full term of the contract;
or, (2) During the joint lives only.

This question was discussed by Mr. Sunderland (*J.I.A.*, xxvii, 90), who concluded that the premium under the first heading was "unsound in theory", and should not be adopted in practice for the following reasons:

- (1) That if the life tenant die early, the reversioner can usually effect the assurance for the remaining term at a lower rate, by reason of the risk having diminished.
- (2) That the last annual premium falls due, on an average, six months before the risk terminates. Therefore, allowing for the days of grace, this premium really covers only five months' risk, and, in consequence, frequently remains unpaid.

When $t=1$, which is generally the case, Mr. Sunderland thinks the simplest course is to make the premium payable during the joint existence only. Sometimes, however, when it is desired to make the premium as small as possible, he recommends charging a final payment of half an annual premium.

This arrangement appears both simple and suitable, and allowing for a final payment of $\frac{1}{2} \cdot P^1_{x,y}$, we have, when $t=1$,

$$R=1-P^1_{x,y}-(P^1_{x,y}+i)\hat{a}_{xy} \quad . \quad . \quad . \quad . \quad (5)$$

When $t>1$, I think it better to make the premium as small as possible, and therefore payable during the entire duration of the contract. Mr. Sunderland objects that the assurance might, in a certain event, be effected elsewhere at a lower rate. Is it not usual, however, to make provision in the deed for tacking any premiums in arrear to the mortgage? Moreover, the cost of repaying the loan by borrowing from another source, would probably far exceed the gain in premium thereby secured, and so effectually prohibit any transfer.

If t be less than five years, the well-known approximate method of obtaining the required premium (with but slight loss of accuracy) is as follows:—Add t years to the expectation of y , the life tenant, and find the age $y-\delta$ corresponding to the increased expectation. Then the required formula will be:

$$R=1-P^1_{x,y-\delta}-(P^1_{x,y-\delta}+i)\hat{a}_{x,y-\delta} \quad . \quad . \quad . \quad . \quad (6)$$

Where $P^1_{x,y-\delta}$ is the annual premium payable during the joint lives of x and y , and for t years longer.

By making the reversionary charge a capital sum rather than an annuity, income tax payable on that portion of the annuity representing capital is thereby avoided.

III.—LIFE INTEREST OF £1 IN POSSESSION.

(a). Let L_x represent the present value of a life interest of £1 in possession, deduction having been made for any charges on the life income such as income tax, &c.

A sufficient margin should be reserved to give the vendor or borrower a permanent interest in the estate, as an inducement to him to come forward from time to time to establish his existence.

This margin would, presumably, depend on the social position and personal character of the life tenant, to some extent.

Due consideration should also be given to the power vested in the trustees to vary the securities.

Now, proceeding to establish a formula, we find that a whole-life policy must be set up to provide for a return of the capital outlaid on the death of the life tenant.

Also, the life income must furnish interest on the total outlay

during his entire existence, and the premiums to cover the outlay, less half a premium. This half premium is, on the average, provided by the life interest.

Hence, for an advance of 1, we should have iL_x on account of interest, and $'P_x + 'P_x \cdot L$ on account of premiums, so that

$$1 = 'P_x + ('P_x + i)L_x$$

and

$$L_x = \frac{1 - 'P_x}{'P_x + i} \quad \dots \quad (6)$$

where $'P$ is the annual premium to assure $1 - \frac{1}{2}$, that is,

$'P = P_x \left(1 - \frac{1}{2}\right)$ and $'P = \frac{2P_x}{2 + P_x}$. Table III was calculated from formula (6).

The sum to be assured for a life interest of 1 is $\frac{1 - \frac{1}{2}'P}{'P + i}$.

The redemption-money immediately after an annual payment of the life interest has been made is $\frac{1}{'P + i}$. Should the life interest be redeemed m months after this period, the redemption-money will be $\frac{1}{'P + i} \left(1 + i' \cdot \frac{m}{12}\right)$ where i' is a rate agreed upon.

The difference between the sum assured and the value of the life interest is $\frac{\frac{1}{2}'P}{'P + i}$, and the ratio of this difference to the life interest value is $\frac{\frac{1}{2}'P}{1 - 'P}$.

This ratio, representing the percentage to be *added* to the life interest to find the sum to be assured, is given in the right-hand column of Table III. If twice this percentage be added instead, we shall obtain the redemption-money $\frac{1}{'P + i}$.

I submit that formula (6) represents the value of a life interest more exactly, and is more easily understood, than any other expression in use. Mr. Sprague (*J.I.A.*, xxvii, 110), adopts Mr. Baden's formula, namely, $\frac{v - \frac{1}{2}P}{P + d}$.

Putting this $= L_x$ as above, we have

$$v - \frac{P}{2} = (P + d)L_x,$$

and

$$1 = \frac{P}{2} (1 + i) + \{P(1 + i) + i\}L_x.$$

Here, the interest provided on the outlay of 1, namely, iL_x , is correct. However, the annual premium introduced, namely, $P(1+i)$, insures $(1+i)$, that is, an excess of one year's interest on the outlay. On the other hand, a charge of half an annual premium only is made in the first instance.

The following comparison may be of interest:

*Specimen Value of a Life Interest of £1 in Possession—Interest
5 per-cent.*

Premium per-cent	$\frac{1}{P+d} - 1$	Sum Assured $\frac{1}{P+d}$	$v - \frac{P}{P+d}$	Sum Assured $v(1 + \frac{1}{2}P)$ $P+d$	Table III	Sum Assured
£						
2	13·788	14·788	13·937	14·225	14·043	14·185
4	10·413	11·413	10·643	11·087	10·772	10·997
6	8·291	9·291	8·570	9·114	8·700	8·977
8	6·845	7·845	7·158	7·771	7·272	7·588

(b). Let it be desired to charge a life interest, immediately subsequent to the first n years, redemption being allowed in the meantime at a rate i' (thus eliminating mortality for the first n years).

Then, denoting by ${}_nL_x$ the present value of a deferred life interest of £1, and setting up the full policy at the outset, we have

$$1 = 'P_x + ('P_x + i')a'_{\mu} + ('P_x + i)_{n'}L_{\mu}$$

$$\text{and } n|L_e = \frac{1 - P_x - (P_x + i')a'_{i'}}{P_x + i'}$$

$$\text{Or, if } i' = i, \quad {}_n L_x = \frac{1 - {}^i P_x}{{}^i P_x + i} - a_{\overline{i}|} \quad , \quad , \quad , \quad , \quad (8)$$

The sum assured and the redemption-money after the first n years are, of course, the same as for an ordinary life interest.

Numerical Example.—If $n=5$ years, and $P_x=.02$, then
 ${}_nL_x=14.043-4.330=9.713$.

This case can be arranged more economically by setting up the full policy only at the end of n years, and assuring for a smaller sum in the interval (see page 25).

(c). Denoting by ${}_nL_x$ the value of a life interest of £1 in a leasehold for n years, we have

$$1 = \phi_x + (\phi_x + i) {}_nL_x$$

and

$${}_nL_x = \frac{1 - \phi_x}{\phi_x + i} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where ϕ_x remains to be determined. The life interest will provide an average surplus payment of $\frac{\phi_x}{2}$ should x die within n years, and a surplus payment of ϕ_x should he survive that period.

Hence ϕ_x is the annual premium for an endowment assurance of $(1 - \frac{\phi_x}{2})$ payable in n years, or previous death, minus the annual premium for a pure endowment of $\frac{\phi_x}{2}$, payable only if x be alive at the end of n years.

That is,

$$\phi_x = P_{x, \overline{n}} \left(1 - \frac{\phi_x}{2}\right) - P_{x, \overline{n}} \cdot \frac{\phi_x}{2}$$

and

$$\phi_x = \frac{2P_{x, \overline{n}}}{2 + P_{x, \overline{n}} + P_{x, \overline{n}}^{-1}}$$

The pure endowment premium, namely, $P_{x, \overline{n}} \cdot \frac{\phi_x}{2}$, is usually very small.

The trifling error caused by neglecting it in the valuation is on the safe side, and may be considered as a set-off against any slight loss caused by delay in re-investment. The annual premium ϕ_x will then be similarly involved in the formula, as the whole-life premium for an ordinary life interest in possession. Therefore, the value of a leasehold of £1 for n years, depending on x , may be found approximately by taking out the numerical value opposite $P_{x, \overline{n}}$ in Table III.

Example.—Find the value of a lease of £1 per annum for 25 years depending on a life assurable at the premium for age 30.

If $P_{30, 25} = \cdot 0355$, we have, taking interest at 6 per-cent, approximate value = 10·172, real value 10·214, subject to the usual deductions for expenses, &c.

Table III will not usually render much assistance in valuing leases on lives, inasmuch as a high rate of interest appears to be generally adopted.

It may be of some interest to compare the formula

$$1 = \phi_x + (\phi_x + i) {}_nL_x$$

with Mr. Jellicoe's formula

$${}_na_x = \frac{1}{P_{x, \overline{n+1}} + d} - 1,$$

from which we get

$$1 - (P_{x, \overline{n+1}} + d) = {}_nL_x \cdot (P_{x, \overline{n+1}} + d)$$

and

$$r = P_{x, \overline{n+1}} + (P_{x, \overline{n+1}} + iv) {}_nL_x.$$

The correct amount of interest, namely, $iv {}_nL_x$, is here allowed for, but the premium provides for a surplus of one year's interest on the outlay of r should death occur within n years. Moreover, the assurance is carried on for a year after the lease has expired.

Taking the same figures as before, Mr. Jellicoe's formula would bring out the value as 9.51.

Mr. Sprague gives (*J.I.A.*, viii, 13) the following rule for valuing a life interest (of, say, r per annum) in connection with a whole-life policy (of, say, s , subject to an annual premium of sP_x).

Determine how much of r is covered by s and find the value of that portion and of the policy together, then value the remainder of r in the ordinary way.

Thus, a policy of $1 - \frac{{}'P_x}{2}$ will cover a life interest of $'P_x + i$, that is (substituting $'P_x = \frac{2P_x}{2 + P_x}$), a policy of $\frac{2}{2 + P_x}$ will cover a life interest of $\frac{2P_x}{2 + P_x} + i$, or a policy of s will cover a life interest of $s\left\{P_x\left(1 + \frac{i}{2}\right) + i\right\}$.

Now, the value of this policy of s , together with the corresponding life interest is

$$s\left\{P_x\left(1 + \frac{i}{2}\right) + i\right\} \times \frac{1 - {}'P_x}{{}'P_x + i}$$

which, by reduction,

$$= s\left\{P_x\left(1 + \frac{i}{2}\right) + i\right\} \times \frac{1 - \frac{P_x}{2}}{P_x\left(1 + \frac{i}{2}\right) + i} = s\left(1 - \frac{P_x}{2}\right)$$

The value of the remainder of the life interest, namely,

$r - s \left\{ P_x \left(1 + \frac{i}{2} \right) + i \right\}$ per annum $= m$ suppose, can be found by multiplying m into the value opposite P_{x+n} in Table III, x being insurable at a premium P_{x+n} at the time of valuation.

Should the policy of s be in excess of that required for the life interest, the surplus should be taken at the office surrender-value.

The value of the policy could be obtained separately by subtracting the value of a life interest of £1 at the date of valuation from the value at the time the policy was effected, and multiplying the difference by s . The life interest of r would in such case have to be obtained from the premium P_{x+n} in Table III.

This method of valuation is not applied here so easily as that described previously.

Numerical Example.—Find the value (interest at 5 per-cent) of a life interest of £323, together with a policy of £1,000, subject to a premium of £25 per annum. The present age of the life being 40, assurable at a premium of, say, £2·17 per-cent.

The policy will cover a life interest of

$$1,000 \left\{ \cdot 025 \left(1 + \frac{\cdot 05}{2} \right) + \cdot 05 \right\} = 75\cdot 625,$$

and the value of these together is $1,000 \left(1 - \frac{\cdot 025}{2} \right)$, that is £987·5.

The remainder of the life interest, namely, 247·375 is worth $247\cdot 38 \times 12\cdot 445 = \text{£}3,078\cdot 6$. The total value is therefore £4,066.

IV.—REVERSIONARY LIFE INTEREST OF £1.

(a). Let $L_{y,x}$ denote the present value of a reversionary life interest of £1 to x after y , all charges on the life income having been deducted and a margin reserved when necessary, as more fully explained under paragraph III.

Here, a whole-life policy should be effected, payable on the death of the reversioner x .

Assume the reversionary life interest to be payable annually, and the first payment receivable therefor 12 months after the death of the life tenant.

If the premium be annual, the reversionary annuity will then commence, on an average, six months after payment of a premium, and, therefore, on an average, the first annual premium furnished by the reversionary life-interest will be provided 18 months after

payment of the premium due at the beginning of the year in which the joint existence terminated. Provision must, therefore, be made for half an annual premium to be payable on the failure of the joint lives. We have, consequently, allowing for interest at a rate i in possession and i' in reversion:

$$1 = 'P_x \left(1 + a'_{xy} + \frac{A'_{xy}}{2} \right) + i' a'_{xy} + L_{y|x} ('P_x + i);$$

that is, $1 = 'P_x + ('P_x + i') a'_{xy} + L_{y|x} ('P_x + i).$

$$\text{And } L_{y|x} = \frac{1 - 'P_x - ('P_x + i') a'_{xy}}{'P_x + i} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$\text{Or, if } i' = i, \quad L_{y|x} = \frac{1 - 'P_x}{'P_x + i} - a_{xy} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Formulas (10) and (11) were adopted for the calculation of Table IV.

Specimen Values of a Reversionary Life Interest of £1.

AGES		TABLE IV MALE LIFE TENANT		TABLE IV FEMALE LIFE TENANT		MR. SPRAGUE'S FORMULA	
Rever- sioner	Life Tenant	5 per-cent through- out	5 ⁰⁰ / ₁₀₀ in possession and 6 ⁰⁰ / ₁₀₀ in reversion	5 per-cent through- out	5 ⁰⁰ / ₁₀₀ in possession and 6 ⁰⁰ / ₁₀₀ in reversion	5 per-cent through- out	5 ⁰⁰ / ₁₀₀ in possession and 6 ⁰⁰ / ₁₀₀ in reversion
30	60	5.416	4.867	4.691	4.145	5.374	4.799
40	70	6.340	5.896	5.856	5.413	6.336	5.859

$$* (1 - \frac{1}{2} d_5) \frac{1 - (P + d_6)(1 + a_{xy})}{P + d_5}. \quad (a_{xy} \text{ being the Carlisle annuity for the true ages.})$$

(b). If the option of redemption within n years be allowed, as before, then, setting up the full policy at the outset, we have

$$1 = 'P_x + ('P_x + i'') a''_n + ('P_x + i') v''^n \cdot \hat{a}'_{x+n, y+n} + ('P_x + i) L_{y|x} \quad (12)$$

whence the value of $L_{y|x}$ is known.

Or, putting $i'' = i'$, we have

$$1 = 'P_x + ('P_x + i') (a'_{|n} + v'^n \cdot \hat{a}'_{x+n, y+n}) + ('P_x + i) L_{y|x}.$$

$$\text{And } L_{y|x} = \frac{1 - 'P_x - ('P_x + i') (a'_{|n} + v'^n \cdot \hat{a}'_{x+n, y+n})}{'P_x + i} \quad . \quad (13)$$

Or, again, putting $i' = i$, and so having one rate of interest throughout,

$$L_{y|x} = \frac{1 - 'P_x}{'P_x + i} - (a_{|n} + v^n \cdot \hat{a}_{x+n, y+n}) \quad . \quad . \quad . \quad (14)$$

If we subtract this last value of L_{yx} from that obtained in formula (11) we shall find the difference which is made in the value by granting power of redemption over n years, assuming the full policy to be set up at the outset. This difference will be represented by the expression

$$(a_n + r^n \cdot \hat{a}_{x+n, y+n}) - \hat{a}_{xy},$$

when one rate of interest throughout is used.

In Table V the values for 5 per-cent have been calculated in this manner. Where redemption is allowed at, say, 5 per-cent, and the remainder of the formula is based on a rate i , the difference will become

$$\frac{P_x + .05 \cdot a_n^{5\%} + (r^n \cdot \hat{a}_{x+n, y+n} - \hat{a}_{xy})}{P_x + i}$$

The remainder of Table V has been framed from this expression.

If an annuity during the joint lives of x and y be preferred to a present sum down, such annuity may be obtained by dividing the value of the reversion (found from the formula) by the complete joint life annuity, calculated at the same rate as that used for \hat{a}_{xy} in the formula.

Passing from the consideration of the above formulas, I will now proceed to offer a few remarks on the practical treatment of reversionary life interests:

- (1). In regard to simple mortgages.
- (2). In reference to purchases, whether by way of reversionary charge, or otherwise (option of re-purchase being allowed over a fixed term of years).

ADVANCES BY WAY OF MORTGAGE ON THE SECURITY OF REVERSIONARY LIFE INTERESTS.

Mr. Sprague has (*J.L.A.*, xvii, 229) clearly pointed out the difficulties to be contended with in making these advances. He shows conclusively that transactions of the kind should be arranged, if possible, by way of reversionary charge, rather than by way of mortgage. Exception should, however, be made under special circumstances, or where a fund exists available for the annual heavy payment required for premiums and interest. If this annual payment be guaranteed, the amount of assurance strictly necessary does not exceed that for a mortgage on a life interest in possession. On the other hand, if no premium or

interest can be relied on, the full policy would appear to be required at the outset, in order to protect the lender completely.

If the full assurance be set up in the first instance, the strain put upon the borrower for the heavy annual premiums is usually very severe, and, in fact, a great incentive to him to allow the premiums to fall into arrear.

Although objection is usually made to giving a reversionary charge in respect of the advance, especially when the life tenant is in bad health, the reversioner may agree that a policy to cover the *actual initial outlay* should be effected at the outset, and that arrears of premium and interest should be treated as further advances by way of *reversionary* charge.

This plan would not remove the difficulty arising by reason of the reversioner becoming, perhaps, uninsurable. It would therefore be necessary to provide for the possible requirement of a suitably increasing deferred assurance, and, consequently, to charge a premium in the first instance for the option of obtaining further assurance at the ordinary rate, corresponding to the increased age of the reversioner.

Should the above course be impracticable, the following arrangement, offering great economy to the reversioner during the term of the joint lives, might be adopted under certain circumstances, and where the security is ample.

Assume both lives to be in existence at the end of t years (the curtate joint expectation of x and y) and x to be then assurable at the ordinary rate, namely, P_{x+t} . Then the value of a reversionary life interest of £1 t years hence, that is, $L_{y+t, x+t}$, will be

$$= \frac{1 - P_{x+t}}{P_{x+t} + i} - a_{x+t, y+t}.$$

In order to find what advance could be made on this deferred life interest, we must deduct from $L_{y+t, x+t}$ the temporary premiums to insure this amount on the death of x , if occurring within t years, and also the interest on the outlay for the same period, together with the single premium (say T) for the option of obtaining a policy of $\frac{1}{P_{x+t} + i}$ at the ordinary rate, after a lapse of t years.

The above temporary premium will be payable only for t years or till the first death, if occur before t years. If, however, we suppose the joint existence to fail before t years, then, if x die first, the policy will provide the total initial outlay, namely, $L_{y+t, x+t}$; whereas, if y die first, x 's interest will become a life

interest in possession, the security being thereby improved. In the latter event, no further assurance would be necessary.

The formula for the present advance on a reversionary life interest of 1 deferred t years is thus

$$= L_{y+t, x+t} \{ 1 - P(1 + {}_{t-1}a_{x,y}) - i {}_t a_{x,y} \} - T \quad (15)$$

Numerical Example of the above arrangement.

What advance could be granted in respect of a reversionary life interest of £1,000 (all deductions allowed for) to M20 after M50?

Taking interest at 5 per-cent in possession and 6 per-cent in reversion, the life interest would be worth £5,687 at the end of 18 years (the joint expectation), and the whole-life policy then required would be about £13,000.

Now, the temporary premium for 18 years at age 20 is, say, 1.08 per-cent, that is, £61. 8s. 5d. for £5,687 assured, and the option is 12s. per-cent, that is, £78 for a deferred assurance of £13,000.

Also, the present value of the deferred reversionary life interest (by Table IX) is £2,037.

Suppose that £2,000 is advanced on the security at $4\frac{1}{2}$ per-cent (inclusive of the first premium). Then the annual payment for premiums and interest for the first 18 years will be £61. 8s. 5d. + £90 = £151. 8s. 5d. (in addition to the single premium for the option, namely, £78); that is, $7\frac{1}{2}$ per-cent on the advance.

At the end of 18 years the full policy of £13,000 will be set up, and the annual charge will then be £335. 16s. + £90 = £425. 16s.

If the £2,000 advance had been made in the ordinary way, a reversionary annuity of £578 would have been required to support it, together with a policy for £8,554, subject to a whole-life annual premium of £144. 11s. If we add to the latter, the annual interest, namely, £90, we shall obtain a yearly payment throughout of £234. 11s., representing about $11\frac{1}{2}$ per-cent on the loan.

So that, leaving the option of £78 out of consideration, a yearly saving of about £83 is made for 18 years, as a set-off for an increase of £191 per annum subsequent to this period. Should, however, the loan be redeemed during the term of the joint expectation, or either life die within this period, the increased annual charge of £191 would not be incurred at all.

This economy of the annual charge during the existence of the life tenant would, I think, be attractive to a borrower.

The advance that could be made as above in respect of a reversionary life interest would usually be between 60 per-cent and 70 per-cent of the loan that could be granted on the same annuity if carried out in the ordinary way.

The practical objection to contend with in arranging the mortgage as above, is the difficulty of obtaining so large an amount of *deferred* assurance, the more especially as the term of deferment is frequently a considerable number of years.

Actuaries naturally feel some uncertainty as to whether an option calculated to provide assurance at the end of 10 or 15 years can be safely relied upon.

The amount of deferred assurance could be much reduced by effecting a *whole-life* policy of $L_{y+t.x+t}$ instead of a *temporary* assurance for t years. Thus, in the foregoing example, the deferred assurance would be cut down from £13,000 to £7,313. On the other hand, the annual charge for the first 18 years would be somewhat increased.

	£	s.	d.
Annual premium	95	19	5
„ interest	90	0	0
Total yearly payment	185	19	5

which represents about $9\frac{1}{4}$ per-cent on the loan.

THE PURCHASE OF A REVERSIONARY LIFE INTEREST (POWER OF RE-PURCHASE BEING ALLOWED OVER A PERIOD OF n YEARS UPON THE TERMS OF ACCUMULATION AT INTEREST i').

The usual method of arranging the assurances in connection with these transactions appears to be as follows:—Effect a policy in the first instance for the total initial outlay only, and set up the remainder of the full policy required at the end of the n th year, if option of re-purchase has not been exercised in the interval. Also, charge a single premium (say T) at the outset for the possible deterioration of the life during the period of n years.

How should the amount of the advance be obtained under this arrangement? The value brought out by formulas (13) and (14) will not apply, for they assume the full policy to be effected at the outset. The present advance, accumulated with interest, and the premiums necessary to protect it, should amount to the value of the reversionary life interest n years hence, namely, $L_{y+n'.x+n}$ (assuming x to be then a healthy life), that is, should

$$= \frac{1 - {}^{\circ}P_{x+n}^{x+n}}{{}^{\circ}P_{x+n} + i} - \dot{a}_{x+n.y+n}.$$

Therefore, if we effect a temporary assurance of $L_{y+n, x+n}$ for n years at a premium of $P.L_{y+n, x+n}$, the present advance will be

$$L_{y+n, x+n} \{1 - P - (P + i')a'_{n-1}\} - T. \quad (16)$$

Example.—What is the purchase value of a reversionary life interest of £1,000 to M30 after M60 (all proper deductions having been made), power of re-purchase being allowed at 5 per-cent, over five years?

$L_{65|35} = £5,407$ (with interest at 5 per-cent in possession and 6 per-cent in reversion).

The short-term premium, age 30, for five years, is 1.100 per-cent. The value of T is 1.000 per-cent, using Mr. Rothery's table (see *J.I.A.*, xxviii, 483) with a loading of about 25 per-cent; and as a policy of £13,500 must be set up at the end of five years, the option to be charged will, therefore, be $135 \times 1.00 = £135$. Hence, the present value of the life interest will be

$$\begin{aligned} &5,407 \{1 - .011 - (.011 + .05)4.33\} - 135 \\ &= £3,785. \end{aligned}$$

from which, of course, legal and other initial expenses, if any, must be deducted to obtain the purchase value.

I believe the usual custom is to effect a *whole-life* policy for the first n years instead of a *temporary* assurance, and to charge for the option in respect of the increase of assurance only.

This plan has the advantage of reducing the amount of deferred assurance required; but, on the other hand, it increases the annual charge for the first five years.

Moreover, in the event of re-purchase during the period allowed, the borrower will suffer to some extent by the surrender of his *whole-life* policy, whereas, a temporary assurance offers the cheapest possible accommodation. The slight increase in the cost of the option will not counterbalance the difference in the annual premiums.

When a *whole-life* policy is effected as above, there appears to be some difficulty in arriving at the *true* present value of the life interest on account of the assurance (after the n th year) being subject to annual premiums corresponding to different ages.

REMARKS IN CONNECTION WITH THE TABLES APPENDED TO THIS PAPER.

We have, unfortunately, no sufficient data from which to form an accurate opinion as to the mortality of those lives on which reversions depend, and are offered for sale or loan. It is generally

admitted that reversions do not yield the calculated rate of interest during the early years after purchase, and this effect seems to be properly attributed to the influence of selection.

In the absence of reliable data, we have the following select tables for consideration:

- (1). Those based on the experience of assured lives: such as those of Mr. Sprague.
- (2). The Government Annuity Tables (1883) of Mr. Finlaison.

The question then arises, how far the mortality represented by them is likely to correspond with that of life tenants?

Tables of assured lives may be considered as a class representing the ordinary community, operated upon by two opposite forces, namely:

- (1). Medical selection on the part of the Assurer.
- (2). Withdrawal of healthy lives on the part of the Assured.

Medical examination, on the one hand, by eliminating most of those lives in an unsound condition, raises the vitality of the class above the average for the first few years. The effect of medical selection is then practically lost, and the lives become worse than the average, a slight recovery taking place after a considerable number of years.

The selection represented by tables of annuity experience appears to be different in character, namely, that exercised by a body of lives believing themselves to be, in all probability, longer lived than the ordinary community. The mortality is shown to be light during the first three or four years after purchase, and then becomes practically normal.

Now, as life tenants undergo no medical examination, and as reversioners would probably in offering the security for sale believe them to be, generally, better lives than the average, I should suggest that the mortality of life tenants would be more closely allied to that of annuitants than to that of assured lives. Reversioners, of course, belong to the latter class.

A further advantage in favour of adopting annuity tables, is that they treat of each sex separately.

(Where a reversioner asks for power of re-purchase, or desires a *loan* only, as opposed to a sale, it would appear that the life tenant has not the same average prospect of longevity.)

As no joint-life annuities seem to have been calculated embracing the double mortality above mentioned, I have adopted the following course in tabulating the complete joint-life annuities appended to this paper (see Table VII):

For the *reversioner*, I used the Carlisle Table without modification. It is generally known that the Carlisle single life annuity-values correspond fairly well with those of the select Institute Tables, although somewhat smaller than the latter for ages 25 to 40, and larger subsequently.

For *life tenants*, the annuity-values for the true ages were taken out from the Government Annuity Experience (males and females separately), and the exact ages corresponding to the same annuity-values in the Carlisle Table were obtained to two decimal places. The new ages of the life tenants thus found were combined with the ages of the reversioners, and the corresponding Carlisle *joint-life* annuities were calculated by interpolation from the values given in *Jones's Annuities*.

The rates of interest selected for tabulation were 4 per-cent, 5 per-cent, and 6 per-cent.

The above mode of proceeding assumes that, if $a_{x,r} = a_{x+\delta,r}$,
Govt. Carl. Carlisle
 then $a_{x,y} = a_{x+\delta,y}$.
Govt. Carl. Carl. Carl.

The error involved in this arbitrary assumption is very small, and may be passed over without consideration.

As the published Government Annuity Tables do not extend below age 40, I have limited my tables to that age so far as life tenants are concerned.

The foregoing method of obtaining joint-life annuities is, I admit, a rough one; but, for the purpose with which they were calculated, the results appear to me more reliable than any existing annuity-values.

A few notes may appropriately be made here as to the rates of interest to be used in the formulas.

Mr. Jellicoe formerly explained the method of valuing reversionary interests considered as isolated cases. He assumed that annuities were actually purchased for premiums and interest, in order to render the reversion absolute. A considerable profit

over and above the interest supposed to be realized was thus concealed in the annuity introduced. Hence, Mr. Jellicoe's formulas do not show the true return on investments of the class, and Mr. Sprague has rightly pronounced them as inappropriate for the use of assurance companies and other societies holding a number of these securities on their books.

Introduction of the office premium also covers a large profit not expressed in the rate of interest used in the formulas. This step is necessary, however, as a policy is always set up to make the security marketable, and a commission is usually paid on the premium.

Exclusive of the profit obtained by use of the loaded premium, it is now well understood that formulas, in which one rate of interest is used throughout, really provide that rate on the security in themselves, and do *not* require, as sometimes remarked, a supply of outside investments yielding the same rate of interest.

In the valuation of reversions, the rates of interest to be used are influenced by competition, the market rate, the amount of insurance effected, and the character of the security, &c., &c.

Ceteris paribus, the rates depend to some extent on the probable duration of the status, on the termination of which the capital is recoverable. The longer the duration the higher should be the rate of interest, for there is liability to wider fluctuation in the value of investments, &c.

The rates now realized point to a considerable decline on those obtained many years ago.

Advances on absolute reversions to the best Stock Exchange securities may now be had without difficulty at $4\frac{1}{2}$ per-cent, or even at 4 per-cent, under special circumstances, together with a policy on the reversioner's life as a collateral security.

Loans may also be obtained at this rate on life interests in the proceeds of *first-class* investments.

The rates to be adopted for reversionary life interests appear rather higher, the value being seldom calculated to yield less than 5 per-cent in possession and 5 or 6 per-cent in reversion. The rates used are, to some extent, influenced by the amount of assurance effected.

There is no means of estimating, with any degree of accuracy, the probable price of an investment when the reversion is expected to fall in—frequently at a time many years distant. Hence, opinion differs largely as to the margin which should be reserved

in pricing the funds and securities embraced in the interest under notice.

In the examples given in this paper, I have assumed that the necessary margins have already been allowed for, and, consequently, have valued the interest as a reversion to so much cash.

If it be the practice of an office to charge an extra premium for a young woman, it is convenient, for the purposes of valuation, to rate up her age a few years, corresponding to the extra premium charged. If 5*s.* per-cent extra premium is charged for female lives, the number of years to be added for a reversionary life interest is roughly as follows:

Age 20	Age 25	Age 30	Age 40	Age 45	Subsequent Ages
7	5	4	3	2	1

If an absolute reversion depend on two lives, both of them will follow the Government Annuity Table. In such cases, one of the ages will require modification, in order to utilize the joint-life annuities appended.

The following list of equivalent ages, as found from the 3 per-cent single life annuity-values, may be serviceable for this purpose:

Government (Male) Age x	Carlisle Equivalent Age	Government (Female) Age x	Carlisle Equivalent Age
Ages 41 to 46	$x + 2$	Below age 64	$x - 4$
„ 47 „ 50	$x + 1$	Ages 65 to 71	$x - 3$
„ 51 „ 57	x	„ 72 & upwards	$x - 4$
„ 58 & upwards	$x - 1$		

Thus $a_{\overline{x+y}} = a_{x_1} + a_{y_1} - a_{x+y}$

$$= a_{x_1} + a_{y_1} - a_{x+3 \cdot y}$$

where x is younger than 40, and a male.

The value $a_{x+3 \cdot y}$ will be found from the joint annuities appended, $x+3$ being the age at the top, and y the age at the side.

The value of the reversion will then be $1 - i a_{\overline{x+y}}$.

EXPLANATION OF THE TABLES, WITH EXAMPLES.

Table I.—Values of an absolute reversion to £1.

$$\text{Formula} \quad R_x = 1 - i a_x,$$

where \bar{a}_x is taken as $a_x + \frac{A_x}{2}$.

The calculations were based on the Government Annuity Tables (1883), male and female lives being separately treated.

The values for 3 per-cent, 4 per-cent, and 5 per-cent, were obtained from the complete annuities calculated direct from the corresponding curtate annuity-values given in Mr. Finlaison's report (pp. 32 to 35). Those for $4\frac{1}{2}$ per-cent, $5\frac{1}{2}$ per-cent, and 6 per-cent, were interpolated from the previous values by means of second differences; final corrections being made with the third differences of the H^M single premiums given in Mr. King's *Text-Book*.

I find the results for ages 40 to 80 may be relied on as correct to three decimal places.

Comparison with the Carlisle single premiums exhibits very remarkable differences, thus:

For male life tenants the values of Table I are about 7 per-cent *greater* than the Carlisle values at age 50, and about 1 per-cent *greater* for ages of 60 and upwards.

For female life tenants the tabular values are about 7 per-cent *less* than those of the Carlisle up to age 60. Above this age the percentage diminishes until age 80, when the value is 1 per-cent *less* than the Carlisle.

The tabular values for males exceeds those for females by about $8\frac{1}{2}$ per-cent at age 60, and by 2 per-cent at age 80.

Example.—What is the value of an absolute reversion to £1,000 on death of F 71, duty 3 per-cent?

Taking interest at 5 per-cent, the value is

$$\begin{array}{rcl} & £644 & \\ - \text{Duty} & \underline{20} & \\ \text{Value} & \underline{£624} & \text{subject to legal expenses.} \end{array}$$

Table II.—Values of a contingent reversion of £1 to x after y .

$$\text{Formula} \quad R_{y \text{ } x}^1 = 1 - P_{y \text{ } x}^1 (1 + a_{xy}) - i a_{xy}.$$

Here males and females are again treated separately, values being given for each age of the life tenant from 40 to 80, and for quinquennial ages of the reversioner from 40 to 60, the reversioner having been assumed as the younger life in every case.

The values have been calculated for rates of interest, 4 per-cent, 5 per-cent, and 6 per-cent. A table of the office premiums used in the calculations follows Table II. These premiums will probably differ slightly from the rates quoted by an individual office.

In order to find the value corresponding to a special premium $P_{x,y}^1 \pm \Delta$, the tabular value should be corrected by $\mp \Delta(1 + a_{xy})$, that is, by $\mp \Delta \cdot \frac{1}{2} + a_{xy} \left(1 + \frac{i}{2}\right)$.

This correction may easily be calculated from the Tables of Complete Annuities appended to the paper.

Example.—What is the value of a contingent reversion to £1,000 on the death of F55, provided F22 survive her—duty, 1 per-cent?

Suppose the contingent premium be 1.487 per-cent (including the extra). Then, as this premium corresponds to $P_{32,55}^1$, the value of the reversion, allowing for interest at 6 per-cent, will be

$$\begin{array}{r} \text{£284} \\ - \text{Duty } \frac{3}{100} \\ \hline \text{£281} \end{array} \quad \text{subject to legal expenses.}$$

Table III.—Values of a life interest of £1.

$$\text{Formula} \quad L_x = \frac{1 - P_x}{P_x + i}.$$

Values have been calculated for ordinary whole-life premiums, ranging from 30s. up to 160s. (proceeding by differences of 1s.), and rates of interest 4 per-cent, $4\frac{1}{2}$ per-cent, 5 per-cent, $5\frac{1}{2}$ per-cent, and 6 per-cent.

A column is given at the right hand showing the percentage to be added to the life interest value, in order to find the corresponding sum to be assured.

Assuming x die midway between two yearly payments of the life income (which he may be expected to do on the average), the excess of the initial outlay over the sum to be assured will be exactly counterbalanced by the half premium furnished from the

life interest. Should x die later in the year, a surplus will be obtained from the life interest over and above the amount (namely, ${}_2^{\text{'P}}$) required to complete the initial outlay. On the other hand, should x die earlier than anticipated, the small deficit then experienced may reasonably be deducted from the interest provided to the day of death.

Example.—What is the value of the life interest of a gentleman aged 40, in £300 per annum, together with a whole-life policy of £2,000 (with reversionary bonus of £280), subject to a premium of £10? Putting future bonuses on one side, we may consider the assurance already on foot as a policy of £2,280, subject to a premium of £40, whence $P = \cdot 01754$.

Now, taking interest at 5 per-cent, the policy will cover a life interest of $2,280 \left\{ \cdot 01754 \left(1 + \frac{\cdot 05}{2} \right) + \cdot 05 \right\} = £155$. And the value of these together is $2,280 \left(1 - \frac{\cdot 01754}{2} \right) = £2,260$.

If the life tenant be now assurable at an annual premium of 2·745 per-cent, the remainder of the life interest, namely, £145 per annum, is worth $145 \times 12\cdot 623 = £1,830$.

Thus, the total value of the security is $£2,260 + £1,830 = £4,090$, subject to legal and other initial expenses.

Table IV.—Values of a reversionary life interest of £1.

$$\left. \begin{array}{l} \text{Formula} \\ \text{Or, when } i' = i, \end{array} \right\} \frac{1 - {}^{\text{'P}}_x - ({}^{\text{'P}}_x + i') a'_{xy}}{{}^{\text{'P}}_x + i} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \frac{1 - {}^{\text{'P}}_x}{{}^{\text{'P}}_x + i} - a_{xy}$$

Males and females are treated separately for the following rates of interest:

In Possession	In Reversion
4 per-cent	5 per-cent
5 "	5 "
5 "	6 "
6 "	6 "

The values are tabulated for every age of the life tenant, from 85 to 40, and for quinquennial ages of the reversioner, from 20 to 60. The borrower is, in every case, assumed to be the younger life.

It may be observed that a difference of five years in the age of the reversioner produces usually a change in the value ranging from 10 per-cent to 15 per-cent of such value, when the reversioner is below age 50.

I subjoin some numerical specimen values of a reversionary life interest of 1 at various rates of interest, and also a table of the whole-life non-profit rates of premium used in the calculations:

Rates per-cent	Male Life Tenant, aged last Birthday	AGE OF THE REVERSIONER NEXT BIRTHDAY			Female Life Tenant aged last Birthday	AGE OF THE REVERSIONER NEXT BIRTHDAY		
		20	40	60		20	40	60
4 & 5	85	13.80	11.12	6.34	85	13.60	10.94	6.19
5	...	11.73	9.68	5.77	...	11.56	9.52	5.63
5 & 6	...	11.39	9.40	5.60	...	11.21	9.23	5.46
6	...	9.98	8.32	5.14	...	9.82	8.17	5.01
4 & 5	65	8.32	6.08	2.54	65	7.55	5.39	2.10
5	...	7.08	5.29	2.31	...	6.42	4.69	1.91
5 & 6	...	6.47	4.83	2.10	...	5.81	4.24	1.71
6	...	5.70	4.27	1.92	...	5.12	3.75	1.56
4 & 5	45	3.94	2.34	...	45	3.07	1.73	...
5	...	3.35	2.04	2.61	1.51	...
5 & 6	...	2.79	1.65	2.12	1.18	...
6	...	2.50	1.46	1.92	1.04	...

Whole-Life Assurance Non-Profit Rates used for Table IV.

Age of the Reversioner next Birthday	Office Annual Premium per-cent			Age of the Reversioner next Birthday	Office Annual Premium per-cent		
	£	s.	d.		£	s.	d.
20	1	13	9	55	4	18	1
25	1	16	3	60	6	3	10
30	2	0	9	65	7	18	6
35	2	6	9	70	10	6	7
40	2	14	10	75	13	7	8
45	3	5	6	80	19	0	0
50	3	19	3

Certain slight and unavoidable irregularities are apparent in the values of reversionary life interests, Table IV. These are due to the faulty graduation of the Carlisle Table, but they are of so slight a nature as in no way to interfere with the practical utility of the table.

Table V.—Showing the differences in the value of a reversionary life interest of £1, when power of re-purchase is allowed at 5 per-cent, during the first five years, and the full policy is set up at the outset.

The reversionary interest has been taken at one rate of interest throughout, namely, 4 per-cent, 5 per-cent, and 6 per-cent, separately for each sex.

The formulas used were—

$$(a_{\bar{5}} + v^5 \cdot a_{x+5, y+5}) - a_{xy} \text{ for 5 per-cent throughout,}$$

$$\text{and } \frac{{}_xP + \cdot 05}{{}_xP + i} \cdot a_{\bar{5}} + \left(v^5 \cdot a_{x+5, y+5} - a_{xy} \right) \text{ for 4 per-cent and 6 per-cent respectively.}$$

It may be noticed that the corresponding difference for a contingent reversion to a capital sum is

$$({}_xP^1 + \cdot 05)(a_{\bar{5}} + v^5 \cdot a_{x+5, y+5} - a_{xy}) \text{ for 5 per-cent throughout.}$$

Examination of this table shows that the difference in question ranges from a little over two years' purchase, down to rather more than one year's purchase, according as the age of the life tenant varies from 80 to 40.

The differences in the case of male life tenants are slightly greater than those for females.

Table VI.—Complete single life annuities.

The rates of interest and ages correspond in every particular with those of Table I.

Table VII.—Complete joint-life annuities.

The manner in which these values have been obtained has already been explained. They are given for 4 per-cent, 5 per-cent, and 6 per-cent, and for males and females separately.

Table VIII.—The difference in the value of a reversionary life interest of £1, assuming the full policy to be set up at the end of five years, and power of re-purchase to be allowed in the interval, at 5 per-cent compound interest.

The single premiums for the options were taken to be the following percentages on the whole deferred assurance:

or, with power of re-purchase, and full policy at end of n years,

$$R_{y,x}^1 = R_{y+n,x+n}^1 (1 - P_{x,y}^1 - (P_{x,y}^1 + i') a'_{-1}) - T$$

$$L_x = \frac{1 - p_x}{p_x + i} \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad (6)$$

$$L_{q,x} = \frac{1 - P_x - (P_x + i')i'_{xy}}{P_x + i} \quad \dots \quad (10)$$

(Using two rates of interest.)

$$I_{\mu}^{\eta}(x) = \frac{1 - p_x}{p_x + i} - \mu_{xy} \dots \dots \dots (11)$$

(Using one rate of interest throughout.)

or, with power of re-purchase, and full policy at end of n years,

$$L_{ij,x} = L_{ij+n,x+n} \{1 - P_x - (P_x + i') a'_{-1}\} - T \quad . \quad . \quad (16)$$

In all the above formulas P has been substituted for 'P'. The error caused thereby is very small, and on the safe side for the purchaser or lender. It may be considered as a set-off against the loss of interest caused by delay in re-investment of capital.

TABLE I.*
Value of an Absolute Reversion to £1.
Male Life Tenant.

Age of Life Tenant	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent
40	·4137	·3782	·3473	·3201	·2956
41	·4209	·3852	·3542	·3267	·3019
42	·4282	·3926	·3612	·3334	·3085
43	·4356	·4000	·3685	·3405	·3154
44	·4431	·4073	·3758	·3478	·3226
45	·4508	·4150	·3834	·3552	·3300
46	·4587	·4229	·3912	·3629	·3375
47	·4668	·4310	·3992	·3708	·3451
48	·4752	·4394	·4074	·3788	·3528
49	·4840	·4482	·4160	·3870	·3607
50	·4929	·4572	·4249	·3957	·3689
51	·5018	·4662	·4340	·4049	·3783
52	·5110	·4755	·4435	·4145	·3879
53	·5207	·4853	·4534	·4243	·3975
54	·5308	·4956	·4636	·4343	·4072
55	·5409	·5060	·4742	·4451	·4186
56	·5511	·5165	·4850	·4563	·4299
57	·5625	·5282	·4965	·4679	·4417
58	·5740	·5400	·5084	·4801	·4537
59	·5855	·5520	·5208	·4928	·4665
60	·5970	·5639	·5336	·5056	·4796
61	·6085	·5760	·5461	·5184	·4927
62	·6201	·5881	·5586	·5313	·5058
63	·6316	·6002	·5711	·5443	·5191
64	·6430	·6122	·5837	·5573	·5326
65	·6544	·6242	·5963	·5699	·5459
66	·6657	·6362	·6089	·5826	·5590
67	·6770	·6481	·6212	·5955	·5721
68	·6882	·6599	·6334	·6086	·5852
69	·6994	·6718	·6458	·6216	·5986
70	·7105	·6836	·6583	·6345	·6121
71	·7216	·6953	·6707	·6473	·6257
72	·7327	·7071	·6830	·6601	·6392
73	·7428	·7182	·6951	·6727	·6526
74	·7530	·7292	·7070	·6852	·6658
75	·7632	·7399	·7183	·6973	·6781
76	·7730	·7505	·7293	·7092	·6901
77	·7825	·7605	·7400	·7206	·7019
78	·7914	·7704	·7505	·7316	·7136
79	·8003	·7800	·7607	·7423	·7246
80	·8091	·7895	·7707	·7527	·7353
81	·8287	·8109	·7938	·7773	·7614
82	·8418	·8252	·8092	·7938	·7788
83	·8537	·8382	·8233	·8088	·7949
84	·8658	·8514	·8376	·8242	·8112
85	·8730	·8594	·8461	·8334	·8210

* For explanation of the bases upon which Tables I-X have been calculated see pp. 31-36.

TABLE I—(continued).
Value of an Absolute Reversion to £1.

Female Life Tenant.

Age of Life Tenant	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent
40	·3568	·3206	·2897	·2632	·2402
41	·3615	·3281	·2968	·2698	·2464
42	·3722	·3357	·3041	·2768	·2531
43	·3803	·3436	·3117	·2841	·2601
44	·3885	·3516	·3196	·2918	·2675
45	·3972	·3601	·3279	·2996	·2753
46	·4062	·3690	·3365	·3079	·2835
47	·4154	·3782	·3455	·3170	·2921
48	·4248	·3876	·3550	·3264	·3011
49	·4346	·3973	·3646	·3359	·3104
50	·4445	·4074	·3746	·3456	·3200
51	·4546	·4175	·3850	·3554	·3298
52	·4649	·4278	·3948	·3653	·3399
53	·4751	·4384	·4055	·3760	·3501
54	·4862	·4491	·4165	·3871	·3605
55	·4973	·4606	·4278	·3983	·3715
56	·5087	·4723	·4395	·4099	·3831
57	·5204	·4843	·4517	·4221	·3951
58	·5323	·4966	·4641	·4347	·4079
59	·5443	·5090	·4768	·4474	·4211
60	·5565	·5214	·4895	·4603	·4347
61	·5688	·5340	·5022	·4732	·4481
62	·5812	·5469	·5154	·4864	·4615
63	·5938	·5601	·5291	·5004	·4750
64	·6066	·5735	·5431	·5151	·4890
65	·6197	·5872	·5574	·5298	·5040
66	·6330	·6013	·5719	·5448	·5195
67	·6464	·6154	·5867	·5600	·5351
68	·6597	·6295	·6014	·5753	·5508
69	·6728	·6434	·6160	·5904	·5666
70	·6855	·6570	·6303	·6054	·5824
71	·6977	·6698	·6438	·6196	·5972
72	·7095	·6823	·6570	·6333	·6110
73	·7210	·6946	·6699	·6466	·6247
74	·7324	·7068	·6826	·6597	·6384
75	·7435	·7187	·6952	·6728	·6517
76	·7543	·7303	·7075	·6858	·6651
77	·7649	·7417	·7196	·6985	·6784
78	·7752	·7528	·7314	·7110	·6916
79	·7853	·7637	·7429	·7232	·7044
80	·7950	·7741	·7542	·7352	·7168
81	·8133	·7941	·7756	·7579	·7408
82	·8279	·8100	·7926	·7761	·7601
83	·8424	·8258	·8098	·7944	·7795
84	·8569	·8417	·8271	·8129	·7992
85	·8657	·8514	·8374	·8240	·8110

TABLE II.

*Value of a Contingent Reversion to £1.—Interest 4 per-cent.***Male Life Tenant.**

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	·839	·840	·838	·835	·828	·819	·811	·794	·763
84	·819	·819	·817	·814	·807	·797	·788	·768	·735
83	·801	·801	·799	·796	·788	·778	·767	·745	·711
82	·786	·786	·784	·780	·772	·761	·749	·725	·690
81	·773	·773	·771	·767	·758	·746	·733	·708	·672
80	·762	·761	·760	·755	·746	·734	·720	·694	·656
79	·751	·750	·749	·744	·734	·722	·707	·679	·641
78	·740	·739	·738	·733	·722	·710	·694	·665	·626
77	·729	·728	·728	·722	·711	·698	·681	·651	·612
76	·718	·717	·717	·711	·701	·686	·667	·637	·597
75	·707	·706	·705	·699	·689	·673	·653	·623	·582
74	·694	·694	·692	·686	·676	·659	·638	·607	·565
73	·681	·681	·679	·673	·663	·644	·622	·590	·547
72	·668	·668	·666	·660	·649	·629	·606	·573	·529
71	·655	·654	·653	·646	·635	·614	·589	·555	·511
70	·642	·641	·640	·632	·622	·600	·573	·538	·492
69	·629	·627	·627	·618	·607	·584	·556	·519	·474
68	·615	·614	·613	·604	·592	·568	·539	·500	·454
67	·601	·601	·599	·590	·577	·552	·521	·481	·434
66	·587	·587	·585	·575	·562	·536	·503	·462	·414
65	·574	·573	·571	·561	·547	·519	·486	·444	·395
64	·560	·559	·557	·546	·532	·502	·469	·426	·376
63	·546	·546	·543	·532	·517	·486	·452	·408	·358
62	·532	·532	·529	·518	·502	·470	·435	·390	·341
61	·518	·518	·515	·504	·487	·454	·418	·373	·325
60	·504	·505	·502	·490	·472	·439	·402	·357	·310
59	·490	·492	·488	·475	·456	·422	·384	·339	...
58	·476	·478	·474	·460	·441	·406	·367	·321	...
57	·462	·464	·460	·446	·426	·390	·350	·304	...
56	·448	·450	·447	·432	·411	·374	·333	·287	...
55	·435	·437	·435	·419	·396	·359	·317	·271	...
54	·422	·424	·422	·405	·381	·343	·301
53	·410	·412	·409	·391	·367	·327	·285
52	·398	·400	·396	·377	·353	·311	·269
51	·386	·388	·383	·364	·339	·296	·253
50	·374	·377	·370	·351	·325	·282	·239
49	·363	·366	·358	·339	·312	·268
48	·352	·355	·347	·328	·300	·255
47	·342	·345	·337	·317	·288	·243
46	·332	·335	·327	·306	·277	·232
45	·322	·325	·318	·296	·267	·223
44	·313	·315	·308	·286	·257
43	·304	·306	·298	·276	·247
42	·295	·297	·288	·266	·237
41	·286	·288	·278	·256	·227
40	·278	·279	·269	·246	·217

TABLE II—(continued).

Value of a Contingent Reversion to £1.—Interest 1 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	·831	·831	·829	·826	·818	·810	·801	·783	·751
84	·809	·809	·807	·803	·795	·786	·777	·756	·723
83	·790	·790	·788	·783	·775	·765	·755	·732	·697
82	·773	·773	·771	·766	·758	·747	·735	·711	·674
81	·758	·758	·756	·751	·743	·731	·717	·692	·654
80	·745	·745	·743	·738	·729	·716	·702	·675	·637
79	·732	·732	·730	·725	·716	·702	·687	·659	·620
78	·720	·720	·718	·712	·703	·689	·672	·643	·604
77	·708	·708	·706	·700	·690	·676	·658	·627	·588
76	·696	·695	·693	·687	·677	·662	·643	·611	·571
75	·683	·683	·681	·674	·665	·648	·628	·596	·555
74	·669	·669	·667	·660	·651	·633	·611	·579	·537
73	·655	·655	·653	·646	·637	·617	·594	·561	·519
72	·641	·641	·639	·632	·622	·601	·577	·543	·500
71	·627	·627	·625	·617	·607	·585	·559	·525	·481
70	·613	·612	·611	·603	·592	·569	·542	·506	·462
69	·597	·597	·596	·587	·576	·551	·523	·486	·441
68	·581	·582	·580	·571	·559	·533	·504	·466	·420
67	·565	·567	·564	·554	·542	·515	·485	·445	·399
66	·549	·551	·548	·538	·525	·497	·466	·425	·378
65	·534	·535	·533	·523	·509	·480	·447	·407	·359
64	·518	·520	·517	·507	·492	·462	·428	·388	·339
63	·503	·505	·502	·491	·475	·445	·410	·369	·320
62	·488	·490	·487	·475	·458	·428	·392	·350	·302
61	·473	·475	·472	·460	·442	·411	·374	·331	·284
60	·458	·461	·458	·446	·427	·394	·357	·314	·269
59	·443	·446	·444	·431	·411	·377	·340	·296	...
58	·428	·432	·430	·416	·396	·360	·323	·279	...
57	·414	·418	·416	·401	·381	·344	·306	·262	...
56	·400	·404	·402	·387	·366	·328	·289	·246	...
55	·387	·390	·389	·373	·351	·313	·273	·230	...
54	·374	·377	·375	·359	·336	·297	·257
53	·361	·364	·361	·345	·321	·282	·241
52	·348	·352	·348	·331	·307	·267	·226
51	·336	·340	·335	·318	·293	·252	·211
50	·325	·329	·323	·305	·280	·238	·198
49	·314	·318	·311	·293	·267	·224
48	·303	·307	·300	·282	·255	·212
47	·292	·297	·290	·271	·244	·202
46	·282	·287	·280	·261	·234	·192
45	·272	·277	·271	·251	·225	·184
44	·262	·267	·261	·241	·215
43	·253	·257	·251	·231	·205
42	·244	·248	·241	·221	·195
41	·235	·239	·232	·211	·185
40	·227	·230	·223	·202	·176

TABLE II—(continued).

*Value of a Contingent Reversion to £1.—Interest 5 per-cent.***Male Life Tenant.**

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	·814	·815	·813	·811	·803	·795	·788	·771	·740
84	·791	·792	·790	·789	·780	·771	·762	·744	·711
83	·772	·772	·770	·769	·760	·750	·739	·720	·686
82	·755	·755	·753	·751	·742	·732	·719	·699	·664
81	·740	·740	·738	·735	·726	·716	·702	·680	·644
80	·727	·727	·725	·721	·713	·701	·688	·664	·627
79	·715	·715	·713	·708	·700	·687	·674	·649	·611
78	·703	·703	·701	·696	·688	·674	·660	·635	·596
77	·691	·691	·689	·685	·676	·662	·646	·621	·581
76	·679	·679	·677	·673	·663	·649	·632	·606	·565
75	·667	·667	·665	·660	·651	·636	·617	·591	·549
74	·653	·653	·651	·646	·637	·621	·601	·574	·532
73	·639	·639	·637	·632	·623	·605	·584	·556	·514
72	·625	·625	·623	·617	·609	·589	·567	·537	·496
71	·610	·610	·609	·602	·594	·573	·549	·519	·477
70	·595	·595	·594	·587	·579	·557	·532	·501	·459
69	·580	·580	·579	·572	·564	·540	·514	·482	·439
68	·565	·565	·564	·557	·548	·523	·496	·463	·419
67	·550	·551	·550	·542	·532	·506	·478	·444	·400
66	·536	·537	·536	·527	·516	·490	·461	·425	·381
65	·523	·524	·522	·513	·501	·474	·444	·406	·363
64	·509	·510	·507	·498	·485	·458	·427	·388	·345
63	·495	·496	·493	·483	·469	·442	·410	·371	·328
62	·481	·482	·479	·468	·454	·426	·393	·354	·311
61	·467	·468	·465	·454	·439	·410	·376	·337	·294
60	·452	·454	·452	·441	·424	·394	·360	·321	·279
59	·437	·439	·438	·426	·408	·377	·343	·303	...
58	·422	·425	·424	·411	·393	·360	·326	·286	...
57	·408	·411	·410	·396	·378	·344	·309	·269	...
56	·394	·397	·396	·382	·363	·328	·292	·253	...
55	·380	·383	·382	·368	·349	·313	·276	·237	...
54	·367	·370	·369	·354	·334	·297	·260
53	·355	·358	·356	·340	·320	·282	·244
52	·343	·346	·343	·327	·306	·267	·229
51	·332	·335	·330	·315	·292	·252	·215
50	·321	·324	·318	·303	·279	·239	·202
49	·311	·313	·306	·292	·266	·226
48	·301	·303	·295	·281	·254	·214
47	·291	·293	·285	·270	·243	·203
46	·281	·283	·276	·259	·232	·192
45	·271	·274	·267	·249	·222	·182
44	·262	·265	·258	·239	·212
43	·253	·256	·248	·230	·202
42	·245	·247	·239	·221	·193
41	·237	·239	·230	·212	·184
40	·229	·232	·222	·204	·176

TABLE II—(continued).

Value of a Contingent Reversion to £1.—Interest 5 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	804	805	803	800	793	784	776	759	728
84	780	781	779	776	769	757	748	730	697
83	759	759	757	754	747	734	724	704	669
82	740	740	738	735	727	714	703	681	644
81	723	723	721	718	709	696	684	662	623
80	709	709	707	702	694	681	668	645	605
79	695	695	693	687	679	665	652	628	587
78	682	682	680	673	665	651	637	612	570
77	669	669	667	660	652	637	622	596	553
76	655	655	653	646	638	622	606	579	537
75	641	641	639	633	625	608	590	562	520
74	626	626	624	618	610	592	572	544	501
73	611	611	609	603	595	575	554	525	482
72	596	596	594	588	579	558	536	506	464
71	581	580	579	572	563	541	518	488	444
70	565	565	563	557	547	525	501	470	425
69	548	549	547	541	530	507	482	450	404
68	531	533	531	524	513	489	463	430	383
67	514	517	515	507	495	471	444	409	362
66	498	501	499	490	477	452	424	389	342
65	482	484	482	474	461	434	405	370	323
64	466	468	466	458	444	416	387	351	304
63	450	453	450	442	427	399	369	332	286
62	434	438	435	426	410	382	351	314	269
61	419	423	420	411	394	365	333	296	253
60	405	408	406	396	379	348	316	279	239
59	390	393	391	381	363	331	299	262	...
58	375	379	377	366	348	314	282	245	...
57	360	365	363	351	333	298	265	228	...
56	346	351	349	336	318	282	249	212	...
55	333	338	336	322	303	267	234	196	...
54	320	325	323	309	288	252	219
53	308	313	310	296	274	237	204
52	296	301	297	283	260	222	189
51	284	289	284	270	246	209	175
50	273	278	272	258	234	196	162
49	263	268	261	246	222	183
48	253	258	251	235	211	171
47	243	248	241	225	201	161
46	233	238	232	216	192	153
45	224	229	224	208	184	146
44	215	220	215	198	175
43	206	211	206	189	165
42	197	202	197	180	156
41	189	193	188	171	148
40	182	186	180	162	140

TABLE II—(continued).

*Value of a Contingent Reversion to £1.—Interest 6 per-cent.***Male Life Tenant.**

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	·791	·791	·789	·787	·781	·772	·765	·750	·720
84	·768	·768	·766	·763	·757	·747	·739	·721	·690
83	·746	·746	·744	·741	·734	·724	·715	·696	·663
82	·726	·726	·724	·721	·711	·704	·693	·673	·639
81	·709	·709	·707	·704	·697	·686	·674	·653	·618
80	·694	·695	·694	·690	·682	·670	·658	·636	·601
79	·680	·682	·681	·676	·668	·655	·642	·620	·585
78	·667	·670	·668	·663	·655	·641	·627	·606	·570
77	·655	·657	·656	·651	·643	·628	·613	·591	·554
76	·643	·644	·643	·638	·630	·615	·598	·575	·538
75	·631	·631	·629	·624	·616	·602	·584	·560	·522
74	·617	·616	·614	·609	·601	·587	·568	·543	·504
73	·602	·601	·599	·594	·586	·571	·551	·525	·486
72	·587	·586	·584	·579	·571	·555	·533	·506	·467
71	·571	·571	·569	·564	·556	·538	·515	·487	·448
70	·555	·555	·555	·549	·540	·521	·497	·468	·430
69	·539	·540	·540	·533	·523	·503	·478	·448	·411
68	·524	·525	·525	·517	·507	·486	·460	·429	·391
67	·509	·510	·510	·501	·491	·469	·442	·410	·371
66	·494	·495	·495	·486	·475	·452	·424	·392	·352
65	·479	·481	·480	·471	·460	·436	·407	·374	·334
64	·464	·467	·465	·456	·444	·420	·390	·356	·316
63	·449	·453	·450	·441	·428	·404	·373	·339	·299
62	·434	·439	·436	·426	·413	·388	·357	·322	·283
61	·420	·425	·422	·412	·398	·372	·341	·306	·268
60	·407	·410	·409	·399	·384	·356	·325	·290	·253
59	·393	·395	·395	·385	·369	·339	·308	·273	...
58	·378	·381	·381	·371	·354	·323	·291	·256	...
57	·364	·367	·367	·356	·339	·307	·274	·239	...
56	·350	·353	·353	·342	·324	·291	·257	·222	...
55	·337	·340	·340	·328	·310	·276	·242	·206	...
54	·324	·327	·327	·314	·295	·261	·226
53	·312	·315	·314	·301	·281	·246	·211
52	·300	·303	·301	·288	·267	·231	·196
51	·288	·292	·289	·275	·254	·217	·183
50	·277	·281	·277	·262	·241	·204	·170
49	·267	·271	·266	·250	·228	·191
48	·257	·261	·255	·239	·216	·179
47	·247	·251	·245	·229	·205	·168
46	·238	·242	·236	·220	·195	·158
45	·230	·234	·228	·212	·187	·150
44	·222	·226	·220	·203	·178
43	·214	·218	·212	·194	·169
42	·206	·210	·204	·185	·160
41	·199	·203	·195	·177	·152
40	·192	·195	·187	·170	·145

TABLE II—(continued).

Value of a Contingent Reversion to £1.—Interest 6 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	·779	·780	·778	·776	·770	·761	·753	·737	·707
84	·753	·754	·752	·749	·743	·732	·723	·705	·675
83	·730	·730	·728	·725	·719	·707	·697	·678	·646
82	·709	·709	·707	·704	·698	·685	·674	·654	·621
81	·691	·691	·689	·686	·679	·666	·654	·633	·598
80	·676	·676	·674	·670	·662	·650	·637	·615	·579
79	·661	·661	·659	·655	·646	·634	·620	·597	·561
78	·647	·647	·645	·640	·631	·619	·604	·580	·544
77	·633	·633	·631	·626	·617	·604	·588	·564	·527
76	·618	·618	·616	·612	·603	·588	·571	·547	·509
75	·603	·603	·602	·597	·589	·572	·555	·530	·492
74	·587	·587	·587	·581	·573	·555	·537	·511	·473
73	·571	·571	·571	·565	·557	·538	·519	·492	·454
72	·555	·555	·555	·549	·541	·521	·501	·473	·435
71	·539	·539	·539	·533	·524	·504	·482	·455	·415
70	·523	·524	·523	·517	·508	·487	·464	·437	·396
69	·506	·508	·506	·500	·491	·468	·445	·417	·375
68	·489	·491	·489	·483	·473	·449	·425	·396	·354
67	·472	·474	·472	·465	·455	·430	·405	·376	·333
66	·455	·457	·455	·448	·437	·412	·386	·356	·313
65	·438	·441	·439	·432	·420	·395	·368	·337	·295
64	·422	·425	·423	·416	·403	·377	·350	·318	·277
63	·406	·409	·407	·400	·386	·360	·332	·300	·260
62	·390	·394	·392	·384	·369	·343	·314	·283	·244
61	·375	·379	·377	·368	·353	·326	·297	·265	·228
60	·360	·364	·362	·353	·338	·310	·281	·248	·213
59	·345	·349	·347	·338	·323	·294	·264	·231	...
58	·330	·335	·333	·323	·308	·278	·247	·214	...
57	·316	·321	·319	·309	·293	·261	·231	·198	...
56	·302	·307	·306	·295	·278	·246	·215	·182	...
55	·289	·294	·293	·282	·264	·231	·200	·167	...
54	·276	·281	·280	·269	·249	·216	·185
53	·264	·269	·267	·256	·235	·201	·171
52	·252	·257	·255	·243	·222	·187	·157
51	·241	·247	·244	·231	·210	·174	·143
50	·231	·237	·233	·219	·199	·163	·131
49	·222	·228	·223	·208	·189	·152
48	·213	·219	·214	·198	·179	·142
47	·204	·210	·205	·189	·170	·133
46	·195	·201	·197	·181	·161	·124
45	·187	·193	·189	·174	·153	·117
44	·179	·185	·180	·165	·144
43	·171	·177	·172	·156	·136
42	·163	·169	·164	·147	·128
41	·156	·161	·156	·139	·120
40	·149	·153	·148	·132	·112

Table of the Office Annual Premiums per-cent for Contingent Assurances adopted in the Calculation of Table II.

Age of Life Tenant	AGE OF REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	1.000	1.008	1.085	1.181	1.404	1.654	1.890	2.495	3.632
84	1.004	1.017	1.092	1.192	1.414	1.665	1.916	2.538	3.672
83	1.009	1.026	1.099	1.204	1.424	1.676	1.942	2.581	3.712
82	1.014	1.035	1.106	1.216	1.434	1.687	1.968	2.625	3.753
81	1.019	1.044	1.114	1.228	1.444	1.698	1.994	2.669	3.794
80	1.024	1.054	1.122	1.240	1.454	1.710	2.020	2.713	3.835
79	1.030	1.062	1.129	1.250	1.461	1.723	2.051	2.755	3.879
78	1.036	1.070	1.137	1.260	1.469	1.736	2.082	2.797	3.923
77	1.043	1.078	1.145	1.270	1.477	1.749	2.113	2.839	3.967
76	1.050	1.086	1.153	1.280	1.485	1.763	2.145	2.881	4.012
75	1.057	1.094	1.161	1.291	1.493	1.778	2.177	2.923	4.057
74	1.064	1.102	1.169	1.301	1.503	1.799	2.212	2.969	4.109
73	1.071	1.111	1.177	1.311	1.513	1.820	2.247	3.015	4.162
72	1.078	1.120	1.186	1.321	1.524	1.841	2.282	3.061	4.215
71	1.085	1.129	1.195	1.331	1.535	1.862	2.318	3.107	4.268
70	1.092	1.138	1.204	1.342	1.546	1.883	2.354	3.154	4.321
69	1.100	1.144	1.213	1.356	1.567	1.914	2.399	3.216	4.403
68	1.108	1.150	1.222	1.370	1.588	1.945	2.441	3.278	4.485
67	1.116	1.157	1.231	1.384	1.610	1.976	2.489	3.341	4.567
66	1.124	1.164	1.241	1.398	1.632	2.007	2.534	3.404	4.650
65	1.133	1.171	1.251	1.413	1.654	2.038	2.579	3.467	4.733
64	1.142	1.179	1.262	1.428	1.678	2.070	2.623	3.525	4.801
63	1.151	1.187	1.273	1.444	1.702	2.102	2.667	3.583	4.869
62	1.160	1.195	1.284	1.460	1.726	2.134	2.711	3.641	4.937
61	1.169	1.204	1.296	1.476	1.750	2.167	2.755	3.699	5.006
60	1.179	1.213	1.308	1.492	1.775	2.200	2.800	3.758	5.075
59	1.188	1.223	1.317	1.507	1.798	2.233	2.844	3.811	...
58	1.197	1.233	1.326	1.523	1.821	2.266	2.888	3.864	...
57	1.206	1.243	1.335	1.539	1.844	2.299	2.932	3.917	...
56	1.215	1.253	1.344	1.555	1.868	2.333	2.976	3.971	...
55	1.225	1.263	1.354	1.571	1.892	2.367	3.021	4.025	...
54	1.234	1.273	1.374	1.596	1.924	2.410	3.071
53	1.243	1.284	1.394	1.621	1.956	2.453	3.122
52	1.252	1.295	1.414	1.646	1.988	2.496	3.173
51	1.261	1.306	1.434	1.671	2.021	2.539	3.224
50	1.271	1.317	1.454	1.696	2.054	2.583	3.275
49	1.280	1.328	1.468	1.716	2.080	2.614
48	1.289	1.339	1.482	1.736	2.107	2.645
47	1.298	1.351	1.496	1.756	2.134	2.676
46	1.307	1.363	1.510	1.776	2.161	2.707
45	1.317	1.375	1.525	1.796	2.188	2.738
44	1.326	1.388	1.544	1.819	2.216
43	1.335	1.401	1.563	1.842	2.244
42	1.344	1.414	1.582	1.865	2.272
41	1.353	1.428	1.601	1.889	2.300
40	1.363	1.442	1.621	1.913	2.329

TABLE III.
Value of a Life Interest of £1.

Office Annual Premium per-cent	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent	Per-cent on Life Interest Value to be added, in order to obtain the Sum Assured
£ s.						
1 10	17·946	16·448	15·181	14·095	13·153	·76
1 11	17·779	16·307	15·061	13·991	13·062	·79
1 12	17·615	16·167	14·942	13·888	12·972	·82
1 13	17·453	16·030	14·824	13·785	12·882	·84
1 14	17·294	15·896	14·707	13·683	12·793	·86
1 15	17·138	15·763	14·593	13·583	12·705	·89
1 16	16·983	15·630	14·480	13·484	12·617	·91
1 17	16·830	15·500	14·368	13·387	12·533	·94
1 18	16·681	15·374	14·258	13·291	12·448	·96
1 19	16·534	15·250	14·150	13·198	12·366	·99
2 0	16·390	15·126	14·043	13·105	12·284	1·01
2 1	16·250	15·005	13·938	13·013	12·203	1·04
2 2	16·111	14·886	13·834	12·922	12·122	1·07
2 3	15·974	14·769	13·733	12·833	12·043	1·10
2 4	15·840	14·653	13·633	12·744	11·965	1·12
2 5	15·707	14·539	13·534	12·657	11·888	1·15
2 6	15·507	14·426	13·435	12·571	11·812	1·18
2 7	15·449	14·316	13·340	12·487	11·737	1·21
2 8	15·324	14·209	13·245	12·404	11·663	1·23
2 9	15·200	14·102	13·152	12·322	11·590	1·26
2 10	15·076	13·995	13·059	12·241	11·517	1·29
2 11	14·956	13·890	12·968	12·160	11·445	1·32
2 12	14·839	13·788	12·878	12·080	11·374	1·35
2 13	14·723	13·687	12·789	12·001	11·305	1·37
2 14	14·607	13·587	12·700	11·924	11·236	1·39
2 15	14·492	13·488	12·615	11·847	11·177	1·42
2 16	14·383	13·392	12·529	11·771	11·099	1·44
2 17	14·271	13·297	12·445	11·696	11·032	1·47
2 18	14·162	13·202	12·362	11·621	10·966	1·49
2 19	14·055	13·106	12·279	11·549	10·901	1·52
3 0	13·951	13·016	12·198	11·477	10·836	1·54
3 1	13·848	12·926	12·118	11·406	10·772	1·56
3 2	13·746	12·836	12·039	11·336	10·709	1·58
3 3	13·646	12·748	11·962	11·267	10·646	1·61
3 4	13·547	12·662	11·885	11·198	10·584	1·64
3 5	13·448	12·575	11·809	11·130	10·524	1·67
3 6	13·351	12·489	11·734	11·063	10·464	1·70
3 7	13·256	12·406	11·659	10·996	10·404	1·73
3 8	13·159	12·324	11·585	10·930	10·345	1·76
3 9	13·071	12·243	11·513	10·866	10·287	1·79
3 10	12·979	12·162	11·441	10·802	10·229	1·82
3 11	12·889	12·082	11·370	10·738	10·172	1·85
3 12	12·799	12·002	11·300	10·675	10·115	1·87
3 13	12·711	11·925	11·231	10·613	10·060	1·89
3 14	12·625	11·849	11·163	10·551	10·005	1·92
3 15	12·540	11·774	11·096	10·491	9·950	1·95
3 16	12·455	11·699	11·030	10·431	9·896	1·97
3 17	12·372	11·626	10·964	10·372	9·843	2·00
3 18	12·291	11·553	10·898	10·314	9·790	2·03
3 19	12·210	11·481	10·834	10·257	9·737	2·06

TABLE III—(continued).
Value of a Life Interest of £1.

Office Annual Premium per-cent	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent	Per-cent on Life Interest Value to be added, in order to obtain the Sum Assured
£ s.						
4 0	12·130	11·410	10·772	10·199	9·685	2·09
4 1	12·050	11·340	10·710	10·112	9·633	2·12
4 2	11·972	11·270	10·647	10·085	9·582	2·14
4 3	11·895	11·201	10·584	10·030	9·531	2·17
4 4	11·819	11·133	10·522	9·975	9·481	2·19
4 5	11·743	11·065	10·462	9·921	9·432	2·22
4 6	11·669	10·999	10·402	9·867	9·383	2·24
4 7	11·596	10·934	10·344	9·814	9·335	2·27
4 8	11·524	10·869	10·286	9·761	9·287	2·30
4 9	11·452	10·805	10·228	9·709	9·240	2·33
4 10	11·381	10·742	10·171	9·657	9·192	2·36
4 11	11·310	10·679	10·114	9·605	9·145	2·39
4 12	11·241	10·617	10·058	9·554	9·099	2·42
4 13	11·173	10·556	10·003	9·504	9·053	2·44
4 14	11·106	10·495	9·948	9·455	9·008	2·47
4 15	11·038	10·435	9·893	9·406	8·963	2·49
4 16	10·971	10·376	9·839	9·357	8·919	2·52
4 17	10·906	10·317	9·786	9·309	8·875	2·51
4 18	10·842	10·259	9·734	9·261	8·831	2·57
4 19	10·778	10·201	9·682	9·214	8·788	2·59
5 0	10·715	10·143	9·630	9·166	8·745	2·62
5 1	10·652	10·086	9·579	9·119	8·702	2·64
5 2	10·589	10·030	9·528	9·073	8·659	2·67
5 3	10·528	9·976	9·478	9·028	8·618	2·69
5 4	10·469	9·922	9·429	8·983	8·577	2·72
5 5	10·410	9·868	9·381	8·939	8·537	2·75
5 6	10·351	9·815	9·332	8·894	8·496	2·78
5 7	10·292	9·762	9·284	8·850	8·456	2·81
5 8	10·233	9·709	9·236	8·806	8·416	2·84
5 9	10·177	9·658	9·189	8·764	8·376	2·87
5 10	10·120	9·607	9·142	8·721	8·337	2·90
5 11	10·063	9·556	9·096	8·679	8·298	2·93
5 12	10·007	9·506	9·051	8·637	8·260	2·96
5 13	9·954	9·457	9·006	8·596	8·223	2·99
5 14	9·900	9·408	8·961	8·555	8·185	3·02
5 15	9·846	9·359	8·917	8·515	8·147	3·05
5 16	9·792	9·310	8·872	8·475	8·109	3·08
5 17	9·739	9·262	8·828	8·435	8·072	3·10
5 18	9·687	9·214	8·785	8·395	8·036	3·13
5 19	9·636	9·167	8·743	8·356	8·000	3·15
6 0	9·585	9·120	8·700	8·316	7·964	3·18
6 1	9·535	9·075	8·658	8·277	7·928	3·20
6 2	9·485	9·030	8·616	8·239	7·893	3·23
6 3	9·435	8·985	8·575	8·201	7·858	3·25
6 4	9·385	8·940	8·534	8·163	7·824	3·28
6 5	9·338	8·896	8·493	8·126	7·789	3·30
6 6	9·290	8·852	8·453	8·089	7·755	3·33
6 7	9·242	8·809	8·414	8·053	7·721	3·36
6 8	9·195	8·766	8·374	8·016	7·688	3·39
6 9	9·148	8·724	8·335	7·980	7·655	3·42

TABLE III—(continued).
Value of a Life Interest of £1.

Office Annual Premium per-cent	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent	Per-cent on Life Interest Value to be added, in order to obtain the Sum Assured
£ s.						
6 10	9.101	8.681	8.296	7.945	7.622	3.45
6 11	9.056	8.639	8.258	7.910	7.589	3.48
6 12	9.011	8.597	8.220	7.875	7.557	3.51
6 13	8.966	8.557	8.182	7.840	7.524	3.54
6 14	8.922	8.516	8.145	7.805	7.492	3.57
6 15	8.877	8.476	8.108	7.771	7.462	3.60
6 16	8.832	8.435	8.071	7.736	7.430	3.64
6 17	8.789	8.395	8.034	7.702	7.398	3.67
6 18	8.746	8.356	7.997	7.669	7.367	3.70
6 19	8.704	8.317	7.961	7.636	7.336	3.73
7 0	8.663	8.278	7.926	7.604	7.305	3.76
7 1	8.621	8.240	7.891	7.571	7.275	3.79
7 2	8.579	8.202	7.856	7.539	7.245	3.82
7 3	8.538	8.164	7.822	7.507	7.215	3.85
7 4	8.498	8.127	7.787	7.475	7.186	3.88
7 5	8.458	8.090	7.753	7.444	7.156	3.91
7 6	8.418	8.053	7.719	7.413	7.127	3.94
7 7	8.379	8.017	7.686	7.381	7.098	3.97
7 8	8.340	7.981	7.653	7.350	7.069	4.00
7 9	8.301	7.946	7.620	7.319	7.041	4.03
7 10	8.262	7.910	7.587	7.289	7.013	4.06
7 11	8.224	7.875	7.555	7.259	6.985	4.09
7 12	8.186	7.840	7.522	7.229	6.958	4.12
7 13	8.148	7.806	7.490	7.199	6.930	4.14
7 14	8.110	7.771	7.458	7.169	6.902	4.17
7 15	8.073	7.737	7.427	7.140	6.874	4.19
7 16	8.037	7.703	7.396	7.111	6.847	4.22
7 17	8.002	7.670	7.365	7.083	6.820	4.24
7 18	7.966	7.636	7.334	7.054	6.797	4.27
7 19	7.930	7.603	7.303	7.026	6.767	4.31
8 0	7.894	7.570	7.272	6.997	6.741	4.35

TABLE IV.

Present Value of a Reversionary Life Interest of £1.—Interest 4 per-cent in possession and 5 per-cent in reversion.

Male Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	17-326	16-937	16-278	15-471	14-504	13-400	12-186	10-836	9-390
	20	25	30	35	40	45	50	55	60
85	13-797	13-434	12-808	12-039	11-122	10-053	8-890	7-649	6-342
84	13-383	13-023	12-404	11-639	10-732	9-668	8-512	7-291	6-014
83	13-014	12-659	12-046	11-285	10-381	9-325	8-176	6-974	5-725
82	12-685	12-333	11-725	10-969	10-074	9-020	7-877	6-696	5-469
81	12-388	12-041	11-437	10-685	9-795	8-747	7-612	6-449	5-242
80	12-119	11-775	11-176	10-429	9-545	8-500	7-373	6-228	5-040
79	11-871	11-532	10-936	10-194	9-314	8-271	7-156	6-029	4-857
78	11-639	11-305	10-712	9-975	9-099	8-063	6-954	5-845	4-691
77	11-417	11-086	10-497	9-765	8-893	7-863	6-764	5-672	4-534
76	11-198	10-873	10-288	9-559	8-692	7-667	6-577	5-504	4-383
75	10-982	10-655	10-076	9-353	8-491	7-469	6-391	5-335	4-232
74	10-753	10-433	9-857	9-139	8-282	7-266	6-198	5-160	4-080
73	10-511	10-195	9-626	8-912	8-061	7-051	5-994	4-974	3-918
72	10-250	9-939	9-376	8-665	7-822	6-820	5-772	4-771	3-744
71	9-963	9-655	9-102	8-391	7-560	6-565	5-529	4-546	3-541
70	9-652	9-344	8-797	8-096	7-283	6-281	5-257	4-295	3-337
69	9-382	9-076	8-536	7-839	7-025	6-040	5-028	4-083	3-161
68	9-114	8-812	8-278	7-588	6-782	5-804	4-804	3-882	2-994
67	8-848	8-552	8-024	7-340	6-543	5-574	4-590	3-691	2-836
66	8-584	8-294	7-773	7-094	6-308	5-349	4-382	3-509	2-685
65	8-324	8-039	7-525	6-853	6-075	5-127	4-179	3-334	2-542
64	8-066	7-785	7-279	6-615	5-846	4-909	3-982	3-166	2-404
63	7-809	7-534	7-036	6-380	5-619	4-696	3-789	3-004	2-271
62	7-554	7-285	6-794	6-146	5-395	4-486	3-601	2-846	2-142
61	7-300	7-038	6-554	5-913	5-173	4-278	3-415	2-690	2-015
60	7-046	6-795	6-316	5-682	4-953	4-070	3-230	2-536	1-893
59	6-794	6-549	6-078	5-453	4-735	3-865	3-049	2-383	...
58	6-543	6-303	5-839	5-221	4-517	3-663	2-867	2-232	...
57	6-291	6-057	5-601	4-996	4-301	3-460	2-687	2-078	...
56	6-039	5-810	5-363	4-766	4-086	3-257	2-506	1-922	...
55	5-789	5-570	5-126	4-536	3-882	3-054	2-321	1-766	...
54	5-567	5-355	4-917	4-338	3-683	2-878	2-169
53	5-355	5-149	4-716	4-148	3-504	2-711	2-025
52	5-149	4-951	4-523	3-966	3-331	2-553	1-889
51	4-952	4-761	4-338	3-792	3-166	2-402	1-762
50	4-764	4-582	4-162	3-627	3-009	2-259	1-611
49	4-583	4-408	3-993	3-469	2-859	2-125
48	4-410	4-242	3-832	3-318	2-717	1-999
47	4-241	4-083	3-679	3-176	2-582	1-879
46	4-087	3-932	3-533	3-040	2-456	1-768
45	3-936	3-789	3-396	2-912	2-338	1-665
44	3-793	3-652	3-265	2-791	2-227
43	3-655	3-522	3-143	2-679	2-125
42	3-526	3-397	3-027	2-575	2-031
41	3-402	3-280	2-920	2-476	1-945
40	3-281	3-168	2-809	2-378	1-865

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 4 per-cent in possession and 5 per-cent in reversion.

Female Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	17-326	16-937	16-278	15-471	14-504	13-400	12-186	10-836	9-390
	20	25	30	35	40	45	50	55	60
85	13-601	13-237	12-617	11-847	10-937	9-872	8-710	7-469	6-190
84	13-156	12-798	12-183	11-421	10-518	9-458	8-305	7-096	5-836
83	12-756	12-403	11-792	11-037	10-141	9-085	7-945	6-762	5-523
82	12-394	12-047	11-439	10-691	9-801	8-751	7-620	6-461	5-243
81	12-065	11-722	11-119	10-377	9-494	8-448	7-327	6-191	4-994
80	11-764	11-425	10-826	10-091	9-214	8-171	7-062	5-946	4-770
79	11-481	11-150	10-556	9-826	8-955	7-917	6-819	5-722	4-568
78	11-221	10-892	10-302	9-577	8-713	7-679	6-593	5-514	4-382
77	10-970	10-644	10-059	9-339	8-481	7-453	6-377	5-317	4-207
76	10-723	10-402	9-822	9-107	8-255	7-232	6-169	5-129	4-041
75	10-478	10-161	9-586	8-877	8-030	7-012	5-963	4-939	3-876
74	10-226	9-911	9-344	8-641	7-800	6-789	5-752	4-759	3-710
73	9-964	9-657	9-092	8-394	7-560	6-556	5-533	4-553	3-536
72	9-685	9-383	8-825	8-132	7-305	6-309	5-300	4-345	3-353
71	9-385	9-088	8-536	7-850	7-029	6-042	5-048	4-121	3-152
70	9-065	8-768	8-220	7-543	6-727	5-748	4-773	3-876	2-931
69	8-759	8-466	7-921	7-255	6-450	5-481	4-527	3-660	2-742
68	8-444	8-167	7-632	6-972	6-178	5-219	4-288	3-451	2-565
67	8-143	7-873	7-345	6-694	5-910	4-964	4-055	3-250	2-400
66	7-846	7-583	7-063	6-419	5-648	4-715	3-829	3-054	2-245
65	7-554	7-297	6-786	6-150	5-391	4-471	3-608	2-864	2-099
64	7-266	7-016	6-513	5-886	5-138	4-233	3-394	2-681	1-960
63	6-984	6-740	6-245	5-627	4-892	4-002	3-185	2-504	1-829
62	6-705	6-466	5-981	5-372	4-649	3-777	2-982	2-331	1-703
61	6-432	6-202	5-723	5-123	4-414	3-556	2-786	2-164	1-582
60	6-164	5-940	5-470	4-889	4-183	3-342	2-596	2-003	1-464
59	5-900	5-684	5-222	4-641	3-958	3-133	2-411	1-846	...
58	5-642	5-433	4-979	4-410	3-738	2-931	2-232	1-695	...
57	5-389	5-188	4-741	4-183	3-525	2-734	2-060	1-548	...
56	5-140	4-948	4-509	3-959	3-317	2-543	1-892	1-405	...
55	4-899	4-715	4-284	3-748	3-115	2-356	1-731	1-268	...
54	4-670	4-494	4-067	3-544	2-921	2-180	1-585
53	4-454	4-285	3-866	3-354	2-743	2-020	1-450
52	4-248	4-087	3-676	3-176	2-579	1-874	1-324
51	4-054	3-900	3-499	3-010	2-428	1-741	1-207
50	3-869	3-723	3-333	2-855	2-289	1-619	1-101
49	3-694	3-556	3-176	2-709	2-162	1-509
48	3-527	3-396	3-029	2-574	2-043	1-406
47	3-368	3-245	2-891	2-445	1-933	1-314
46	3-216	3-101	2-759	2-323	1-831	1-228
45	3-072	2-963	2-633	2-208	1-734	1-148
44	2-932	2-831	2-512	2-098	1-643
43	2-795	2-703	2-395	1-992	1-556
42	2-663	2-579	2-282	1-890	1-471
41	2-535	2-458	2-171	1-792	1-388
40	2-412	2-341	2-067	1-692	1-303

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 5 per-cent throughout.

Male Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	14·730	14·446	13·959	13·359	12·624	11·772	10·815	9·729	8·538
	20	25	30	35	40	45	50	55	60
85	11·730	11·458	10·983	10·396	9·681	8·832	7·890	6·868	5·767
84	11·378	11·108	10·637	10·051	9·341	8·494	7·554	6·546	5·469
83	11·064	10·797	10·330	9·745	9·039	8·193	7·256	6·262	5·206
82	10·784	10·519	10·055	9·472	8·769	7·925	6·991	6·012	4·973
81	10·532	10·270	9·808	9·227	8·526	7·685	6·756	5·790	4·767
80	10·303	10·043	9·584	9·006	8·308	7·468	6·544	5·592	4·583
79	10·092	9·836	9·378	8·803	8·107	7·269	6·351	5·413	4·417
78	9·895	9·642	9·186	8·614	7·920	7·084	6·172	5·248	4·266
77	9·706	9·456	9·002	8·433	7·741	6·908	6·003	5·093	4·123
76	9·520	9·274	8·822	8·255	7·566	6·736	5·837	4·942	3·986
75	9·336	9·088	8·641	8·077	7·391	6·562	5·672	4·790	3·849
74	9·152	8·909	8·453	7·892	7·209	6·384	5·501	4·633	3·710
73	8·966	8·696	8·255	7·696	7·017	6·195	5·320	4·466	3·563
72	8·774	8·477	8·040	7·483	6·809	5·992	5·123	4·284	3·405
71	8·470	8·236	7·805	7·249	6·581	5·768	4·907	4·082	3·220
70	8·206	7·970	7·544	6·992	6·340	5·519	4·666	3·856	3·035
69	7·976	7·741	7·320	6·770	6·115	5·307	4·462	3·666	2·875
68	7·748	7·516	7·099	6·553	5·904	5·100	4·264	3·486	2·723
67	7·522	7·294	6·881	6·339	5·696	4·898	4·074	3·314	2·579
66	7·298	7·074	6·666	6·127	5·491	4·700	3·889	3·151	2·442
65	7·077	6·857	6·453	5·919	5·289	4·505	3·709	2·994	2·312
64	6·857	6·640	6·242	5·713	5·089	4·314	3·534	2·843	2·187
63	6·639	6·426	6·034	5·510	4·892	4·127	3·363	2·697	2·066
62	6·422	6·214	5·826	5·308	4·697	3·942	3·196	2·555	1·948
61	6·206	6·003	5·620	5·107	4·504	3·759	3·031	2·415	1·833
60	5·990	5·796	5·416	4·908	4·312	3·577	2·867	2·277	1·722
59	5·776	5·586	5·212	4·710	4·122	3·397	2·706	2·140	...
58	5·562	5·376	5·007	4·512	3·933	3·219	2·545	2·004	...
57	5·348	5·166	4·803	4·315	3·745	3·041	2·385	1·866	...
56	5·134	4·956	4·599	4·117	3·558	2·863	2·224	1·762	...
55	4·921	4·751	4·396	3·918	3·380	2·684	2·060	1·586	...
54	4·733	4·568	4·217	3·747	3·207	2·530	1·925
53	4·552	4·392	4·044	3·583	3·051	2·383	1·797
52	4·377	4·223	3·879	3·426	2·901	2·244	1·677
51	4·210	4·061	3·720	3·276	2·757	2·112	1·564
50	4·050	3·908	3·569	3·133	2·620	1·986	1·457
49	3·896	3·760	3·424	2·997	2·490	1·868
48	3·749	3·618	3·286	2·867	2·366	1·756
47	3·608	3·483	3·155	2·744	2·249	1·652
46	3·474	3·354	3·030	2·627	2·139	1·555
45	3·346	3·232	2·912	2·516	2·036	1·464
44	3·224	3·115	2·800	2·412	1·940
43	3·107	3·004	2·695	2·315	1·851
42	2·997	2·898	2·596	2·225	1·769
41	2·892	2·798	2·504	2·140	1·694
40	2·789	2·702	2·409	2·055	1·625

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 5 per-cent throughout

Female Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the Head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	14:730	14:446	13:959	13:359	12:624	11:772	10:815	9:729	8:538
	20	25	30	35	40	45	50	55	60
85	11:563	11:290	10:820	10:230	9:520	8:673	7:730	6:706	5:629
84	11:185	10:916	10:447	9:862	9:155	8:309	7:371	6:371	5:307
83	10:845	10:579	10:112	9:531	8:827	7:982	7:051	6:071	5:022
82	10:537	10:275	9:809	9:232	8:531	7:688	6:763	5:801	4:768
81	10:257	9:998	9:535	8:961	8:264	7:422	6:503	5:559	4:541
80	10:001	9:745	9:284	8:714	8:020	7:179	6:268	5:339	4:338
79	9:763	9:510	9:052	8:485	7:795	6:956	6:052	5:138	4:154
78	9:540	9:290	8:834	8:270	7:584	6:747	5:851	4:951	3:985
77	9:326	9:079	8:626	8:065	7:382	6:548	5:660	4:774	3:826
76	9:116	8:872	8:423	7:865	7:186	6:354	5:475	4:604	3:675
75	8:908	8:667	8:220	7:666	6:990	6:161	5:292	4:435	3:525
74	8:694	8:456	8:013	7:462	6:790	5:965	5:105	4:265	3:374
73	8:471	8:237	7:797	7:249	6:581	5:760	4:911	4:088	3:216
72	8:234	8:003	7:568	7:023	6:359	5:543	4:704	3:901	3:049
71	7:979	7:752	7:320	6:779	6:119	5:309	4:480	3:700	2:867
70	7:707	7:479	7:049	6:514	5:856	5:051	4:236	3:480	2:666
69	7:439	7:221	6:795	6:266	5:615	4:816	4:018	3:286	2:494
68	7:179	6:966	6:545	6:021	5:375	4:586	3:806	3:099	2:333
67	6:923	6:715	6:299	5:781	5:145	4:362	3:599	2:918	2:183
66	6:670	6:468	6:057	5:544	4:917	4:143	3:398	2:742	2:042
65	6:422	6:224	5:819	5:312	4:693	3:929	3:202	2:572	1:909
64	6:177	5:984	5:585	5:084	4:473	3:720	3:012	2:407	1:783
63	5:937	5:749	5:355	4:860	4:259	3:517	2:827	2:248	1:664
62	5:700	5:517	5:129	4:640	4:048	3:319	2:647	2:093	1:549
61	5:468	5:290	4:908	4:425	3:843	3:125	2:473	1:943	1:439
60	5:240	5:067	4:691	4:215	3:642	2:937	2:304	1:799	1:332
59	5:016	4:848	4:478	4:009	3:446	2:754	2:140	1:658	...
58	4:796	4:634	4:270	3:800	3:255	2:576	1:981	1:522	...
57	4:581	4:425	4:066	3:613	3:069	2:403	1:828	1:390	...
56	4:370	4:221	3:867	3:423	2:888	2:235	1:679	1:262	...
55	4:165	4:022	3:674	3:238	2:713	2:071	1:536	1:139	...
54	3:970	3:833	3:488	3:062	2:544	1:917	1:497
53	3:786	3:655	3:315	2:898	2:389	1:776	1:287
52	3:611	3:486	3:152	2:744	2:246	1:648	1:175
51	3:446	3:327	3:001	2:601	2:115	1:531	1:071
50	3:289	3:176	2:858	2:467	1:994	1:424	977
49	3:140	3:033	2:724	2:341	1:883	1:327
48	2:998	2:897	2:598	2:224	1:780	1:237
47	2:863	2:768	2:479	2:113	1:684	1:156
46	2:734	2:645	2:366	2:008	1:595	1:080
45	2:611	2:528	2:258	1:908	1:511	1:010
44	2:492	2:415	2:154	1:813	1:432
43	2:376	2:306	2:054	1:722	1:356
42	2:264	2:200	1:957	1:634	1:282
41	2:155	2:097	1:862	1:549	1:210
40	2:050	1:997	1:773	1:463	1:136

TABLE IV—(continued).

*Present Value of a Reversionary Life Interest of £1.—Interest
5 per-cent in possession and 6 per-cent in reversion.*

Male Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	14·730	14·446	13·959	13·359	12·624	11·772	10·815	9·729	8·538
	20	25	30	35	40	45	50	55	60
85	11·389	11·126	10·666	10·095	9·403	8·578	7·660	6·669	5·604
84	11·005	10·745	10·292	9·724	9·038	8·218	7·305	6·332	5·294
83	10·663	10·408	9·959	9·395	8·714	7·898	6·992	6·036	5·021
82	10·360	10·108	9·663	9·102	8·426	7·615	6·714	5·775	4·781
81	10·088	9·838	9·398	8·842	8·169	7·362	6·468	5·545	4·570
80	9·843	9·596	9·159	8·606	7·939	7·134	6·248	5·341	4·381
79	9·619	9·375	8·941	8·393	7·729	6·927	6·048	5·157	4·212
78	9·410	9·168	8·738	8·194	7·533	6·734	5·863	4·989	4·057
77	9·211	8·972	8·544	8·005	7·347	6·551	5·689	4·831	3·913
76	9·017	8·780	8·355	7·821	7·166	6·372	5·519	4·677	3·774
75	8·821	8·587	8·167	7·633	6·985	6·196	5·349	4·522	3·637
74	8·619	8·387	7·970	7·442	6·798	6·012	5·177	4·364	3·496
73	8·403	8·174	7·762	7·239	6·599	5·817	4·991	4·195	3·349
72	8·170	7·944	7·537	7·018	6·384	5·607	4·790	4·010	3·189
71	7·914	7·691	7·290	6·775	6·147	5·376	4·568	3·805	3·013
70	7·643	7·425	7·030	6·512	5·896	5·133	4·322	3·583	2·826
69	7·390	7·174	6·785	6·271	5·665	4·904	4·114	3·380	2·655
68	7·153	6·940	6·558	6·047	5·449	4·694	3·915	3·200	2·503
67	6·920	6·712	6·335	5·829	5·238	4·490	3·724	3·029	2·362
66	6·693	6·488	6·116	5·617	5·032	4·292	3·539	2·866	2·226
65	6·467	6·268	5·901	5·407	4·830	4·099	3·361	2·711	2·099
64	6·246	6·051	5·690	5·202	4·631	3·909	3·189	2·563	1·977
63	6·026	5·837	5·481	5·000	4·434	3·724	3·020	2·420	1·859
62	5·809	5·626	5·275	4·800	4·240	3·541	2·855	2·282	1·745
61	5·594	5·416	5·072	4·603	4·051	3·364	2·694	2·145	1·635
60	5·380	5·207	4·867	4·406	3·863	3·186	2·535	2·012	1·525
59	5·166	5·000	4·665	4·210	3·674	3·009	2·377	1·878	...
58	4·953	4·793	4·464	4·015	3·487	2·833	2·219	1·744	...
57	4·740	4·585	4·262	3·819	3·300	2·655	2·061	1·607	...
56	4·527	4·378	4·059	3·623	3·113	2·478	1·901	1·468	...
55	4·323	4·177	3·860	3·435	2·943	2·311	1·750	1·327	...
54	4·129	3·986	3·677	3·257	2·773	2·154	1·609
53	3·948	3·813	3·508	3·096	2·620	2·011	1·485
52	3·778	3·649	3·345	2·944	2·474	1·877	1·369
51	3·615	3·492	3·192	2·799	2·335	1·749	1·260
50	3·460	3·342	3·045	2·660	2·204	1·628	1·156
49	3·311	3·200	2·906	2·528	2·079	1·515
48	3·170	3·064	2·774	2·404	1·961	1·408
47	3·036	2·937	2·650	2·287	1·851	1·308
46	2·910	2·815	2·533	2·177	1·747	1·216
45	2·789	2·700	2·422	2·073	1·651	1·132
44	2·677	2·592	2·318	1·977	1·560
43	2·570	2·491	2·222	1·887	1·478
42	2·470	2·397	2·132	1·803	1·402
41	2·376	2·308	2·049	1·727	1·334
40	2·283	2·220	1·964	1·654	1·276

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 5 per cent in possession and 6 per cent in reversion.

Female Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of Life Interest of £1 in Possession								
	14:730	14:416	13:959	13:359	12:624	11:772	10:815	9:729	8:538
	20	25	30	35	40	45	50	55	60
85	11:207	10:943	10:487	9:917	9:229	8:406	7:492	6:501	5:461
84	10:796	10:539	10:085	9:522	8:839	8:021	7:116	6:150	5:127
83	10:427	10:175	9:725	9:168	8:490	7:676	6:780	5:838	4:832
82	10:095	9:848	9:401	8:850	8:177	7:366	6:478	5:559	4:571
81	9:795	9:552	9:107	8:562	7:895	7:087	6:209	5:308	4:338
80	9:521	9:282	8:841	8:299	7:638	6:833	5:964	5:082	4:131
79	9:268	9:033	8:595	8:058	7:402	6:600	5:741	4:876	3:943
78	9:033	8:799	8:365	7:833	7:182	6:384	5:534	4:685	3:772
77	8:807	8:576	8:147	7:620	6:973	6:178	5:338	4:506	3:613
76	8:588	8:361	7:935	7:412	6:770	5:978	5:149	4:334	3:460
75	8:369	8:145	7:725	7:203	6:569	5:781	4:962	4:163	3:311
74	8:146	7:925	7:509	6:993	6:363	5:581	4:772	3:991	3:159
73	7:914	7:696	7:285	6:774	6:147	5:371	4:574	3:812	3:001
72	7:667	7:452	7:046	6:541	5:919	5:149	4:363	3:623	2:832
71	7:399	7:191	6:788	6:290	5:672	4:909	4:135	3:419	2:649
70	7:118	6:916	6:519	6:024	5:413	4:659	3:896	3:205	2:458
69	6:839	6:642	6:250	5:762	5:160	4:413	3:669	3:002	2:278
68	6:576	6:383	5:996	5:515	4:922	4:184	3:459	2:817	2:121
67	6:316	6:128	5:748	5:271	4:690	3:961	3:255	2:639	1:973
66	6:060	5:879	5:505	5:037	4:464	3:743	3:056	2:466	1:835
65	5:810	5:634	5:266	4:805	4:240	3:531	2:863	2:298	1:705
64	5:564	5:393	5:032	4:578	4:022	3:325	2:675	2:136	1:582
63	5:322	5:158	4:803	4:354	3:810	3:123	2:492	1:980	1:465
62	5:085	4:926	4:579	4:137	3:602	2:927	2:315	1:828	1:354
61	4:854	4:700	4:360	3:923	3:400	2:737	2:144	1:681	1:246
60	4:627	4:480	4:145	3:716	3:202	2:552	1:978	1:540	1:141
59	4:405	4:264	3:935	3:514	3:010	2:373	1:818	1:402	...
58	4:188	4:053	3:730	3:316	2:822	2:198	1:662	1:269	...
57	3:977	3:848	3:531	3:125	2:640	2:029	1:513	1:140	...
56	3:771	3:647	3:335	2:938	2:464	1:866	1:369	1:015	...
55	3:570	3:453	3:146	2:760	2:292	1:709	1:231	895	...
54	3:379	3:269	2:967	2:588	2:130	1:561	1:107
53	3:202	3:098	2:800	2:430	1:982	1:426	991
52	3:035	2:937	2:647	2:285	1:847	1:305	885
51	2:880	2:788	2:505	2:151	1:724	1:196	788
50	2:735	2:648	2:372	2:027	1:612	1:096	699
49	2:598	2:517	2:249	1:912	1:511	1:006
48	2:471	2:393	2:134	1:805	1:417	926
47	2:349	2:277	2:027	1:705	1:331	852
46	2:234	2:166	1:925	1:612	1:251	784
45	2:121	2:062	1:828	1:523	1:175	720
44	2:018	1:960	1:736	1:438	1:104
43	1:914	1:862	1:645	1:356	1:035
42	1:813	1:766	1:557	1:275	967
41	1:713	1:671	1:471	1:195	901
40	1:614	1:578	1:392	1:116	831

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 6 per-cent throughout.

Male Life Tenant.

Age of Life Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	12·881	12·592	12·219	11·752	11·173	10·494	9·721	8·827	7·827
	20	25	30	35	40	45	50	55	60
85	9·975	9·698	9·337	8·881	8·322	7·646	6·885	6·051	5·137
84	9·641	9·366	9·009	8·554	7·999	7·325	6·566	5·745	4·853
83	9·344	9·072	8·718	8·265	7·712	7·040	6·285	5·476	4·603
82	9·080	8·810	8·459	8·007	7·457	6·788	6·035	5·240	4·383
81	8·844	8·575	8·227	7·778	7·230	6·562	5·814	5·031	4·189
80	8·631	8·364	8·018	7·571	7·026	6·359	5·616	4·846	4·016
79	8·436	8·171	7·827	7·383	6·840	6·174	5·436	4·679	3·861
78	8·254	7·991	7·649	7·208	6·667	6·002	5·270	4·526	3·719
77	8·081	7·820	7·479	7·042	6·502	5·839	5·113	4·383	3·587
76	7·912	7·653	7·314	6·880	6·342	5·680	4·961	4·243	3·460
75	7·742	7·484	7·149	6·715	6·182	5·523	4·808	4·103	3·334
74	7·566	7·310	6·977	6·547	6·016	5·359	4·653	3·959	3·205
73	7·378	7·124	6·795	6·368	5·840	5·185	4·486	3·806	3·070
72	7·176	6·924	6·598	6·174	5·650	4·998	4·305	3·638	2·923
71	6·953	6·703	6·382	5·960	5·440	4·792	4·106	3·452	2·762
70	6·717	6·471	6·154	5·729	5·218	4·575	3·885	3·251	2·591
69	6·497	6·253	5·940	5·517	5·013	4·371	3·698	3·067	2·434
68	6·291	6·049	5·741	5·320	4·822	4·184	3·519	2·903	2·295
67	6·089	5·850	5·546	5·128	4·635	4·002	3·347	2·748	2·165
66	5·891	5·655	5·354	4·941	4·453	3·825	3·181	2·600	2·041
65	5·695	5·463	5·166	4·757	4·274	3·653	3·021	2·460	1·924
64	5·502	5·274	4·981	4·576	4·098	3·484	2·866	2·325	1·812
63	5·311	5·087	4·799	4·399	3·924	3·319	2·714	2·196	1·704
62	5·122	4·903	4·618	4·223	3·752	3·156	2·566	2·070	1·609
61	4·935	4·720	4·440	4·049	3·585	2·998	2·421	1·946	1·499
60	4·749	4·538	4·261	3·876	3·418	2·839	2·278	1·825	1·398
59	4·563	4·357	4·084	3·704	3·251	2·681	2·136	1·704	...
58	4·378	4·177	3·908	3·532	3·085	2·524	1·994	1·582	...
57	4·193	3·996	3·731	3·360	2·920	2·366	1·852	1·458	...
56	4·007	3·815	3·554	3·187	2·754	2·208	1·709	1·332	...
55	3·830	3·640	3·379	3·022	2·604	2·059	1·573	1·204	...
54	3·661	3·473	3·219	2·865	2·453	1·919	1·446
53	3·504	3·323	3·071	2·724	2·318	1·792	1·335
52	3·356	3·180	2·929	2·590	2·189	1·672	1·230
51	3·214	3·043	2·795	2·462	2·066	1·558	1·132
50	3·079	2·912	2·666	2·342	1·950	1·450	1·039
49	2·950	2·788	2·544	2·224	1·839	1·349
48	2·827	2·670	2·429	2·115	1·735	1·254
47	2·711	2·559	2·320	2·012	1·637	1·165
46	2·601	2·453	2·218	1·915	1·545	1·083
45	2·496	2·352	2·121	1·824	1·460	1·008
44	2·398	2·258	2·030	1·739	1·380
43	2·305	2·170	1·946	1·660	1·307
42	2·218	2·088	1·867	1·586	1·240
41	2·137	2·011	1·794	1·519	1·180
40	2·056	1·934	1·720	1·455	1·128

TABLE IV—(continued).

Present Value of a Reversionary Life Interest of £1.—Interest 6 per-cent throughout.

Female Life Tenant.

Age of Life-Tenant	The Age of the Reversioner is at the head of each Column, with the corresponding Value of a Life Interest of £1 in Possession								
	12:881	12:592	12:219	11:752	11:173	10:494	9:721	8:827	7:827
	20	25	30	35	40	45	50	55	60
85	9:817	9:538	9:180	8:724	8:168	7:493	6:734	5:898	5:006
84	9:460	9:186	8:828	8:377	7:823	7:150	6:396	5:580	4:700
83	9:139	8:869	8:513	8:065	7:511	6:842	6:091	5:297	4:430
82	8:850	8:584	8:229	7:785	7:237	6:566	5:823	5:044	4:190
81	8:589	8:326	7:972	7:532	6:987	6:317	5:581	4:816	3:977
80	8:351	8:090	7:739	7:301	6:760	6:091	5:361	4:611	3:787
79	8:131	7:873	7:521	7:080	6:551	5:883	5:160	4:424	3:615
78	7:926	7:669	7:323	6:891	6:356	5:690	4:974	4:251	3:458
77	7:730	7:475	7:132	6:703	6:171	5:507	4:798	4:088	3:312
76	7:539	7:287	6:946	6:520	5:991	5:329	4:628	3:932	3:172
75	7:349	7:099	6:762	6:337	5:813	5:153	4:460	3:777	3:035
74	7:155	6:907	6:573	6:152	5:631	4:971	4:280	3:621	2:896
73	6:953	6:708	6:377	5:959	5:440	4:787	4:111	3:459	2:751
72	6:738	6:495	6:168	5:754	5:238	4:589	3:922	3:287	2:596
71	6:505	6:267	5:942	5:533	5:019	4:375	3:717	3:102	2:428
70	6:261	6:028	5:707	5:299	4:990	4:352	3:592	2:908	2:253
69	6:018	5:789	5:471	5:069	4:566	3:933	3:298	2:724	2:088
68	5:789	5:563	5:249	4:852	4:356	3:729	3:109	2:556	1:944
67	5:563	5:341	5:032	4:640	4:150	3:530	2:926	2:394	1:809
66	5:341	5:124	4:819	4:431	3:949	3:336	2:747	2:237	1:682
65	5:123	4:910	4:610	4:227	3:752	3:147	2:573	2:085	1:563
64	4:909	4:700	4:405	4:027	3:559	2:963	2:404	1:938	1:450
63	4:699	4:495	4:205	3:830	3:371	2:783	2:240	1:796	1:343
62	4:493	4:293	4:009	3:639	3:187	2:608	2:081	1:658	1:241
61	4:292	4:096	3:817	3:451	3:008	2:439	1:927	1:525	1:142
60	4:094	3:904	3:629	3:269	2:833	2:274	1:778	1:397	1:046
59	3:901	3:716	3:445	3:091	2:663	2:114	1:634	1:272	...
58	3:713	3:532	3:266	2:917	2:497	1:958	1:494	1:151	...
57	3:529	3:353	3:091	2:749	2:336	1:808	1:360	1:034	...
56	3:350	3:178	2:920	2:585	2:180	1:662	1:230	.921	...
55	3:175	3:009	2:754	2:428	2:028	1:522	1:106	.812	...
54	3:009	2:848	2:597	2:277	1:884	1:390	.995
53	2:855	2:699	2:452	2:138	1:753	1:270	.891
52	2:710	2:559	2:317	2:010	1:634	1:162	.795
51	2:575	2:429	2:193	1:892	1:525	1:064	.708
50	2:449	2:307	2:077	1:783	1:426	.976	.628
49	2:330	2:193	1:969	1:682	1:336	.896
48	2:219	2:085	1:860	1:588	1:253	.824
47	2:113	1:984	1:775	1:500	1:177	.758
46	2:013	1:887	1:686	1:418	1:106	.698
45	1:917	1:796	1:601	1:340	1:030	.641
44	1:825	1:707	1:520	1:265	.976
43	1:735	1:622	1:441	1:193	.915
42	1:647	1:538	1:364	1:122	.855
41	1:560	1:455	1:288	1:051	.796
40	1:474	1:374	1:219	.982	.734

TABLE V.

Showing the Difference in the Number of Years' Purchase of a Reversionary Life Annuity, when Power of Re-purchase over a Period of 5 Years is allowed (at 5 per-cent Compound Interest), and the full Assurance required is set up at the outset.

Male Life Tenant.

Age of Life-Tenant	Equal Ages	DIFFERENCE OF AGE BETWEEN THE LIFE-TENANT AND THE REVERSIONER											
		5 Years	10 Years	15 Years	20 Years	25 Years	30 Years	35 Years	40 Years	45 Years	50 Years	55 Years	60 Years
Interest 5 per-cent throughout													
80	2489	2891	2657	2505	2488	2364	2301	2318	2318	2283	2277	2359	2244
75	2496	2755	2624	2619	2490	2428	2406	2403	2430	2423	2400	2386	...
70	2473	2297	2311	2431	2451	2407	2428	2463	2454	2421	2404
65	2453	2160	1907	1826	1881	1894	1814	1813	1767	1751
60	2457	1756	1659	1703	1712	1626	1616	1561	1436
55	1528	1411	1519	1406	1402	1389	1329	1299
50	1352	1404	1417	1347	1337	1285	1246
45	1354	1410	1325	1295	1257	1203
40	1408	1322	1276	1242	1176
Interest 6 per-cent throughout													
80	2851	2716	2452	2268	2211	2056	1962	1918	1917	1862	1839	1805	1784
75	2821	2554	2394	2350	2191	2008	2106	2082	2021	1906	1959	1910	...
70	2290	2102	2078	1873	1757	1791	1773	1697	1672	1626	1604
65	1970	1935	1665	1561	1593	1572	1480	1456	1398	1376
60	1864	1552	1438	1456	1438	1338	1313	1247	1220
55	1343	1230	1258	1262	1155	1121	1058	1026
50	1183	1201	1219	1111	1077	1021	978
45	1178	1206	1107	1067	1020	966
40	1221	1122	1070	1029	964

TABLE V—(continued).

Showing the Difference in the Number of Years' Purchase of a Reversionary Life Annuity, when Power of Re-purchase over a Period of 5 Years is allowed (at 5 per-cent Compound Interest), and the full Assurance required is set up at the outset.

Female Life Tenant.

Age of Life-Tenant	Equal Ages	DIFFERENCE OF AGE BETWEEN THE LIFE-TENANT AND THE REVERSIONER											
		5 Years	10 Years	15 Years	20 Years	25 Years	30 Years	35 Years	40 Years	45 Years	50 Years	55 Years	60 Years
Interest 5 per-cent throughout													
80	2-9-46	2-8-07	2-5-84	2-3-33	2-2-51	2-2-19	2-1-52	2-1-51	2-1-54	2-1-17	2-1-07	2-0-80	2-0-74
75	2-9-02	2-6-70	2-4-51	2-4-07	2-3-27	2-2-47	2-2-82	2-2-05	2-2-41	2-2-30	2-2-14	2-1-91	...
70	2-3-98	2-1-63	2-1-70	2-0-09	1-8-99	1-9-36	1-9-58	1-8-99	1-8-81	1-8-60	1-8-35
65	2-0-17	1-9-00	1-7-74	1-6-13	1-6-42	1-6-55	1-5-86	1-5-53	1-5-22	1-4-81
60	1-9-04	1-5-91	1-4-27	1-4-60	1-4-93	1-4-00	1-3-67	1-3-29	1-2-82
55	1-3-86	1-2-61	1-2-98	1-3-11	1-2-47	1-2-10	1-1-68	1-1-11
50	1-2-23	1-2-52	1-3-01	1-2-04	1-1-59	1-1-19	1-0-57
45	1-2-76	1-3-25	1-2-08	1-1-63	1-1-10	9-61
40	1-2-74	1-4-41	1-1-16	1-0-49	9-88
Interest 6 per-cent throughout													
80	2-8-13	2-6-35	2-3-86	2-1-66	2-0-84	1-9-23	1-8-27	1-7-08	1-7-71	1-7-13	1-6-83	1-6-54	1-6-29
75	2-7-31	2-4-75	2-2-32	2-2-11	2-0-41	1-9-34	1-9-36	1-9-23	1-8-52	1-8-20	1-7-92	1-7-61	...
70	2-2-24	1-9-79	1-9-53	1-7-65	1-6-30	1-6-14	1-6-35	1-5-36	1-5-21	1-4-86	1-4-50
65	1-8-19	1-7-90	1-5-54	1-3-82	1-3-87	1-3-81	1-2-85	1-2-37	1-1-95	1-1-51
60	1-7-26	1-4-07	1-2-32	1-2-12	1-2-50	1-2-41	1-0-96	1-0-19	9-98
55	1-2-27	1-0-99	1-1-13	1-1-29	1-0-20	9-72	9-23	8-67
50	1-0-79	1-0-93	1-1-16	1-0-05	9-54	9-06	8-13
45	1-4-33	1-1-57	1-0-34	9-83	9-26	8-61
40	1-1-14	9-79	9-18	8-77	8-18

TABLE VI.
Complete Single-Life Annuity-Values.

Male Life.

Age	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent
40	14·657	13·818	13·054	12·362	11·740
41	14·478	13·662	12·916	12·242	11·635
42	14·295	13·498	12·776	12·120	11·525
43	14·110	13·333	12·630	11·991	11·410
44	13·923	13·171	12·484	11·858	11·290
45	13·730	13·000	12·332	11·724	11·167
46	13·533	12·824	12·176	11·584	11·042
47	13·330	12·644	12·016	11·440	10·915
48	13·120	12·458	11·852	11·295	10·787
49	12·900	12·262	11·680	11·145	10·655
50	12·678	12·062	11·502	10·987	10·518
51	12·455	11·862	11·320	10·820	10·362
52	12·225	11·656	11·130	10·645	10·202
53	11·983	11·438	10·932	10·467	10·042
54	11·730	11·209	10·728	10·285	9·880
55	11·478	10·978	10·516	10·089	9·690
56	11·223	10·744	10·300	9·885	9·502
57	10·938	10·481	10·070	9·675	9·305
58	10·650	10·222	9·832	9·453	9·105
59	10·363	9·955	9·584	9·220	8·892
60	10·075	9·691	9·328	8·989	8·673
61	9·788	9·422	9·078	8·756	8·455
62	9·498	9·153	8·828	8·522	8·237
63	9·210	8·884	8·578	8·285	8·015
64	8·925	8·618	8·326	8·049	7·790
65	8·640	8·351	8·074	7·820	7·568
66	8·358	8·084	7·822	7·589	7·350
67	8·075	7·820	7·576	7·355	7·132
68	7·795	7·558	7·332	7·116	6·913
69	7·515	7·293	7·084	6·880	6·690
70	7·238	7·031	6·834	6·645	6·465
71	6·960	6·771	6·586	6·413	6·238
72	6·683	6·509	6·340	6·180	6·013
73	6·430	6·262	6·098	5·951	5·790
74	6·175	6·018	5·860	5·724	5·570
75	5·920	5·780	5·634	5·504	5·365
76	5·675	5·544	5·414	5·287	5·165
77	5·438	5·322	5·200	5·080	4·968
78	5·215	5·102	4·990	4·880	4·773
79	4·993	4·889	4·786	4·685	4·590
80	4·773	4·678	4·586	4·498	4·412
81	4·282	4·202	4·124	4·048	3·976
82	3·955	3·885	3·816	3·750	3·686
83	3·657	3·596	3·535	3·476	3·420
84	3·356	3·301	3·248	3·196	3·147
85	3·174	3·125	3·077	3·030	2·985

TABLE VI—(continued).

*Present Value of a Complete Single-Life Annuity-Value.***Female Life.**

Age	4 per-cent	4½ per-cent	5 per-cent	5½ per-cent	6 per-cent
40	16·080	15·098	14·206	13·396	12·663
41	15·888	14·931	14·064	13·276	12·560
42	15·695	14·762	13·918	13·149	12·448
43	15·493	14·587	13·766	13·016	12·332
44	15·288	14·409	13·608	12·876	12·208
45	15·070	14·220	13·442	12·735	12·078
46	14·845	14·022	13·270	12·581	11·942
47	14·615	13·818	13·090	12·418	11·782
48	14·380	13·609	12·900	12·247	11·648
49	14·135	13·393	12·708	12·075	11·493
50	13·888	13·169	12·508	11·898	11·333
51	13·635	12·944	12·300	11·729	11·170
52	13·378	12·716	12·104	11·540	11·002
53	13·115	12·480	11·890	11·345	10·832
54	12·845	12·236	11·670	11·144	10·658
55	12·568	11·988	11·444	10·940	10·475
56	12·283	11·727	11·210	10·720	10·282
57	11·990	11·460	10·966	10·507	10·082
58	11·693	11·188	10·718	10·277	9·868
59	11·393	10·911	10·464	10·047	9·648
60	11·088	10·636	10·210	9·813	9·422
61	10·780	10·356	9·956	9·578	9·199
62	10·470	10·069	9·692	9·338	8·975
63	10·155	9·775	9·418	9·084	8·750
64	9·835	9·478	9·138	8·816	8·520
65	9·508	9·173	8·852	8·549	8·267
66	9·175	8·860	8·562	8·276	8·008
67	8·840	8·547	8·266	8·000	7·748
68	8·508	8·233	7·972	7·722	7·487
69	8·180	7·925	7·680	7·447	7·223
70	7·863	7·622	7·394	7·175	6·960
71	7·558	7·338	7·124	6·916	6·713
72	7·263	7·060	6·860	6·667	6·483
73	6·975	6·787	6·602	6·425	6·255
74	6·690	6·516	6·348	6·187	6·027
75	6·413	6·251	6·096	5·949	5·805
76	6·143	5·993	5·850	5·713	5·582
77	5·878	5·740	5·608	5·482	5·360
78	5·620	5·493	5·372	5·255	5·140
79	5·368	5·251	5·142	5·035	4·927
80	5·125	5·020	4·916	4·815	4·720
81	4·869	4·776	4·687	4·402	4·320
82	4·604	4·523	4·446	4·070	3·998
83	4·340	4·271	4·203	3·738	3·675
84	4·077	4·017	3·958	3·401	3·347
85	3·857	3·803	3·751	3·200	3·151

TABLE VII.

Complete Joint-Life Annuity-Values.—Interest 4 per-cent.

Male Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	3.099	3.091	3.071	3.063	3.040	3.033	3.022	2.960	2.843
84	3.471	3.460	3.437	3.426	3.398	3.390	3.376	3.300	3.159
83	3.803	3.790	3.764	3.751	3.718	3.708	3.690	3.600	3.438
82	4.102	4.086	4.057	4.041	4.005	3.992	3.971	3.867	3.684
81	4.371	4.353	4.322	4.303	4.263	4.248	4.222	4.104	3.902
80	4.617	4.596	4.562	4.541	4.496	4.481	4.449	4.317	4.097
79	4.844	4.821	4.784	4.760	4.712	4.695	4.657	4.511	4.273
78	5.058	5.033	4.992	4.966	4.914	4.896	4.851	4.690	4.435
77	5.263	5.236	5.193	5.164	5.109	5.088	5.035	4.860	4.587
76	5.466	5.436	5.389	5.358	5.299	5.276	5.216	5.025	4.734
75	5.671	5.638	5.588	5.554	5.491	5.465	5.397	5.191	4.881
74	5.883	5.848	5.794	5.757	5.690	5.661	5.584	5.362	5.032
73	6.107	6.069	6.012	5.971	5.901	5.869	5.782	5.543	5.192
72	6.349	6.309	6.247	6.203	6.128	6.092	5.995	5.739	5.366
71	6.614	6.571	6.505	6.457	6.377	6.337	6.229	5.955	5.558
70	6.899	6.849	6.780	6.730	6.643	6.597	6.481	6.186	5.761
69	7.164	7.110	7.036	6.983	6.891	6.841	6.715	6.399	5.950
68	7.418	7.360	7.282	7.225	7.126	7.070	6.933	6.595	6.119
67	7.671	7.609	7.526	7.464	7.360	7.297	7.145	6.783	6.280
66	7.924	7.857	7.769	7.702	7.591	7.520	7.352	6.965	6.433
65	8.176	8.104	8.011	7.939	7.820	7.740	7.555	7.141	6.580
64	8.428	8.350	8.252	8.174	8.047	7.957	7.754	7.312	6.721
63	8.679	8.596	8.492	8.408	8.273	8.172	7.948	7.479	6.857
62	8.930	8.841	8.731	8.641	8.497	8.385	8.140	7.642	6.989
61	9.180	9.085	8.970	8.873	8.719	8.595	8.329	7.802	7.118
60	9.429	9.329	9.207	9.103	8.940	8.803	8.516	7.960	7.244
59	9.678	9.572	9.444	9.333	9.160	9.010	8.702	8.116	...
58	9.926	9.815	9.680	9.562	9.378	9.215	8.886	8.272	...
57	10.174	10.057	9.915	9.790	9.596	9.419	9.070	8.427	...
56	10.421	10.298	10.149	10.018	9.812	9.622	9.253	8.582	...
55	10.658	10.534	10.373	10.234	10.023	9.813	9.431	8.741	...
54	10.889	10.754	10.592	10.445	10.217	9.999	9.593
53	11.102	10.962	10.793	10.638	10.398	10.167	9.741
52	11.309	11.163	10.987	10.823	10.573	10.328	9.882
51	11.508	11.356	11.174	11.002	10.740	10.481	10.014
50	11.699	11.542	11.353	11.173	10.899	10.627	10.139
49	11.885	11.721	11.525	11.337	11.051	10.765
48	12.063	11.894	11.691	11.494	11.197	10.896
47	12.236	12.059	11.850	11.644	11.334	11.019
46	12.401	12.218	12.001	11.787	11.465	11.135
45	12.559	12.371	12.146	11.922	11.588	11.243
44	12.712	12.517	12.284	12.051	11.704
43	12.857	12.657	12.416	12.173	11.814
42	12.998	12.791	12.542	12.289	11.916
41	13.132	12.919	12.661	12.398	12.011
40	13.266	13.047	12.779	12.504	12.097

TABLE VII —(continued).

Complete Joint-Life Annuity-Values.—Interest 4 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	3·276	3·265	3·245	3·236	3·209	3·202	3·190	3·123	2·996
84	3·677	3·661	3·639	3·627	3·595	3·586	3·569	3·484	3·330
83	4·039	4·023	3·995	3·980	3·941	3·933	3·910	3·807	3·628
82	4·368	4·350	4·317	4·300	4·258	4·245	4·217	4·096	3·894
81	4·668	4·647	4·611	4·590	4·545	4·530	4·495	4·357	4·133
80	4·944	4·920	4·881	4·857	4·807	4·790	4·748	4·594	4·348
79	5·201	5·171	5·132	5·105	5·050	5·031	4·981	4·811	4·544
78	5·444	5·411	5·368	5·339	5·279	5·258	5·200	5·013	4·725
77	5·677	5·645	5·596	5·563	5·499	5·475	5·408	5·205	4·896
76	5·907	5·872	5·818	4·783	5·714	5·686	5·610	5·390	5·060
75	6·137	6·098	6·042	6·002	5·929	5·898	5·812	5·574	5·222
74	6·373	6·330	6·270	6·227	6·149	6·114	6·017	5·761	5·386
73	6·619	6·573	6·508	6·462	6·379	6·339	6·231	5·955	5·556
72	6·881	6·831	6·761	6·712	6·623	6·578	6·459	6·161	5·736
71	7·163	7·109	7·033	6·981	6·886	6·835	6·704	6·383	5·931
70	7·470	7·411	7·331	7·273	7·170	7·116	6·972	6·622	6·140
69	7·768	7·703	7·617	7·553	7·446	7·380	7·218	6·842	6·328
68	8·062	7·992	7·901	7·830	7·715	7·638	7·457	7·051	6·503
67	8·354	8·278	8·180	8·103	7·979	7·892	7·691	7·254	6·671
66	8·642	8·560	8·456	8·372	8·239	8·141	7·919	7·452	6·834
65	8·927	8·839	8·729	8·638	8·495	8·384	8·141	7·643	6·991
64	9·209	9·115	8·998	8·900	8·747	8·622	8·357	7·829	7·142
63	9·487	9·387	9·262	9·158	8·994	8·855	8·568	8·009	7·288
62	9·761	9·655	9·523	9·411	9·236	9·083	8·773	8·183	7·428
61	10·031	9·919	9·780	9·660	9·473	9·305	8·972	8·351	7·563
60	10·297	10·178	10·032	9·904	9·705	9·522	9·166	8·514	7·693
59	10·559	10·431	10·280	10·144	9·932	9·733	9·354	8·671	...
58	10·817	10·685	10·523	10·379	10·154	9·939	9·536	8·823	...
57	11·070	10·931	10·761	10·609	10·371	10·138	9·712	8·970	...
56	11·319	11·173	10·995	10·833	10·582	10·333	9·883	9·110	...
55	11·561	11·409	11·224	11·052	10·786	10·523	10·050	9·246	...
54	11·793	11·634	11·442	11·260	10·982	10·703	10·199
53	12·016	11·848	11·649	11·457	11·165	10·869	10·338
52	12·229	12·054	11·845	11·642	11·335	11·021	10·468
51	12·434	12·250	12·032	11·817	11·495	11·161	10·588
50	12·630	12·438	12·210	11·982	11·643	11·290	10·697
49	12·818	12·618	12·379	12·138	11·782	11·409
48	12·999	12·790	12·541	12·285	11·912	11·518
47	13·173	12·955	12·694	12·425	12·034	11·620
46	13·340	13·113	12·841	12·558	12·150	11·715
45	13·500	13·265	12·981	12·684	12·258	11·804
44	13·655	13·411	13·115	12·805	12·361
43	13·804	13·554	13·244	12·921	12·460
42	13·948	13·686	13·368	13·032	12·555
41	14·088	13·816	13·488	13·140	12·648
40	14·222	13·943	13·603	13·244	12·737

TABLE VII—(continued).

Complete Joint-Life Annuity-Values.—Interest 5 per-cent.

Male Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	3.000	2.988	2.976	2.963	2.943	2.940	2.925	2.861	2.771
84	3.352	3.338	3.322	3.308	3.283	3.278	3.261	3.183	3.069
83	3.666	3.649	3.629	3.614	3.585	3.579	3.559	3.467	3.333
82	3.946	3.927	3.904	3.887	3.855	3.847	3.824	3.717	3.565
81	4.198	4.176	4.151	4.132	4.098	4.087	4.059	3.939	3.771
80	4.427	4.403	4.375	4.353	4.316	4.304	4.271	4.137	3.955
79	4.638	4.610	4.581	4.556	4.517	4.503	4.464	4.316	4.121
78	4.835	4.804	4.773	4.745	4.704	4.688	4.643	4.481	4.272
77	5.024	4.990	4.957	4.926	4.883	4.864	4.812	4.636	4.415
76	5.210	5.172	5.137	5.104	5.058	5.036	4.978	4.787	4.552
75	5.391	5.358	5.318	5.282	5.233	5.210	5.143	4.939	4.689
74	5.588	5.547	5.506	5.467	5.415	5.388	5.314	5.096	4.828
73	5.794	5.750	5.704	5.663	5.607	5.577	5.495	5.263	4.975
72	6.016	5.969	5.919	5.876	5.815	5.780	5.692	5.445	5.133
71	6.260	6.210	6.151	6.110	6.043	6.004	5.908	5.647	5.318
70	6.524	6.476	6.415	6.367	6.284	6.253	6.149	5.873	5.503
69	6.754	6.705	6.639	6.589	6.509	6.465	6.353	6.063	5.663
68	6.982	6.930	6.860	6.806	6.720	6.672	6.551	6.243	5.815
67	7.208	7.152	7.078	7.020	6.928	6.874	6.741	6.415	5.959
66	7.432	7.372	7.293	7.232	7.133	7.072	6.926	6.578	6.096
65	7.653	7.589	7.506	7.440	7.335	7.267	7.106	6.735	6.226
64	7.873	7.806	7.717	7.646	7.535	7.458	7.281	6.886	6.351
63	8.091	8.020	7.925	7.849	7.732	7.645	7.452	7.032	6.472
62	8.308	8.232	8.133	8.051	7.927	7.830	7.619	7.174	6.590
61	8.524	8.443	8.339	8.252	8.120	8.013	7.784	7.314	6.705
60	8.740	8.650	8.543	8.451	8.312	8.195	7.948	7.452	6.816
59	8.954	8.860	8.747	8.649	8.502	8.375	8.109	7.589	...
58	9.168	9.070	8.952	8.847	8.691	8.553	8.270	7.725	...
57	9.382	9.280	9.156	9.044	8.879	8.731	8.430	7.863	...
56	9.596	9.490	9.360	9.242	9.066	8.909	8.591	8.003	...
55	9.809	9.695	9.563	9.441	9.244	9.088	8.755	8.143	...
54	9.997	9.878	9.742	9.612	9.417	9.242	8.890
53	10.178	10.054	9.915	9.776	9.573	9.389	9.018
52	10.353	10.223	10.080	9.933	9.723	9.528	9.138
51	10.520	10.385	10.239	10.083	9.867	9.660	9.251
50	10.680	10.538	10.390	10.226	10.004	9.786	9.358
49	10.834	10.686	10.535	10.362	10.134	9.904
48	10.981	10.828	10.673	10.492	10.258	10.016
47	11.122	10.963	10.804	10.615	10.375	10.120
46	11.256	11.092	10.929	10.732	10.485	10.217
45	11.384	11.214	11.047	10.843	10.588	10.308
44	11.506	11.331	11.159	10.947	10.684
43	11.623	11.442	11.264	11.044	10.773
42	11.733	11.548	11.363	11.134	10.855
41	11.838	11.648	11.455	11.219	10.930
40	11.941	11.744	11.550	11.304	10.999

TABLE VII—(continued).

Complete Joint-Life Annuity-Values.—Interest 5 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	3.167	3.156	3.139	3.129	3.104	3.099	3.085	3.023	2.909
84	3.545	3.530	3.512	3.497	3.469	3.463	3.444	3.358	3.231
83	3.885	3.867	3.847	3.828	3.797	3.790	3.764	3.658	3.516
82	4.193	4.171	4.150	4.127	4.093	4.084	4.052	3.928	3.770
81	4.473	4.448	4.424	4.398	4.360	4.350	4.312	4.170	3.997
80	4.729	4.701	4.675	4.645	4.604	4.593	4.547	4.390	4.200
79	4.967	4.936	4.907	4.874	4.829	4.816	4.763	4.591	4.384
78	5.190	5.156	5.125	5.089	5.040	5.025	4.964	4.778	4.553
77	5.404	5.367	5.333	5.294	5.242	5.224	5.155	4.955	4.712
76	5.614	5.574	5.536	5.494	5.438	5.418	5.340	5.125	4.863
75	5.822	5.779	5.739	5.693	5.634	5.611	5.523	5.294	5.043
74	6.036	5.990	5.946	5.897	5.834	5.807	5.710	5.464	5.164
73	6.259	6.209	6.162	6.110	6.043	6.012	5.904	5.641	5.322
72	6.496	6.443	6.391	6.336	6.265	6.229	6.111	5.828	5.489
71	6.751	6.694	6.639	6.580	6.505	6.463	6.335	6.029	5.671
70	7.023	6.967	6.910	6.845	6.768	6.721	6.579	6.249	5.872
69	7.291	7.225	7.164	7.093	7.009	6.956	6.797	6.443	6.044
68	7.551	7.480	7.414	7.338	7.246	7.186	7.009	6.630	6.205
67	7.807	7.731	7.660	7.578	7.479	7.410	7.216	6.811	6.355
66	8.060	7.978	7.902	7.815	7.707	7.629	7.417	6.987	6.496
65	8.308	8.222	8.140	8.047	7.931	7.843	7.613	7.157	6.629
64	8.553	8.462	8.374	8.275	8.151	8.052	7.803	7.322	6.755
63	8.793	8.697	8.604	8.499	8.365	8.255	7.988	7.481	6.874
62	9.030	8.929	8.830	8.719	8.576	8.453	8.168	7.636	6.989
61	9.262	9.156	9.051	8.934	8.781	8.647	8.342	7.786	7.099
60	9.490	9.379	9.268	9.144	8.982	8.835	8.511	7.930	7.206
59	9.714	9.598	9.481	9.350	9.178	9.018	8.675	8.071	...
58	9.934	9.812	9.689	9.550	9.369	9.196	8.834	8.207	...
57	10.149	10.021	9.893	9.746	9.555	9.369	8.987	8.339	...
56	10.360	10.225	10.092	9.936	9.736	9.537	9.136	8.467	...
55	10.565	10.424	10.285	10.121	9.911	9.701	9.279	8.590	...
54	10.760	10.613	10.471	10.297	10.080	9.855	9.408
53	10.944	10.791	10.644	10.461	10.235	9.996	9.528
52	11.119	10.960	10.807	10.615	10.378	10.124	9.640
51	11.284	11.119	10.958	10.758	10.509	10.241	9.744
50	11.441	11.270	11.101	10.892	10.630	10.348	9.838
49	11.590	11.413	11.235	11.018	10.741	10.445
48	11.732	11.549	11.361	11.135	10.844	10.535
47	11.867	11.678	11.480	11.246	10.940	10.616
46	11.996	11.801	11.593	11.351	11.029	10.692
45	12.119	11.918	11.701	11.451	11.113	10.762
44	12.238	12.031	11.805	11.546	11.192
43	12.354	12.140	11.905	11.637	11.268
42	12.466	12.246	12.002	11.725	11.342
41	12.575	12.349	12.097	11.810	11.414
40	12.680	12.449	12.186	11.896	11.488

TABLE VII—(continued).

Complete Joint-Life Annuity-Values.—Interest 6 per-cent.

Male Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	2.906	2.894	2.882	2.871	2.851	2.848	2.836	2.776	2.690
84	3.340	3.226	3.210	3.198	3.174	3.169	3.155	3.082	2.974
83	3.537	3.520	3.501	3.487	3.461	3.454	3.436	3.351	3.224
82	3.801	3.782	3.760	3.745	3.716	3.706	3.686	3.587	3.444
81	4.037	4.017	3.992	3.974	3.943	3.932	3.907	3.796	3.638
80	4.250	4.228	4.201	4.181	4.147	4.135	4.105	3.981	3.811
79	4.445	4.421	4.392	4.369	4.333	4.320	4.285	4.148	3.966
78	4.627	4.601	4.570	4.544	4.506	4.492	4.451	4.301	4.108
77	4.800	4.772	4.740	4.710	4.671	4.655	4.608	4.444	4.240
76	4.969	4.939	4.905	4.872	4.831	4.814	4.760	4.584	4.367
75	5.139	5.108	5.070	5.037	4.991	4.971	3.913	4.724	4.493
74	5.315	5.282	5.242	5.205	5.157	5.135	5.068	4.868	4.622
73	5.503	5.468	5.424	5.384	5.333	5.309	5.235	5.021	4.757
72	5.705	5.668	5.621	5.578	5.523	5.496	5.416	5.189	4.904
71	5.928	5.889	5.837	5.792	5.733	5.702	5.615	5.375	5.065
70	6.164	6.121	6.065	6.023	5.955	5.919	5.836	5.576	5.236
69	6.384	6.339	6.279	6.235	6.160	6.123	6.023	5.760	5.393
68	6.590	6.543	6.478	6.432	6.351	6.310	6.202	5.924	5.532
67	6.792	6.742	6.673	6.624	6.538	6.492	6.374	6.079	5.662
66	6.990	6.937	6.865	6.811	6.720	6.669	6.540	6.227	5.786
65	6.186	7.129	7.053	6.995	6.899	6.841	6.700	6.367	5.903
64	7.379	7.318	7.238	7.176	7.075	7.010	6.855	6.502	6.015
63	7.570	7.505	7.420	7.353	7.249	7.175	7.007	6.631	6.123
62	7.759	7.689	7.601	7.529	7.421	7.338	7.155	6.757	6.227
61	7.946	7.872	7.779	7.703	7.588	7.496	7.300	6.881	6.328
60	8.132	8.054	7.958	7.876	7.755	7.655	7.443	7.002	6.429
59	8.318	8.235	8.135	8.048	7.922	7.813	7.585	7.123	...
58	8.503	8.415	8.311	8.220	8.088	7.970	7.727	7.245	...
57	8.688	8.596	8.488	8.392	8.253	8.128	7.869	7.369	...
56	8.874	8.777	8.665	8.565	8.419	8.286	8.012	7.495	...
55	9.051	8.952	8.840	8.730	8.569	8.435	8.148	7.623	...
54	9.220	9.119	9.000	8.887	8.720	8.575	8.275
53	9.377	9.269	9.148	9.028	8.855	8.702	8.386
52	9.525	9.412	9.290	9.162	8.984	8.822	8.491
51	9.667	9.549	9.424	9.290	9.107	8.936	8.589
50	9.802	9.680	9.553	9.412	9.223	9.044	8.682
49	9.931	9.804	9.675	9.528	9.334	9.145
48	10.054	9.922	9.790	9.637	9.438	9.240
47	10.170	10.033	9.899	9.740	9.536	9.329
46	10.280	10.139	10.001	9.837	9.628	9.411
45	10.385	10.240	10.098	9.928	9.713	9.486
44	10.483	10.334	10.189	10.013	9.793
43	10.576	10.422	10.273	10.092	9.866
42	10.663	10.504	10.352	10.166	9.933
41	10.744	10.581	10.425	10.233	9.993
40	10.825	10.658	10.499	10.297	10.045

TABLE VII—(continued).

Complete Joint-Life Annuity-Values.—Interest 6 per-cent.

Female Life Tenant.

Age of Life Tenant	AGE OF THE REVERSIONER								
	20	25	30	35	40	45	50	55	60
85	3.064	3.054	3.039	3.028	3.005	3.001	2.987	2.929	2.821
84	3.121	3.106	3.091	3.075	3.050	3.044	3.025	3.047	3.127
83	3.742	3.723	3.706	3.687	3.659	3.652	3.627	3.530	3.397
82	4.031	4.008	3.990	3.967	3.936	3.928	3.898	3.783	3.637
81	4.292	4.266	4.247	4.220	4.186	4.177	4.140	4.011	3.850
80	4.530	4.502	4.480	4.451	4.413	4.403	4.360	4.216	4.040
79	4.750	4.719	4.695	4.663	4.622	4.611	4.561	4.403	4.212
78	4.955	4.923	4.896	4.861	4.817	4.804	4.747	4.576	4.369
77	5.151	5.117	5.087	5.049	5.002	4.987	4.923	4.739	4.515
76	5.342	5.305	5.273	5.232	5.182	5.165	5.093	4.895	4.655
75	5.532	5.493	5.457	5.415	5.360	5.341	5.261	5.050	4.792
74	5.726	5.685	5.646	5.600	5.542	5.520	5.432	5.206	4.931
73	5.928	5.884	5.842	5.793	5.733	5.707	5.610	5.368	5.076
72	6.143	6.097	6.051	5.998	5.935	5.905	5.799	5.540	5.231
71	6.376	6.325	6.277	6.219	6.154	6.119	6.004	5.725	5.399
70	6.620	6.564	6.512	6.453	6.383	6.342	6.219	5.919	5.574
69	6.863	6.803	6.748	6.683	6.607	6.561	6.423	6.103	5.739
68	7.092	7.029	6.970	6.900	6.817	6.765	6.612	6.271	5.883
67	7.318	7.251	7.187	7.112	7.023	6.964	6.795	6.433	6.018
66	7.540	7.468	7.400	7.321	7.224	7.158	6.974	6.590	6.145
65	7.758	7.682	7.609	7.525	7.421	7.347	7.148	6.742	6.264
64	7.972	7.892	7.814	7.725	7.614	7.531	7.317	6.889	6.377
63	8.182	8.097	8.014	7.922	7.802	7.711	7.481	7.031	6.484
62	8.388	8.299	8.210	8.113	7.986	7.886	7.640	7.169	6.586
61	8.589	8.496	8.402	8.301	8.165	8.055	7.794	7.302	6.685
60	8.787	8.688	8.590	8.483	8.340	8.220	7.943	7.430	6.781
59	8.980	8.876	8.774	8.661	8.510	8.380	8.087	7.555	...
58	9.168	9.060	8.953	8.835	8.676	8.536	8.227	7.676	...
57	9.352	9.239	9.128	9.003	8.837	8.686	8.361	7.793	...
56	9.531	9.414	9.299	9.167	8.993	8.832	8.491	7.906	...
55	9.706	9.583	9.465	9.324	9.145	8.972	8.615	8.015	...
54	9.872	9.744	9.622	9.475	9.289	9.104	8.726
53	10.026	9.893	9.767	9.614	9.420	9.224	8.830
52	10.171	10.033	9.902	9.742	9.539	9.332	8.926
51	10.306	10.163	10.026	9.860	9.648	9.430	9.013
50	10.432	10.285	10.142	9.969	9.747	9.518	9.093
49	10.551	10.399	10.250	10.070	9.837	9.598
48	10.662	10.507	10.350	10.164	9.920	9.670
47	10.768	10.608	10.444	10.252	9.996	9.736
46	10.868	10.705	10.533	10.334	10.067	9.796
45	10.964	10.796	10.618	10.412	10.134	9.853
44	11.056	10.885	10.699	10.487	10.197
43	11.146	10.970	10.778	10.559	10.258
42	11.234	11.054	10.855	10.630	10.318
41	11.321	11.137	10.931	10.701	10.377
40	11.407	11.218	11.000	10.770	10.439

TABLE VIII.

Present Value of a Reversionary Life Interest of £1, with Power of Re-purchase over a Period of 5 Years, at 5 per-cent Compound Interest. The full Assurance to be set up at the end of the Term of 5 Years.—Interest 5 per-cent in Possession, and 6 per-cent in Reversion.

Male Life Tenant.

Age of Life Tenant	20		25		30		35		40		45		50		55		60	
	Policy . 11-576 Option . 146		Policy . 11-100 Option . 141		Policy . 13-519 Option . 135		Policy . 12-803 Option . 128		Policy . 11-970 Option . 180		Policy . 11-039 Option . 221		Policy . 9-989 Option . 200		Policy . 8-818 Option . 220		Policy . 7-664 Option . 229	
	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ	Present Value	Δ
80	7-923	1-920	7-535	2-001	7-180	1-979	6-661	1-915	5-953	1-986	5-161	1-970	4-362	1-886	3-439	1-902	2-466	1-915
75	6-811	2-010	6-499	2-088	6-104	2-063	5-601	2-029	4-921	2-061	4-171	2-025	3-453	1-896	2-611	1-881	1-787	1-850
70	6-080	1-563	5-780	1-615	5-399	1-631	4-915	1-597	4-250	1-646	3-539	1-594	2-893	1-429	2-155	1-428	1-398	1-428
65	5-237	1-230	4-956	1-312	4-586	1-315	4-129	1-278	3-490	1-340	2-817	1-282	2-251	1-110	1-625	1-086	965	1-131
60	4-398	982	4-137	1-070	3-785	1-082	3-359	1-017	3-751	1-112	2-112	1-044	1-654	881	1-151	861	589	936
55	3-629	694	3-388	789	3-059	801	2-661	774	2-698	815	1-561	750	1-176	574	776	551
50	2-882	578	2-658	681	2-355	690	1-997	663	1-172	732	1-009	619	708	448
45	2-277	512	2-067	633	1-794	628	1-465	608	984	667	592	540
40	1-812	471	1-615	605	1-368	596	1-064	590	629	617

In the above table, Δ represents the difference between the present value tabulated above, and the corresponding value of a Reversionary Life Interest of £1, when no option of re-purchase is permitted.

At the head of each column is given:

- (1) The full policy to be set up at the end of 5 years.
- (2) The single premium for the option of obtaining this deferred policy at the ordinary rate of premium for an age 5 years older.

TABLE IX.

The Present Value of a Reversionary Life Interest of £1, Deferred during the Term of the Joint-Life Expectation.
Male Life Tenant.

Reversioner aged 20 next Birthday

Age of Life Tenant	Joint Life Expectation, t Years	Option per cent	Policy required at end of t Years	Single Premium for the Option	Term Premium per cent, Age x for t Years	Initial Policy required, namely, $L_{y+t} x : t$	Annual Premium payable for first t Years	Present Advance	L_{y+x}	Per cent of Advance to L_{y+x}
80	4	1.0	11.79	.118	.958	10.797	.1031	8.744	9.843	.89
70	8	.7	11.47	.101	.986	8.910	.0878	5.918	7.643	.77
60	12	.6	11.03	.084	1.028	7.330	.0755	3.861	5.380	.72
50	18	.6	13.28	.080	1.080	5.987	.0615	2.035	3.160	.59
40	22	.5	12.67	.063	1.118	3.960	.0444	1.087	2.283	.48

Reversioner aged 40 next Birthday

80	4	1.5	12.35	.185	1.125	8.382	.1198	6.022	7.930	.83
70	7	1.5	11.82	.177	1.186	6.207	.0925	4.017	5.896	.69
60	11	1.5	11.06	.166	1.626	4.415	.0718	2.280	3.863	.59

Reversioner aged 60 next Birthday

80	1	3.0	8.10	.213	3.990	4.418	.1764	3.050	1.381	.70
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TABLE IX—(continued).

The Present Value of a Reversionary Life Interest of £1, Deferred during the Term of the Joint-Life Expectation.
Female Life Tenant.

Reversioner aged 20 next Birthday									
Age of Life Tenant	Joint-Child Expectation, t Years	Option percent	Policy required at end of t Years	Single Premium for the Option	Term Premium percent, Age x for t Years	Initial Policy required, namely, $L_{y+t,x,t}$	Annual Premium payable for first t Years	Present Advance	Percent of Advance to $L_{y,x}$
80	5	1.0	14.71	.117	.966	10943	.0980	8.434	.89
70	10	.6	14.25	.086	1.013	8.844	.0844	5.262	.74
60	14	.6	13.79	.083	1.013	7.096	.0688	3.283	.71
50	20	.5	12.99	.065	1.100	5.113	.0554	1.712	.63
40	24	.5	12.35	.062	1.133	3.464	.0376	.830	.52
Reversioner aged 40 next Birthday									
80	5	1.5	12.18	.183	1.414	8.406	.1060	6.295	.83
70	9	1.5	11.34	.170	1.577	5.913	.0872	3.390	.63
60	13	1.5	10.65	.160	1.633	4.117	.0620	1.734	.56
Reversioner aged 60 next Birthday									
80	4	3.0	8.10	.213	3.990	4.261	.1420	2.879	.70

DISCUSSION.

The PRESIDENT (Mr. B. Newbatt) having called upon the referees, Mr. Manly and Mr. Rothery, to open the discussion.

Mr. H. W. MANLY said he wished to congratulate Mr. Nightingale for his courage in attacking a question which some present might consider had already been thoroughly threshed out. It should, however, be understood that Mr. Nightingale did not so much come before them to teach new methods as to supply what might be termed an "Actuaries' Ready Reckoner", so that when reversionary transactions were submitted to them, they might only have to open the pages of the *Journal* in order to quote a value straight off. But that would not necessarily be the result, for probably each of them would have a different opinion as to the manner in which many of those complicated interests should be calculated; but the tables would at least be useful as affording a check upon their own calculations. In recent years there had been a good deal of competition amongst the offices for reversionary business. Years ago, when they only had Mr. Jellicoe's formulas, and all followed the same or very similar rules, the prices quoted were so much alike that it was not deemed necessary to enquire at more than one or two offices. Recently it had become the custom to send a circular to a great number of offices asking for tenders, and he heard that those tenders varied considerably. Naturally, when the public found that there was a large difference between the prices quoted, the practice would extend and would undoubtedly result in a permanent diminution of the profit to be made out of such transactions. The author suggested that in dealing with a contingent reversion in order to cover the risk for " t years" after the death of the life tenant, instead of calculating the contingent premium as they had previously done, by Mr. Curtis Otter's formula, they should adopt the plan of adding t years to the expectation of y (the life tenant), and then finding the age $y - \delta$ corresponding to the increased expectation. He (Mr. Manly) did not like, as a rule, the function "expectation" being brought into their calculations at all. He would like to ask the author if he had compared the values produced by that method with the values produced by the strict formula. The author, in discussing the value of a reversionary life interest, following in the footsteps of Mr. Sunderland, suggested that they should take a temporary assurance for the curtate joint expectation of the life tenant and the reversioner, with an option of continuing the assurance at the end of the period, giving as an example a case where such expectation was about eighteen years. Eighteen years was a very long time for an option, and he could not quite see the reason for introducing the curtate expectation, or how they could introduce that value into calculations for single individuals, and he did not think it was scientific to do so. He would like to give expression to some views of his own with regard to the question of insurance in connection with contingent reversionary interests. Mr. Sutton once said that when these transactions were carried out, and offices were asked to re-assure portions of the risk, he thought that not only

should the assurances be offered, but a portion of the original transaction also, so that the re-assuring offices should not suffer all the loss, and the office effecting the assurances obtain all the profit in the event of early death. Some life offices had fattened upon the losses of others. He could not see why these policies as between office and office should not be considered in the form of an indemnity. To take an example, suppose £10,000 was advanced, and it was desired to effect assurances to the extent of £40,000, in order to cover the accumulations of interest and premiums; suppose those assurances were divided amongst 10 offices, then each office should be asked to assure £1,000 and the amount of an annuity of $(i+P)$ up to the end of the year in which the joint existence of the life tenant and reversioner failed. Such a policy would be an indemnity to the office for the loss it would incur by the death of the reversioner, and there would be no unpleasant feeling that one office had made excessive profits out of the losses of others. It would, in his opinion, be far preferable to assuring the life for a term of 18 years and undertaking to assure it for a larger amount at the end of the 18 years for some present small consideration.

Mr. H. J. ROTHERY said that the main feature of the author's tables was that they were based upon "complete" annuities. He approved of the principle, but could not help thinking that the formulas so produced were more complicated than those with which they had become familiar. No doubt most of them used Mr. Sprague's formulas, and he should hesitate to attack a complicated reversionary interest problem by some of the author's formulas. It was satisfactory to see that the author's comparison between the results obtained by his method and those obtained by the ordinary system, using the Carlisle Table, showed close agreement if the life tenant was of a fairly advanced age. The results for male life tenants were nearly identical, and, if the life was a female, they were also very close if the rule was followed of deducting a few years from the age. The author would admit that formulas and tables were only to be considered as tools to be employed. Some members seemed to fear that an improper use would be made of the tables if published, but he thought that persons looking at them in connection with the paper would be rather frightened of cutting their own fingers. The author spoke of principles to be borne in mind, one being that the "total outlay was to be recovered in any event." That was not necessarily a *principle*. In contingent reversions it did not seem necessary that the outlay should in each case be recovered. It seemed only to be *convenient* because these transactions were carried out by life assurance companies, and gave rise to insurance business, and thus provided a margin for expenses, &c. Speaking of life interests, it was important to state very clearly that the subject was being dealt with in a theoretical, and not entirely in a practical manner. There were many points that occurred in practice, and would be clear to gentlemen who had these transactions before them, that might be lost sight of, and the fact that they were not dealt with in the paper should be emphasized. The author stated that "a sufficient margin" should be reserved to give the vendor or borrower a permanent "interest in the estate, as an inducement to him to come forward

"from time to time to establish his existence." Unfortunately, even that margin was not sometimes sufficient in the case of a loan, because the life tenant might still *sell* his equity of redemption. In that case he had no beneficial interest whatever in the life estate, and serious difficulty might arise in trying to produce the life tenant. An important question arose in cases where a loan was granted on a life estate in landed property which was in course of development. In such a case the present income might not be sufficient to meet interest and premiums, but it might be clear that in a few years, if new leases were granted, the margin would be ample. A difficulty in such a case arose under the Settled Lands Act, which provided that a life tenant could, under no circumstances, divest himself of certain powers that he took under the settlement, one being the power to grant leases. Cases had arisen in practice where a life tenant had sold his equity of redemption, but, although he had not a single pennyworth of interest in the life estate, no new leases could be granted without him. It seemed, however, possible that as the life tenant was held under the Act to be a *trustee*, the Court might order him to grant leases if for the benefit of the estate by virtue of that office, but, of course, that involved legal process. As to the options for re-purchase he thought it would be undesirable to lengthen the term of years. They were rarely exercised except when a transaction was being transferred to another company, and were necessarily very expensive for the vendor.

Mr. G. H. RYAN said that although it might be necessary to take up a critical attitude towards some of the formulas and tables given by Mr. Nightingale, what was most noticeable was the spirit of zeal and thoroughness which pervaded the paper. With regard to the publication of the tables, it had been said that by such means they were spoiling the actuary's market. He could not take that view at all. It appeared to him that the author had not made the subject so simple or attractive as to lead outsiders to trespass upon this dangerous field of work. They would remember that that famous work, *Every Man his Own Lawyer*, was credited with having created a vast amount of legal work, and so it seemed to him not unlikely that these tables might, so far from diminishing the actuary's consultation work, rather increase it. In discussing the question generally, one of the most important points to bear in mind was the proper limits of the transactions—in other words, the determination of the maximum amount which ought to be undertaken on individual risks. In dealing with these matters practically, it had always appeared to him to be most difficult to fix the amount to which an office could properly and safely go, and he was inclined to think that this question did not, at any rate, receive adequate consideration. As to the rate of mortality to which life tenants were subject, the author assumed that it would be equivalent to that of Government annuities, and in the absence of actual information that assumption was a reasonable and fair one. What, however, he would like to submit was whether it would be feasible to approach the reversionary societies, when the Institute next took up the question of collecting new data—a matter which he felt sure would not escape the attention of the Council—and ask them to join in the work which the Institute was seeking to

do by contributing their experience. The French offices, when lately analyzing the rate of mortality to which assured lives were subject, also took up the experience of their companies regarding annuities. That might be a useful addendum to the Institute's work, as well as the special experience of the reversionary societies, if the latter were not considered outside their scope. Many of the author's tables would, in his opinion, be of considerable use in the future. It was most desirable that they should have such a means of verifying their actual calculations in cases that came before them, and also of giving a rough estimate of the amount of loan which certain transactions would appear to bear. But in some respects the author was a little ahead of the times. He had taken up and investigated the old-fashioned custom, in dealing with contingent reversionary annuities, of effecting the total amount of the requisite whole-life assurance at the commencement of the transaction, and from that he had passed directly to a novel arrangement—a temporary insurance only was set up in the early years, coupled with a deferred whole-life assurance subsequently. By passing from one extreme to the other he had left out a practical method which was in frequent use, namely, that by which a half of the necessary insurance (or some other proportion) was effected at the outset on the whole-life scale and the balance arranged by means of deferred assurances, an immediate fine being paid for the option of taking up the latter. Had the author seen his way to include amongst his tables one which gave effect to this arrangement, it would have been of far greater value than the table he had actually given, on the assumption that only temporary insurances were in the first instance effected. He should hesitate in recommending the acceptance of deferred assurances for large sums for so long a period as 18 years, and could not help thinking that such transactions in the present state of actuarial opinion would be impracticable.

Mr. A. H. BAILEY said that there was one feature in reversionary transactions which had not been referred to, but which was of essential importance. It was that, when a reversion fell in, the terms on which the transaction was entered into frequently became the subject of dispute in the Court of Chancery, and then the evidence required was invariably what was the *market* value of the interest. The Court took no interest in elaborate formulas, or Mr. A and Mr. B's ideas on the subject, but the question was, what price could have been realized in the market. The plan first suggested by Mr. Griffith Davies, and extended by Mr. Jellicoe and others, was to show how, by buying annuities and effecting assurances, a purchaser of reversionary interests could realize a reasonable rate of interest and recover his capital, and the value so determined was held to be the market value. In years gone by it was a common practice in reversionary life interest cases to effect participating assurances in what were thought to be good offices, in the belief that the reversionary bonuses in future years would provide material increase to the amount, so that assurances for considerably less than the actual sum required were effected in the first instance. He had never heard of anyone buying an absolute reversion and at the same time purchasing an annuity during the lifetime of the life tenant. The

consequence was, the author's formulas did not represent the market values. He therefore must confess to being somewhat alarmed at the idea of publishing these tables on the ground that they would lead to the very mischief that Mr. Ryan had spoken of—of every man being his own lawyer. The author said that a considerable profit over and above the interest supposed to be realized was thus concealed in the annuities introduced. That was an entire mistake. The Government, for reasons of their own, granted annuities at prime cost. The grantors of the annuity could not realize a profit, and, as a matter of fact, Mr. Jellicoe's practice was to value the annuities on the Carlisle $3\frac{1}{2}$ per-cent Table with no loading. The author said, "If the reversioner is a young woman, it is convenient for the purpose of valuation to rate up her age a few years", but evidence showed that the average duration of life in young women was greater than that in young men, and the rating-up was therefore wrong. As he had said before, owing to the cases in which reversionary transactions had been taken into Court, and seeing that even if the companies did succeed it was not pleasant to be dragged into Chancery, he had become a little less inclined for such transactions than in former years.

Mr. D. A. BUMSTED agreed that it was very dangerous to deal with reversions, as they were very speculative in their nature. He regarded the paper as an interesting study, especially for students, bringing before them, as it did, various formulas in different ways, and showing how they might be worked out. What was wanted in practice was some simple formula which would obviate their working out an elaborate calculation, and he thought Mr. Jellicoe's formula, upon the whole, was the best that they could have to value reversions. They could only give approximate results after all. There were so many questions to be considered, such as rate of interest, depreciation of securities, &c., that it would prevent even the most accurate calculation coming right. As to the question of granting leases, referred to by Mr. Rothery, he had heard of cases where a tenant for life, after getting rid of all his property, made a good income by granting leases. In regard to another matter, the author said, "Equity would require that the policy, or the surrender-value thereof, should be handed over on redemption." Equity did not require anything of the kind. In the first place, the lender, if he insured, did it at his own risk. There were many risks. For instance, the policy might lapse, or the reversioner might disappear: the evidence itself might be defective: the proof of age might not be forthcoming; and, lastly, the office itself might fail. These were some of the contingencies affecting the case, so that equity did not require the policy to be handed over on redemption. The policy was effected by the man who lent the money, and he was entitled to it. The risks could be guarded against by proper care, which was part of the lender's stock-in-trade, for which he ought to be paid. No doubt the question of mortality was an important one. They could not depend on a rate of mortality at the extreme ages. In a case of a reversion on a life at 90, what table would furnish sufficient data to calculate the expectation? In such cases they must form their own judgment. In a case of which he knew the rate of mortality was investigated

at two different periods, the result being that the mortality of males seemed to be about the same as that of the Carlisle Table, while that of females was considerably under the Carlisle rate; and as reversions were nearly always on female lives, except in cases of landed estates, it would appear that the mortality was less than the average. The author had calculated the various powers of redemption by means of reducing the sum assured, but if they reduced the sum assured at the beginning of the term, and had to increase it at the end, they could not advance as much as under the ordinary method, and borrowers always wanted as much as they could get. It was no use offering terms which would not be acceptable.

Mr. GEORGE KING said the author, in referring to a statement made by Mr. Sprague with regard to Jellicoe's formulas, said that Mr. Sprague had rightly pronounced them as "inappropriate for the use of insurance companies, and other societies holding a number of these securities." He had not time to refer to Mr. Sprague's own words, but he thought they could scarcely be so emphatic. If they were, then he was afraid that he (Mr. King) must go against that great authority, because he confessed to preferring Mr. Jellicoe's formulas as a rule to all others. Mr. Rothery had said that, theoretically, there was no need to set up a policy, but for various practical reasons, policies were always set up. One reason which Mr. Rothery had not mentioned was the necessity of saying how much of the transaction they should re-assure. Companies only held a certain amount upon one life, and as these reversionary transactions involved large policies, it was the universal custom to re-assure a sufficient portion. He thought the same principle should be applied to the annuities. The company selling annuities to the public would have a limit as to the amount it would grant on one life. For instance, it would not, perhaps, grant more than £500, but that very company would enter upon a reversionary transaction, involving assurances for £50,000 or £100,000, and correspondingly large annuities. Now, by Jellicoe's formulas, the amounts of the annuities, whether they were set up or not, are clearly displayed, and the necessity of re-assuring them enforced. As to the deferred assurance, he could not agree with the author's method of a term policy running over a considerable period of time, and then a deferred policy for the full amount. That was practically impossible. As to the amount of premiums required as a fine for a deferred assurance, he did not agree that an assurance deferred for a long period of years was necessarily a dangerous risk. The time required for terms of more than two or three years almost ceased to be a function of the deferred period, and became to a great extent a function of the age at which the deferred policy would have to be granted. Moreover, if the term increased the fine actually diminished, because, the policy being only granted if the life survived, the chance of the life surviving diminished, and therefore that factor in the calculation diminished, and the fine diminished also. It was, perhaps, not generally known that the probabilities of life for ages below 40, according to the new Government experience, were given in one edition of the Post Office Annuity Rates. He had prepared

monetary tables based upon them. and hoped they would one day be published.

Mr. G. F. HARDY said it had been remarked that the main feature of the paper was the substitution of complete for curtate annuities. He thought, on the whole, that the former were correct in theory, but that curtate formulas were best in practice. He was not sure that it was altogether correct in theory to go into this great exactitude by using complete formulas. For the case of simple life interests the old formulas met all the practical requirements of the case. If they adopted a formula for complete life annuities they practically gave, at the commencement of the transaction, half a year's purchase for the small fraction of the annuity which would be paid in the year in which the tenant for life died. If the tenant for life happened to die in the first half of the year the purchaser actually made a loss on the transaction. These complete formulas assumed that on the failure of the life the money was at once paid. As a matter of fact, that was not done. Sometimes the tenant for life might be at a considerable distance from the office. It was not unusual for a man who had sold his life interest to go abroad. The life interest ceased at the moment of death, but they did not necessarily get the sum assured at that moment; they might have to wait some months before the transaction was closed. Another point was that when the life tenant died they were practically closing an investment without any notice such as they usually got in the case of loans. They might, therefore, fairly put that small fraction of a year against the fact that they were paid off without notice and often had to wait some time before getting the sum assured. One of the more interesting parts of the paper dealt with the formulas for various options for increasing assurances after five years or whatever the case might be. He did not think, however, they would be of advantage to life offices. If, instead of charging the whole-life loaded premium, the office agreed to take a large part of the insurance by way of temporary premium, they were foregoing a large possible profit, because the actual loading on the temporary premiums was much smaller than that on whole-life premiums. He thought it would be safer to grant options for long periods than for short ones: especially would that be the case when they came to consider the class of lives in respect of which these options would be granted. With regard to the separation of male and female life, women did decidedly live longer than men who were life tenants, and that fact ought to be taken into account in calculating the values of these benefits. The author referred to the rate of interest on which loans on reversions were sometimes granted, and mentioned 4 and $4\frac{1}{2}$ per-cent. In many cases they were bought to pay barely 4 per-cent, but he thought that was not a rate at which they ought to invest in what was really a speculative security.

Mr. NIGHTINGALE, in reply, referred to one or two of the points mentioned, and said, in reply to Mr. Manly's query as to formula 6, that he had not made any elaborate comparison of the differences brought out by using the arbitrary premium, $P^1_{x,y-\delta}$, in lieu of the true premium, but had tried one or two examples, and found the differences very slight. With regard to the introduction of the curtate expectation in the formula for the amount that could be advanced on a

deferred reversionary life interest, there was no particular reason for adopting the curtate expectation; any suitable fixed number of years would answer the same purpose for the principle in view. The object was to reduce the annual charge during a limited number of years. The temporary premium was introduced to obtain the cheapest possible annual charge during the average duration of the joint lives. Although he recognized that the whole-life premiums would be more acceptable to the insurance companies, if they could get them, competition might force a company to adopt the less favourable course. The values by his tables in the case of male life tenants were practically the same as those obtained from the formulas generally adopted. The main differences were due to the sexes being separately treated. He did not consider his formulas complicated, and thought that a careful comparison with those now used would show them to be as simple as the latter, and more easily obtained.

On the Formulae for determining the Value of Benefits, according to the Principle of Collective Assurance. By RALPH PRICE HARDY, Fellow of the Institute of Actuaries, Actuary to the Briton Medical and General Life Association, Limited, and to the United Kingdom Temperance and General Provident Institution.

[Read before the Institute, 25 January 1892.]

1.—Amongst the various current proposals for the application of the principle of Collectivity to the needs of life, those that relate to the provision for a minimum aid towards sustenance in that period of permanent incapacity known as Old Age, and for the burial of the dead, appear to be the least open to objection on political grounds; whilst the objects in view appeal so strongly to the circumstances of our common Humanity that few are disinclined to lend their assent to this partial recognition of claims that, in their wider extent, they would strongly resist. Fortunately, the question of the cost of these arrangements can be considered apart from all political grounds; and, since the results of its investigation disclose some novel and curious modifications of the formulæ for the ordinary Benefits, it has appeared to me that, by limiting the enquiry to the terms shown on the title to this Paper, the subject might be brought before this Meeting, without any breach of the benevolent neutrality

that has so largely contributed to the preservation of our personal harmony amidst the many inevitable divergencies of opinion on subjects of close personal interest to all.

2.—But, I may go further, and say that there are not wanting indications that the vast social forces, now segregated or else only partially organized, will be combined and directed towards some of the objects that already fall within the scope of our daily professional purview: and, consequently, that we should begin to consider what new treatment should be applied to the old problems, such being based upon an advanced form of Individualism, so that they may be extended to meet the developments implied in the probable principle of the future, namely, Collectivism.

3.—I venture to submit that this practical utilization of the accumulated stock of our technical knowledge, so largely an inheritance from those past Masters in our Profession whose memories are still revered within these walls, is in accordance with the highest traditions of this Institute, and will afford another, out of our already many recognitions of the public responsibility involved in that signal mark of confidence reposed in us by the grant of a royal Charter.

4.—Actuated by these views, I ask the indulgence and co-operation of the Meeting, while I endeavour to reduce to formulæ the values of the chief Benefits likely to be called for under the system of Collective Assurance—and, which I have dealt with upon the basis of a Population distributed as shown by the l_x column, leaving a direct application to the community as at present distributed until the recent Census Returns have been published.

5. ASSURANCES.—If the Population be constituted with l_x persons at age x , l_{x+1} at age $x+1$, and so on down to the extremity of life, the claims of the first year will be Σd_x , Σd_{x+1} for the second, and so on. Therefore, the total fund to be raised to provide for an assurance of 1 on the death of each member will be

$$\begin{aligned}\Sigma l_x A_x &= v l_x + v^2 l_{x+1} + v^3 l_{x+2} + \&c. \\ &= l_x v (1 + a_x) = l_x (a_x + A_x) \quad . \quad . \quad . \quad (1)\end{aligned}$$

Each individual's contribution to the above would be

$$\frac{\Sigma l_x A_x}{\Sigma l_x} = \frac{l_x (a_x + A_x)}{l_x (1 + e_x)} = \frac{a_x + A_x}{1 + e_x} = v \frac{1 + a_x}{1 + e_x} \quad . \quad . \quad (2)$$

6.—If the assurances are not to extend beyond age $x+n$, the fund to be provided will be

$$\Sigma l_x \Lambda_x - \Sigma l_{x+n} \Lambda_{x+n} - l_{x+n} \Lambda_{x+n} \frac{1-v^n}{i} \quad \dots \quad (3)$$

where $l_{x+n} \Lambda_{x+n}$ represents the then value of the assurances that determine at age $x+n$, their present value being

$$\begin{aligned} l_{x+n} \Lambda_{x+n} (v + v^2 + \dots + v^n) \\ = l_x \frac{1 - {}_n\Lambda_x - {}_np_x}{i} \quad \dots \quad (4) \end{aligned}$$

Each individual's contribution to the above would be

$$\frac{l_x(1 - {}_n\Lambda_x - {}_np_x)}{i(\Sigma l_x - \Sigma l_{x+n})} = \frac{1 - {}_n\Lambda_x - {}_np_x}{i(1 + {}_{n-1}e_x)} \quad \dots \quad (5)$$

7. ANNUITIES-DUE.—In like manner, the annuity-due fund would be represented by

$$\begin{aligned} \Sigma l_x(1 + a_x) &= \Sigma l_x \frac{1 - \Lambda_x}{d} \\ &= \frac{l_x}{d} \{1 + e_x - (a_x + \Lambda_x)\} \\ &= l_x \left(1 + e_x + \frac{e_x - a_x}{i}\right) \quad \dots \quad (6) \end{aligned}$$

Each individual's contribution to the above would be

$$= \frac{l_x \left(1 + e_x + \frac{e_x - a_x}{i}\right)}{\Sigma l_x} = 1 + \frac{e_x - a_x}{i(1 + e_x)} \quad \dots \quad (7)$$

8.—If the survivors of the group at each age be entitled to receive payment (in advance) up to age $x+n$ and no further, the fund to be raised would be

$$l_x \left(1 + {}_{n-1}e_x + \frac{{}_{n-1}e_x - {}_{n-1}a_x}{i}\right) \quad \dots \quad (8)$$

Each individual's contribution to the above would be

$$1 + \frac{{}_{n-1}e_x - {}_{n-1}a_x}{i(1 + {}_{n-1}e_x)} \quad \dots \quad (9)$$

9. DEFERRED ANNUITIES.—If the annuity be deferred to age $x+n$, the first payment being then due, the capitalized sum that would be required to provide for the same would be $l_{x+n}(1 + a_{x+n})$

emerging at the end of the first year, and a like sum at the end of each of the $n-1$ succeeding years: the value of which would be

$$\begin{aligned} & l_{x+n}(1+a_{x+n})(v+v^2+v^3+\dots+v^n) \\ &= l_{x+n}(1+a_{x+n})\frac{1-v^n}{i} \\ &= l_x \frac{(1+i)^n-1}{i} {}_{n-1}a_x \quad \dots \quad (10) \end{aligned}$$

Each individual's contribution to the above would be

$$\frac{l_x \frac{(1+i)^n-1}{i} {}_{n-1}a_x}{\Sigma l_x - \Sigma l_{x+n}} = \frac{(1+i)^n-1}{i} \frac{{}_{n-1}a_x}{1+{}_{n-1}e_x} \quad (11)$$

10. ANNUAL PREMIUMS.—The annual Premiums may be obtained by equating the Assurance or Deferred-annuity funds, gross or individual, to the whole-term or temporary correlative Annuity funds. Thus, the annual Premiums, upon the collective principle, would be as under:

Whole-term Assurance: Premiums payable for life:

$$\begin{aligned} (1) \quad \text{or} \quad (2) &= v \frac{1-A_x}{(1-A_x) + (e_x-a_x)} \quad \dots \quad (12) \\ (6) \quad \text{or} \quad (7) & \end{aligned}$$

Whole-term Assurances: Premiums limited to n :

$$\begin{aligned} (1) \quad \text{or} \quad (2) &= d \frac{(1+a_x)(1+{}_{n-1}e_x)}{(1+i)(1+{}_{n-1}e_x) - (1+{}_{n-1}a_x)} \quad \dots \quad (13) \\ (8) \quad \text{or} \quad (9) & \end{aligned}$$

Assurances ceasing at age $x+n$: Premiums payable throughout the period:

$$\begin{aligned} (4) \quad \text{or} \quad (5) &= \frac{1-{}_nA_x - {}_n p_x}{(1+i)(1+{}_{n-1}e_x) - (1+{}_{n-1}a_x)} \quad \dots \quad (14) \\ (8) \quad \text{or} \quad (9) & \end{aligned}$$

Deferred-Annuities: Premiums payable throughout the period:

$$\begin{aligned} (10) \quad \text{or} \quad (11) &= [(1+i)^n-1] \frac{{}_{n-1}a_x}{(1+i)(1+{}_{n-1}e_x) - (1+{}_{n-1}a_x)} \quad (15) \\ (8) \quad \text{or} \quad (9) & \end{aligned}$$

11.—The forms for other Benefits can be deduced in like manner. Where the status is complete, the individual single and annual Premiums can be obtained by entering Orchard's Tables with the individual annuity-value, in the usual way.

12.—The individual values as above determined, though general in form, have relation to a Population distributed as shown by the

column l_x . While, therefore, they cannot be applied strictly to any other Population, they supply, in my opinion, a fair approximation to the results that would probably eventuate from their use in the case of most European communities, the composing lives of such being reasonably distributed over the several ages. It should be noted that they make no allowance for either emigration or immigration, nor for any expense of administration.

13.—Below will be found Tables, supplying numerical illustrations of the effect of the more important of the foregoing formulæ, upon the neutral basis of Carlisle 3 per-cent, from which an estimate, sufficient for present purposes, can be formed of the tendency of the cost of the several Benefits to a community adopting the principle of Collective Assurance.

14.—These Tables may be read thus:

A Population, constituted as defined, and where the ages range from birth to the extremity of life, would have to raise collectively the gross sum of £177,860 as the fund for providing a whole-term assurance of £1 upon each death; where the youngest member is 20, the amount to be raised would be £134,190; and so on; the individual's contribution thereto being $\cdot4535$ and $\cdot5252$ respectively. The increased cost to the individual (excepting where the youngest age is 0, which is special), by the collective principle, over and above the amount that would be sufficient by the ordinary premiums, will be seen from the amounts of the latter as tabulated. In like manner, for the groups as above, the gross annuity-fund would be £7,359,400 and £4,165,700 respectively; each individual's share being 18.76 and 16.30, and the annual premiums for the same assurances being $\cdot0242$ and $\cdot0322$ respectively. These several results may be compared with the ordinary functions as tabulated. Similarly, where the Benefit is a Deferred Annuity, or Pension, vesting at age 65 (first payment due), the fund to be raised, for the gross Pensions as above, would be £851,600 and £733,850 respectively; the single and annual contributions for each individual up to the attainment of age 65, being $\cdot44$ and 1.30, and $\cdot1366$ and $\cdot2325$ respectively. Again, these results may be compared with those tabulated for the ordinary functions.

15.—The financial effect of the Collective system is to distribute the burthen of purchase over the whole Population, lightening the pressure upon the older and transferring the same to the younger. Such a system, when deliberately adopted by an existing Population, implies either that the aged are unable to bear the cost of a Benefit deemed to be necessary for their protection as some of the composing social units, and as a part of their natural heritage in the common stock; or, else, that the young, when passing down, will succeed to the stock, or means, left behind by those retiring from the activities of life. If the former, the young are called upon to sustain a considerable portion of the cost of a past neglect, personal or social, or else, that of one of the ultimate effects of a long-continued and vast Pauperism. If the latter, there would be no agreement with the actual facts of life as exhibited in the present, or (as I believe) in any possible social arrangements—property and income not now accruing, and probably never will accrue to individuals upon the Tontine system. It is more likely that the former represents the circumstances; but, the propriety of the application of such principles depends upon political considerations that cannot here be discussed.

Carlisle

ASSURANCES AND WHOLE-LIFE ANNUITIES-DUE.

3 per-cent

POPULATION, Ages x to ω		WHOLE-TERM ASSURANCES					WHOLE-LIFE ANNUITIES-DUE		
x	Number Σl_x	Gross Assurance Fund $\Sigma l_x A_x$	Individual's Contribution to the last $\Sigma l_x A_x$ Σl_x	A_x	Annual Premiums		Gross Annuity Fund $\Sigma l_x(1+a_x)$	Individual's Contribution to the last $\frac{\Sigma l_x(1+a_x)}{\Sigma l_x}$	$1+a_x$
					$\frac{\Sigma l_x A_x}{\Sigma l_x(1+a_x)}$	P_x			
0	392,213	177,860	·4535	·4664	·0242	·0255	7,359,400	18·76	18·30
5	351,701	162,940	·4633	·2808	·0251	·0114	6,480,800	18·43	24·69
10	318,605	153,730	·4825	·2861	·0272	·0117	5,661,000	17·77	24·51
15	286,611	144,240	·5033	·3132	·0295	·0133	4,888,100	17·06	23·58
20	255,522	134,190	·5252	·3390	·0322	·0149	4,165,700	16·30	22·70
25	225,496	123,660	·5484	·3690	·0354	·0170	3,496,400	15·51	21·67
30	196,542	112,600	·5729	·4013	·0391	·0195	2,882,000	14·66	20·56
35	168,898	101,170	·5990	·4340	·0435	·0223	2,325,400	13·77	19·43
40	142,648	89,392	·6267	·4716	·0489	·0260	1,828,500	12·82	18·14
45	117,957	77,391	·6561	·5089	·0556	·0302	1,392,800	11·81	16·86
50	95,006	65,327	·6876	·5543	·0641	·0362	1,019,000	10·73	15·30
55	73,641	53,021	·7200	·6095	·0749	·0455	707,960	9·61	13·41
60	54,053	40,645	·7520	·6653	·0883	·0579	460,340	8·52	11·49

Carlisle DEFERRED AND TEMPORARY ANNUITIES, WHERE $x+n=65$. 3 per-cent

[For typographical reasons, the headings to some of the columns are given as foot-notes, with reference numbers to indicate their proper position.]

POPULATION, Ages x to 64		DEFERRED ANNUITIES, FIRST PAYMENT AT AGE 65				TEMPORARY ANNUITIES (IN ADVANCE TO AGE 65)			
x	Number $\Sigma l_x - \Sigma l_{x+n}$	Gross Deferred Annuity Fund	Indi- vidual's Contri- bution to the last	Annual Premiums		Gross Temporary Annuity Fund	Indi- vidual's Contri- bution to the last	$1 + \frac{65-(x+1)}{i} a_x$	
				${}^{65-(x+1)}a_x$	$\frac{{}^{65-(x+1)}a_x}{1 + \frac{65-(x+1)}{i} a_x}$				
		[1]	[2]	[3]		[4]	[5]		
0	355,130	851,600	2.40	.44	.1366	.0245	6,232,310	17.55	17.88
5	314,618	828,360	2.63	.75	.1541	.0312	5,376,950	17.09	23.95
10	281,522	801,360	2.85	.91	.1718	.0386	4,581,150	16.28	23.60
15	249,528	770,110	3.09	1.08	.2004	.0482	3,842,500	15.40	22.50
20	218,439	733,850	3.36	1.30	.2325	.0607	3,156,360	14.45	21.40
25	188,413	691,850	3.67	1.56	.2736	.0776	2,529,060	13.42	20.11
30	159,459	643,120	4.03	1.89	.3276	.1010	1,963,390	12.31	18.67
35	131,815	586,640	4.45	2.30	.4009	.1342	1,463,270	11.10	17.13
40	105,565	521,180	4.94	2.82	.5051	.1838	1,031,830	9.77	15.33
45	80,874	445,290	5.51	3.51	.6626	.2625	672,020	8.31	13.36
50	57,923	357,310	6.17	4.37	.9252	.3996	386,200	6.67	10.93
55	36,558	255,310	6.98	5.47	1.4412	.6886	177,160	4.85	7.94
60	16,970	137,070	8.08	7.09	2.8688	1.6091	47,780	2.82	4.40

$$[1] \quad l_x \frac{(1+i)^n - 1}{i} {}^{65-(x+1)}a_x$$

$$[2] \quad \frac{(1+i)^n - 1}{i} \times \frac{{}^{65-(x+1)}a_x}{1 + \frac{65-(x+1)}{i} e_x}$$

$$[3] \quad [(1+i)^n - 1] \frac{{}^{65-(x+1)}a_x}{(1+i)(1 + \frac{65-(x+1)}{i} e) - (1 + \frac{65-(x+1)}{i} e_x)}$$

$$[4] \quad l_x \left(1 + \frac{{}^{65-(x+1)}e_x + \frac{{}^{65-(x+1)}e_x - {}^{65-(x+1)}a_x}{i}}{1 + \frac{65-(x+1)}{i} e_x} \right)$$

$$[5] \quad 1 + \frac{{}^{65-(x+1)}e_x - {}^{65-(x+1)}a_x}{i(1 + \frac{65-(x+1)}{i} e_x)}$$

16.—Apart from these latter considerations, there would be grave practical difficulties in working a system of compulsory Assurance according to an annual contribution rated by this particular form of the principle.

1st. The general forms provide for only the then existing Population, and do not—and cannot, so far as I can see—take into account the fresh accessions due to

Births, which in most Countries are on an increasing scale.

2nd. Since the necessary contributions can be met only by adults, and, generally, only during the earning period of life, the assessment would vary, and would have to be periodically re-determined.

3rd. A Reserve Fund would have to be created, and kept invested at the assumed rate of interest: this latter would impose serious obligations upon the State.

4th. The expense of administration would be enormous.

17.—A simpler form of the principle and one requiring no machinery for collection, or for investment, would be to levy yearly the amounts required to discharge the emerged Death-payments and Pensions, such being temporarily advanced by the Local Authorities. The cost of administration would still be heavy; but it would be far less than that incurred under the annual contribution method. In many of the Friendly Societies, the practice of providing for the Funeral allowances by direct levy still survives from simpler times; but, I must add that, in these cases, the principle is wrongly applied, since the levy can be made only upon the survivors, and not over an outside Population. The same principle is also employed to meet exceptional Benefits, such as Lying-in allowances, Fire Insurance of Trade Tools, &c.

18.—Benefits thus provided for have the whole of the present and future property of the Nation for their security; and, the yearly levies, to be met out of public resources specially raised for the purpose, do not constitute Taxation in its full sense—some portion being an enforced saving on the individual's part, the other representing more truly a Tax to defray the cost of a neglect in the past, which the expectant beneficiaries are unable entirely to discharge.

19.—In a recent Paper* on the subject of Old Age Pensions, I proposed this form of the principle for dealing with the provision for Old Age, and I have there given the reasons, as they appeared to me, against the various other proposals. One of the chief

* *Old Age Pensioning: with reference to certain proposed Schemes, and to the existing Poor Law provision.* London: Knight & Co.

points was that already a considerable sum was paid really on that account to the impecunious aged, but under the designation of Poor Law relief, and which all existing property has adjusted its price to bear. Consequently, the mere conversion of the Poor Law allowance into a Pension for the impecunious aged would not impose any true additional burthen on the Taxpayer. An extension of the qualifications for claim—and not one, in my opinion, that ought to affect our political forecasts—would enable those lying on the border-land of Official Pauperism to be brought within the scope of such a measure.

20.—Mr. Charles Booth, in his Paper on the same subject, employs the same principle, *i.e.*, of the payment of the yearly outcome, and not of annual Premium contributions: but, his plan seeks to provide a Pension for all the Aged—rich and poor, without exception—and necessarily involves a very large annual outlay. This charge, as intimated above, does not, in its entirety, represent Taxation; but, is partially an enforced saving for the personal eventual benefit of the contributory.

21.—I thought that this Paper would be incomplete without some specific reference to the two most recent proposals for the application of the system of Collective Assurance. But, the scope of the present enquiry, as well as the paramount necessity for avoiding all discussion of political questions, compels me to refrain from examining their supposed merits, or demerits.

DISCUSSION.

The PRESIDENT (Mr. B. Newbatt) said this was a paper which, in a variety of ways, bore the impress in a very marked degree of its author, with regard to sentiment, arrangement, and compactness. He would, indeed, venture to suggest that it lost, possibly, something of its value both to themselves and to the public by that condensation which, out of compliment to the technical knowledge of those whom he was addressing, Mr. Hardy had allowed himself to use. Possibly there were some present who had come in anticipation of hearing a general discussion on the question of "Old Age Pensions." He, however, shared to a certain extent the feelings which had constrained Mr. Hardy to be, in many respects, very brief and reticent as to a question which required to be dealt with in that room with great discretion. They had on the occasion of Mr. Young's paper a very animated discussion, but the question there dealt with was that of thrift and the voluntary submission to certain laws which, if it were not contradictory, he would say would compel

a man to be thrifty. The underlying principle of Mr. Hardy's paper was that there should be some great interference on the part of the State in favour of those whom they might regard as the aged poor, and that was naturally a political question, and not one so obviously within the scope of their discussion. Having said that, there was still room for a great deal of expression of opinion. Possibly it might not be considered necessary very closely to criticize or discuss the formulas which he had presented, although, as Mr. Hardy himself had said, any remarks would be welcomed on that point. Mr. Hardy might be trusted to have made no mistake on questions of that kind. Members might usefully address themselves to the financial considerations which underlay the enormous proposal hinted at in the paper—the financial burden upon the State, its economic aspects—and they might also discuss some questions which he regretted that Mr. Hardy had not dealt with. In a recent paper on the subject of "Old Age Pensions", he had proposed a scheme for dealing with the provision for old age, and had there given the reasons as they appeared to him, against the various other proposals. That would be a very fair question for discussion. He need not ask gentlemen to take care that they did not depart so largely from the line of argument to which they were accustomed in that room as to introduce anything which was purely and simply a question of State policy, but, with that restriction, the field was open. He would, first of all, call upon Mr. Young and Mr. A. Hendriks, the referees of the paper, to open the discussion.

Mr. T. E. YOUNG, B.A., said it was with special pleasure that he addressed the meeting, with equal reference to his sincere personal respect for the author and to his deep interest in the general subject. It was a very happy coincidence when the air was thick with social reforms, involving almost universally the crudest conceptions and the most alarming disregard of facts, of inductions from facts, and of financial results, that men of practical ability, well versed in these special enquiries, like Mr. Hardy and Mr. Charles Booth, had dealt with this perplexing problem. They had considered the subject with a cautious regard to facts, and with a comparative exhaustiveness of treatment, and it was especially fortunate that Mr. Hardy, with his actuarial discipline and practicalness of mind, and Mr. Charles Booth, with his constant appeal to statistics, had brought to bear legitimate and sounder methods of enquiry. For under the impulsive stress of this social agitation they saw repeated the permanent bane which always attached to philanthropic projects no less than to general and special science, the rash ascent to wider generalization without first securing adequate minor generalizations, and hence the hasty erection of crumbling social and financial structures upon imperfect and ill-digested facts. Mr. Hardy had taught them the only accurate method of forming an approximate estimate of the result which these collective schemes involved, and the need, therefore, of patience and wise delay before the public were committed to any particular scheme. He wished first to take the opportunity of emphasizing the statement of the author that the adoption of the Carlisle Table

was in no degree intended to represent the actual facts with which they had to deal. He rightly employed that table as a convenient and well-known specimen and illustration of the method of work that must be pursued in those enquiries, but a specimen and illustration only. As a single example of comparison, and without minute calculation or admitting that Dr. Farr's No. 3 Table was a representative measure, he should imagine that the population shown in the second table between the ages of 20 and 64 would require to be multiplied by about 50 in order to represent the corresponding figures of Dr. Farr's, based upon the population data which he had selected. If any scheme of this collective character was to be seriously considered, the search for appropriate mortality bases must be laborious and difficult. Moreover, the important elements of birth-rate and the distribution of population affected the figures and required to be separately scrutinized in each case, since no doubt the constitution of the population was obviously affected by the varying social, economic, and political conditions of each nation. He suspected also that the introduction of any such scheme would have an effect upon the future birth-rate, and thus render more variable that variable element in our calculations. The formulas at which Mr. Hardy had arrived were extremely curious and interesting, but their interpretation was not at present at all obvious. He, indeed, found it at present impossible to extract the full meaning from the mathematical forms and verbally to express their *rationale*, as they were enabled to do with the various mathematical transformations of their usual expressions. But the results were consistent; their investigation from first principles was clear, and, moreover, their analogy with the more customary symbolic forms was a further attestation of their accuracy. He was particularly interested in the expressions for "deferred annuities", since this form of provision permeated all the current social propositions. He need not, however, dwell upon that point, as the results were sufficiently indicated in the table, with the exception, perhaps, of repeating for the warning of rash reformers that its results were simply an *indication* of the appropriate method of work, since, in every plan that might be seriously considered, the financial results would require to be materially increased. He had intended making a few remarks upon the schemes which in their nature differed from those which Mr. Hardy had very properly called collective schemes—schemes, namely, involving the creation of a specific fund, but he thought that a consideration of those schemes which might be termed specific would, perhaps, involve a deviation from the track which the President had rightly laid down. He consequently contented himself with simply expressing the individual view that the general conclusion at which he had clearly arrived was that the plan of a specific fund must be abandoned as baseless and unworkable. The grounds upon which those conclusions were based were both social and financial, but he would not discuss them now in any detail. The schemes which had been termed by the author "collective", depended upon a general levy upon the population when the pensions actually became due, without the formation of a specific fund and without the payment of direct

contributions. Mr. Charles Booth's plan applied to the entire population without restriction of social rank, and apart altogether from the ever-increasing cost of administration—a most serious factor, which had been generally disregarded in all popular schemes hitherto promulgated—the expenditure involved in such a scheme would amount, he believed, to about £15,000,000 a year in England and Wales alone. In Mr. Hardy's scheme, the general outlines were the same, but the application was restricted to the case of the necessitous poor. He wished to emphasize the essential question of the cost of management, and the important elements of emigration and immigration. He did not know whether it could yet be accepted as sound economic doctrine, but he considered that without very stringent provisions against immigration to these shores, which might be most difficult to adopt, the tendency of any scheme like that of Mr. Charles Booth would be to induce an extended and comparatively pauper immigration by the stimulus of this absolute 5s. a week. It would not be fitting, looking to the scope of the paper and the constitution of the Institute, to delineate in detail the features of the scheme that appeared to him (Mr. Young) to be most suited to the times and to the necessities of the case. He would, therefore, simply express the view that no popular scheme should command approval which tended in any degree to affect detrimentally existing friendly societies. Those bodies, with all their deficiencies, had proved most potent instruments in the teaching of thrift and foresight, and afforded great examples of the spontaneous development of self-denial without the intervention of State aid, when they were left to the people themselves. He wished to tender his cordial thanks to Mr. Hardy for his able and helpful contribution to the subject in a direction where authoritative and skilled aid had hitherto been entirely wanting and was urgently required.

Mr. AUGUSTUS HENDRIKS said he wished to refer more particularly to the statement made by the author that a reserve fund would have to be created and kept invested at an assumed rate of interest. This would no doubt be the case if the Government, in their wisdom, decided upon creating a reserve; but he thought it was much more likely that they would adopt, as the author said, "a simpler form of the principle, and one requiring no machinery for collection or for investment", namely, "to levy yearly the amounts required to discharge the emerged death payments and pensions, such being temporarily advanced by the local authorities." He took note of the word "temporarily", and would express his opinion as to what would take place when that temporary discharge of the function of local authorities had been exercised. He saw no way by which the Government could collect a contribution, as a forced contribution, other than by the machinery of the income tax, and when they considered that even in some of the least bold plans which had been put forward for the consideration of the nation it would mean a very considerable augmentation in the income tax, that part of it would have to be approached with a great deal of circumspection. Taking only the benefits set forth in the paper, it might easily be demonstrated that an income tax for such purposes would be wanted of, say, 1s. 6d. in

the £. Adding to that the present rate, namely, 6*d.*, they might take it that 2*s.* in the £ would have to be levied upon such part of the public as was in the enjoyment of income. Many incomes were very small, and yet had to be levied upon for income-tax purposes. When they remembered how great a burden the 1*s.* 3*d.* of the period of the Crimean War was considered, even for a temporary purpose, they could well imagine that there would be a great deal of discontent if the income tax was at least 2*s.* in the £, or 10 per-cent. subject to indefinite increase. If the Government were to have a reserve fund by the ordinary methods of an insurance company, that reserve, in course of time, would amount to some hundreds of millions, and, in the words of Mr. Hardy, would impose serious obligations upon the State. Not only so, but also upon the insurance companies, because, whether a large reserve were created or an increased income tax were imposed, the already marked tendency of reduction in the rate of interest earned by investments would be accelerated, and, consequently, might in future even jeopardize the stability of insurance companies. If the proposal ever passed into law, it would lead them to say that blessed were those who reduced the rate of interest at which they assessed their obligations. In past days a great many schemes had been brought forward, sometimes outside and sometimes within the precincts of the House of Commons. More than a century ago, Baron Maseres had a scheme for alleviating the distress of the poorer population. That scheme was taken up in the House of Commons; it passed that House, but was negatived in the House of Lords. They also knew that Mr. Pitt showed, at one period of his brilliant career, that he had far greater faith in thrift, as understood by the public, namely, in self-aid, than in any aid which could be given by the Government. He (Mr. Hendriks) was led to this conclusion, that now and for very many years to come, possibly beyond the ken of any of those present, certainly beyond the ken of those who had reached the mystic age of 65, they would see history repeat itself, and that these discussions, however interesting, would be still within the range, and not outside the range of academic discussions as to what was best for the indigent members of the British public. He said that with some amount of regret, because no doubt all of them in the present enlightened generation had every wish that everyone should be assisted. Still, he did not think that all those who were in the condition of wanting assistance should be assisted by the State, and he believed that self-help and thrift would do more for them than any scheme which could be devised, either in that room or in the higher tribunal of the House of Commons.

Mr. M. N. ADLER said that the paper which had been placed before them was one of considerable value and originality, and required a good deal of thought. He did not fully grasp the significance of the formulas which had been arrived at. The tables were based upon the Carlisle Table, which had for its radix 10,000 persons born passing from infancy through the different years of life. He presumed the figure given at the top of the first column of the first table represented the entire population at one moment under the

Carlisle Table, taking 10,000 births as the radix. The entire population of the United Kingdom, according to the last census, was more than 37 millions, which meant just about 100 times as many as the entire population under the Carlisle Table. Hence, to provide a deferred annuity of £1 for those 37 millions, excluding those people who have already attained the age of 65, would require a fund of 100 times £51,000, or about 55 millions. To provide a pension of £13 per annum or 5s. a week for the present population when they attain 65, would represent 13 times £5,000,000, or about £1,000,000,000. It was right to work out these figures that they might grasp what they really meant. In addition to that large sum there must be certain annual contributions to provide for fresh comers. The figures were so vast as actually to stagger one, and gave, indeed, food for reflection. Of course, his figures were based upon the Carlisle Table of mortality, which did not apply to the general population, and no doubt the estimate under a true table would be much less. He was quite in sympathy with what had been said with reference to friendly societies. Those societies had been at work for years, giving various benefits to members, and the legislature ought, in the first instance, to see what could be done to improve them. In any case the question could not be fully dealt with without bearing in mind those potent agencies, originated by the people themselves. As to the question of provision for old age, it was a subject so far-reaching and so beset with difficulties that he, for one, should hardly dare to grapple with it at the present moment.

Mr. GEORGE KING did not think anyone need be ashamed of speaking of the difficulty of the subject. It was not the fault of the author, but because the subject brought forward was very novel, and it was not easy at a moment's notice to take in all its bearings. The chief novelty of the paper was that instead of taking each age at entry by itself, the ages were combined, and then they had to find for all the ages so combined what were the necessary single and annual contributions for each specific benefit. This produced some very interesting formulas, the exact meaning of which it would take time to discover. But it was particularly interesting to notice the analogy that these formulas bore to those to which they were already accustomed. In dealing with this question practically, the great difficulty had been to find how to start such a scheme as would be required, how to deal with those who were beyond the age of youth, and whose contributions therefore for deferred annuities must necessarily be heavy. Mr. Adler brought out the thousand millions as a lump sum, but ignored the annual payments which all those below the age of 65 would have to provide. The author gave them a means of measuring what it would cost to start a scheme. He did not suppose he intended that a young man of 20 should actually pay the contributions brought out in that way, but it only showed what somebody would have to pay. Mr. Hardy had shown how to start the scheme, but he had not discussed what was to be done after the scheme was started, or how they were to deal with the new entrants

who were always coming in at the age fixed upon as the minimum. He hoped that Mr. Hardy would see his way to continue his paper when the population-distribution, as shown in the recent census, was ascertained. The paper brought out in some contrast the method of providing a fund by contributions, out of which the benefits might be paid when they came to be entered upon; and the other method of making the payments of the benefits a charge upon each year, in fact, making the taxpayer, or whoever else is to find the money, provide each year the amount of the benefits actually payable in that year; and he seemed to advocate, if such a scheme was to be carried out at all, the adoption of the second method, making each year bear its own charge, and not to provide a special fund for the purpose. Even granting that they were to have a scheme of pensions or other benefits, he (Mr. King) was not at present prepared to support either method in preference to the other, but he would point out a considerable danger that might arise from making each year carry its own charge, and from not providing a fund for the purpose, namely, that in these democratic days the power of voting must to a great extent lie with those who would benefit from these arrangements, and they would outnumber the others who would to a greater extent have to pay for them. If, therefore, the vote was to be for each year's charge, supposing they fixed 10*s.* a week, why not 15*s.* or 20*s.*? There was no difference of principle between the different amounts. It would be merely a matter of voting how much the annuities were to be at any particular time; whereas, if a fund were provided by annual contribution, no matter whether the contributions were to be paid by those who were to benefit, or by employers, or by the public, or partly from one source and partly from the others, the reply to any demand for increase of the annual benefit would be that the funds would not provide for it—that the contribution would provide only 10*s.* a week and no more. He thought from this point of view, the method of accumulating a fund out of contributions was the safer and more conservative of the two.

Mr. A. H. BAILEY said he might refer to a remark that had been made as to the working of friendly societies. The bulk of the work of those societies was payment in sickness and at death. Annuities in old age were only to a slight extent granted by them. It was rather curious to hear William Pitt cited as an authority on the advantages of thrift, because he himself was a man who was in debt all his life, and died heavily in debt. With regard to the point raised by Mr. King as to whether the annuities in old age should be provided for by annual contribution or by annual charge out of the taxes, having had some little experience of such funds founded on a small scale by railway and other companies, he found that with hardly an exception they had worked unsatisfactorily. He was satisfied that the attempt to create a fund for providing everyone at the age of 65 with 5*s.* a week for the remainder of life would be impracticable. The real question was the cost. Mr. Young said 15 millions annually, but Mr. Booth's calculation was erroneous in that it was based on the census of 1881, and no allowance was made for the expense, which must be enormous. It was well within the mark

to say that the cost would be considerably over 20 millions annually. Then came the question out of what source was this annual outlay to come? Was it to come from the taxes? It might be to some extent satisfactory to have the burden transferred from the rates to the taxes, from the householder to the general revenue, and as householders considered themselves rather hardly treated something might be said for this transfer. But how was the money to be raised? It was suggested many years ago and he should be glad to see it adopted, that the income tax should be levied upon everybody, that there should be no limitation to incomes of £150 a year, (except the limit at which it was not worth the expense of collection); that the anomaly whereby a man with an income of £401 paid on £401, and the man with an income of £399 paid on £279, should cease; and that the income tax should be collected from the recipients of weekly wages weekly. There was a society which collected more than £3,600,000 a year in premiums averaging 2*d.* a week, of which by far the largest number were one penny. That would give some idea of what could be collected for income tax purposes if it was levied on the recipients of weekly wages. Of course, it was quite outside the Institute of Actuaries to go into political questions; but if these pensions were made a charge upon the revenue, he thought that such might be a practicable scheme, but to attempt to raise a fund for the purpose by contributions was wholly impracticable.

The PRESIDENT said that he would urge in opposition to Mr. Hendriks' illustrations from the past that they had to deal with a totally different condition of things at the present moment. The days of Mr. Pitt were not the days of Queen Victoria in 1892. Since then they had changed their masters, and those who in the main were to be benefited by any scheme, whatever it might be, of a pension, were the persons who had the power to enforce their will. Therefore, to say that thrift was the proper solution of this question would not, he feared, meet the case. Before they were many years older, the question in some shape or another would have to be solved. Whether it would be solved in the fashion of Mr. Hardy or in the fashion which they would learn next month from Mr. Chamberlain in the *National Review*, was a question which, of course, they could not there debate. He did not undertake to endorse Mr. Adler's figures, but when he told them that to create a fund for the existing population without reference to those who were to come after would necessitate something like £1,000,000,000, it showed at least the enormous nature of the burden which, if not in the shape of a fund, yet in the shape of an annual contribution the country would have to bear if this scheme was carried out. This knowledge would in itself be an advantage—it would sober some minds. This question of deferred pensions had been treated very much as a light and airy thing—that they only had to pass an Act of Parliament, and the whole thing was done. They now saw that it would only be done, whatever shape it took, at an enormous cost. The real question they would have to consider politically would be, who was to bear the burden.

Mr. ADLER said that Mr. Booth stated in his paper that "at £13 per annum each (5*s.* a week) a universal pension list would reach

£17,000,000." In that he only provided for the population of England and Wales, and if Scotland and Ireland were included it would, roughly speaking, represent £25,000,000 per annum. Of course, they must take into account that there would be a certain saving in the maintenance of aged paupers which, following Mr. Booth's estimate, might represent about £5,000,000.

Mr. R. P. HARDY, in reply, expressed his opinion that any system which proposed to proceed by way of annual contribution, and which necessarily therefore required the creation of a reserve fund, was practically impossible. That would effectually dispose of the scheme in the air commonly known as the Chamberlain and Hunter scheme: and the working men themselves would not listen to Chamberlain and Hunter on that line at all. In reply to Mr. Young, he said that he had found it impossible to select any table that would measure at once the death rate, the rate of emigration and immigration, and all the other circumstances that would affect a moving population like that of Great Britain. They must, therefore, in his view, fall back upon the principle which he suggested, namely, measuring the cost after the events had been determined. That was simplicity itself, and reduced the expenses of working to a minimum. With reference to Mr. Hendriks' observation, his (Mr. Hardy's) suggestion was to distribute the benefits through the present poor-law officials, who had an existing machinery, which must be retained, and could therefore be employed with very little additional expense. He was rather startled to hear Mr. Hendriks refer to Mr. Pitt's "faith" in leaving the people of his country to take care of their own future. He wondered what that eminent gentleman would say if he could be told that 40 per-cent of his compatriots failed in justifying his "faith", namely, that nearly one-half of those who were spared to reach the semi-patriarchal age of 65 would have to stand with hat in hand and beg of their brethren the means to finish the limited portion of their lives. Such a fact was simply appalling, and he welcomed any scheme that would remove or lighten the burden that now fell upon the most miserable poor. Mr. King wished a fund to be created with a view of meeting any possible spoliation of the future. He (Mr. Hardy) had some faith in the honesty of his countrymen. He did not believe they would attempt to violate a solemnly entered into contract of that nature. He might remind Mr. King that if a fund was created it was equally easy to steal consols and mortgages as it was to refuse yearly supplies to meet the annual burden of pensions. If any attempt at compulsion was made it must be remembered that this was not a German nation. They had not been drilled into obedience or dragooned into civility, and he should pity any tax-gatherer who endeavoured to levy a weekly tax upon the working men of this country in the airy way suggested by Mr. Bailey: he would not retain the appointment for long. He differed also from Mr. Bailey in thinking that on the whole the working of the railway superannuation funds had been unsatisfactory. It had done this, and that was a great thing, it had steadied the service. It was the fact that since these funds had been in full working upon an adjusted

financial basis the secessions or rate of leaving had gone down one-half. It was not, he was informed, the intention to raise the money required by means of income tax: a death duty was the means to be resorted to. The working man had made up his mind, and it was astonishing to find how much he knew about the subject, and that he would submit to no further taxation for those purposes that fell upon labour; he was aware that a death duty did not, and that was the particular form which the additional tax would probably take. The death duty was a very fair one, and he did not think anybody should object to it: it did not touch the living earner at all.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

An Attempt to measure the Extra Risk arising from a Consumptive Family History when the Life proposed for Assurance is physically Sound and Healthy. By HENRY WILLIAM MANLY, Actuary of the Mutual Life Assurance Society, and Honorary Treasurer of the Institute of Actuaries.

[Read before the Institute. 29 February 1892.]

IT has been said that there is nothing so delusive as figures, especially in the form of statistics; and to a very large extent this is true. There is a real science of statistics which very few are acquainted with; and the consequence is that enormous labour is often expended on the collection of "Facts" which are not only useless but misleading. So-called "Facts" are often used as a crushing answer to some statement which may be perfectly true; and sometimes with effect, simply because the materials are not at hand to measure correctly the value of the statistics used. The most common offence is that of attempting to draw general conclusions from partial or imperfect observations. For instance, what is the value of the statistics of a coroner respecting the proportion of children murdered, out of the number which come under his observation, when the millions who do not come under his observation are not taken into consideration? Yet such statistics were gravely used to prove that two millions of people insured their children's lives with a view to murdering them, and that therefore such insurances ought to be suppressed. If any weight were to be attached to such statistics we might have it gravely stated, on the evidence of a coroner, that nearly every one died a sudden or violent death.

Again, what value can be attached to the statistics of, say, the Consumption Hospital, with regard to the hereditary nature of phthisis, when the thousands who are born of consumptive

parents, but who never suffer from tubercular disease, and consequently never find their way to the Consumption Hospital, are omitted from the account altogether?

Statistics are of great or little value according as the data are collected from complete or partial observations; and whether such observations embrace sufficient details to furnish material for a scientific investigation of every possible question relating to the subject under consideration, or are merely limited to a single collection of special features. For example, the number of deaths occurring from year to year in a population would give no idea of the rate of mortality prevailing in that population unless the numbers living in each year are known; but even when those two general facts are recorded, the information they afford is exceedingly vague, because the population may be a shifting one, and at one time may contain mostly young persons, and at another mostly old persons. To make the observations complete, and therefore of real value, it is necessary to have the numbers living and dying at each year of age. It is a favourite amusement with officers of health to compare the mortality in their districts with that prevailing in adjoining districts, by measuring the rate of mortality as so many deaths per thousand of the inhabitants, omitting altogether from their observations the ages of the inhabitants in their respective districts, and such disturbing elements as hospitals, infirmaries, and alms-houses.

I have commenced with these few stray observations on statistics in order to emphasize the limited nature and value of the statistics I propose to present to you to-night. They are drawn from a peculiarly select class, and cannot, therefore, apply to any but a similarly select class; and as the greatest care has been taken in their collection to make the observations complete in every detail, I hope that the results will be deemed of some value in reference to our treatment of applicants belonging to that class.

The origin of the investigation was a feeling that many of the rules prevailing amongst medical advisers with regard to persons having a consumptive family history were either altogether empirical, or, if based upon observations at all, those observations were so limited in character as to be of little value. The Mutual Life Assurance Society having had an experience of over 50 years, and having during that time taken many cases coming under the head of healthy lives with a consumptive family history, it was considered that a scientific investigation of all the cases would not only prove interesting, but, perhaps, extremely useful, as a guide to future practice. With this view, Dr. T. Glover Lyon, one of

the medical advisers of the society, examined all the medical reports from the year 1847 (when family history was first considered of sufficient importance to be recorded in the medical form), and selected every case that had any history of consumption or allied diseases. As death in childbirth is now generally considered to arise from consumption when no other evidence is forthcoming, it was determined to collect all such cases, and treat them as a separate class. It is well known that many artifices are resorted to in order to hide the unpleasant fact that a relative has died of consumption; and the rule governing the selection of cases was this: "Supposing that case were to come before you to-day, would you consider the cause of death there recorded to be primarily consumption?"

The particulars contained in the reports so collected were recorded on cards of the following form:

Date of Policy	Age at Entry ...		
Date of Birth	Age at Exit ...		
NAME			
OCCUPATION			
Policy No.	Amount	Annual Premium	
FAMILY HISTORY.			
Relation	Age if Living	Cause of Death	Age at Death
Father		
Mother		
Brothers		
Sisters		
Number of years added to Age			

SUMMARY OF OBSERVATIONS.

RELATIONS WHO DIED OF CONSUMPTION (OR CHILDREN)	Entered		Years of Life	Died		Expected by H.
	Males	Females		Males	Females	
LINEALS.						
1. Father only	67	2	1,115.5	18	2	16.091
2. Mother only—Consumption	71	12	1,077.5	18	1	17.570
3. Mother only—Childbirth	208	18	2,759.5	63	5	53.086
4. Father and Mother only—Consumption	8	1	117.5	2	1	3.821
	354	33	5,100.0	101	12	90.568
LINEALS AND COLLATERALS.						
5. Father and Brother or Sister—Consumption	17	3	81.5	2	0	2.180
6. Mother and Brother or Sister—Consumption	35	2	498.5	9	0	9.919
7. Mother—Childbirth; and Sister—Childbirth	8	1	93.5	1	0	2.316
8. Mother—Childbirth	6	1	81.5	2	0	2.056
9. Brother or Sister—Consumption	2	0	22.0	0	0	—
10. Father and Mother, and Sister or Brother—Consumption	68	7	783.0	14	0	17.043
COLLATERALS.						
10. Sister—Childbirth	108	12	1,381.0	31	2	29.569
11. One Brother or Sister—Consumption	257	25	4,119.5	67	6	71.510
12. Two or more Brothers or Sisters—Consumption	76	5	1,180.5	20	2	22.620
	441	42	6,714.0	118	10	123.699
TOTALS	863	82	12,597.0	233	22	231.310

Average duration of Policy, 13½ years.

It has always been the custom of the society to take none but first-class lives; and in cases where there was any history of consumption it may be assumed that a more than usually strict examination was made. Out of the whole 945 cases above mentioned, there were only about half a dozen rated up, and they were mostly recent cases. The statistics, therefore, relate to a particularly select class, and have reference only to the mortality prevailing amongst those persons of good physique and perfect health, who were considered eligible for assurance at ordinary rates, notwithstanding that they had a history of consumption in their family.

The cards having been completed and sorted, and duplicates eliminated, it was decided to collate the information in exactly the same way as the Institute Mortality Experience was done, as the intention was to compare the results of the observations with the Institute Tables, and so compare like with like.

It is unnecessary to explain the process, as that will be found in the work published by the Institute; and, to avoid occupying unnecessary space, the details of much of the preliminary work have been omitted from the following Tables of Observation:

TABLE OF OBSERVATION—I.
Fathers only.

Age	Entered	Exposed to Risk	Died	Expected by H ^M
11	...	5	...	1.287
12	1	1	...	
13	...	1.5	...	
14	1	2	...	
15	...	2	...	
16	...	2	...	
17	...	2.5	...	
18	1	3.5	...	
19	1	4	...	
20	...	4.5	...	
21	1	8	...	
22	7	11	...	
23	1	13.5	...	2.806
24	5	17	...	
25	2	20	...	
26	5	23	...	
27	4	25.5	...	
28	1	26	...	
29	1	27	2	
30	2	26	...	
31	4	27	...	
32	1	27.5	...	
33	2	29.5	...	
34	5	32	...	3.890
35	1	32.5	1	
36	4	33.5	...	
37	1	35	...	
38	2	36.5	1	
39	1	37	1	
40	4	36.5	...	
41	1	37.5	1	
42	1	37.5	...	
43	2	35.5	1	
44	...	34.5	2	
45	1	32.5	...	3.674
46	1	30.5	1	
47	1	29.5	1	
48	...	25.5	...	
49	...	24	...	
50	...	20	...	
51	...	20	...	
52	...	20	...	
53	...	19.5	...	
54	1	18	...	
55	...	18	...	
56	...	17	...	
57	...	16.5	...	3.484
58	1	15	...	
59	...	15	...	
60	...	15	2	
61	...	13	...	
62	...	12	...	
63	...	10.5	1	
64	1	9	...	
65	...	7	1	
66	...	6.5	...	
67	1	7	1	
68	...	5	1	.950
69	...	4	...	
70	...	4	...	
71	...	3	...	
72	...	3	2	
73	...	1	...	
74	...	1	...	
75	...	1	1	
Totals	69	1,115.5	20	16.091

TABLE OF OBSERVATION—II.
Mothers only—Consumption.

Age	Entered	Exposed to Risk	Died	Expected by H ^M
15	...	5	...	1.039
16	1	1.5	...	
17	1	3	...	
18	2	4	...	
19	...	4	...	
20	...	6	...	
21	4	8.5	...	
22	2	10.5	...	
23	4	13	...	
24	3	16	...	
25	4	17	...	
26	1	17	...	2.556
27	2	16.5	...	
28	4	18	...	
29	2	20	...	
30	4	22	1	
31	2	22	...	
32	4	25	...	
33	2	27.5	...	
34	5	29.5	...	
35	2	31.5	1	
36	4	32.5	...	3.415
37	3	34	1	
38	2	32.5	...	
39	1	31.5	1	
40	2	30	...	
41	...	29.5	...	
42	1	29	1	
43	2	30	1	
44	2	29	2	
45	2	28.5	...	
46	2	29	...	4.660
47	1	27	...	
48	1	24.5	...	
49	...	24.5	...	
50	1	25	1	
51	1	24.5	...	
52	1	24	...	
53	2	25.5	...	
54	1	26	1	
55	1	25	2	
56	2	23	1	
57	...	21	1	4.092
58	...	19	...	
59	...	15	1	
60	...	14.5	...	
61	1	14	...	
62	...	14	...	
63	...	12.5	1	
64	1	11	1	
65	...	9	...	
66	...	9	...	
67	...	8	1	1.808
68	...	6	...	
69	...	5	1	
70	...	4	...	
71	...	4	...	
72	...	3	...	
73	...	3	...	
74	...	3	1	
75	...	2	1	
76	...	1	...	
77	...	1	...	
78	...	1	1	
Totals	83	1,077.5	22	17.570

TABLE OF OBSERVATION—III.

Mother only—Childbirth.

TABLE OF OBSERVATION—IV.

Father and Mother only—

Consumption.

Age	Entered	Exposed to Risk	Died	Expected by H ^a
14	...	1	...	1.776
15	2	2.5	...	
16	1	2	...	
17	...	1.5	...	
18	2	2.5	...	
19	1	4.5	...	
20	3	7	...	
21	2	11.5	...	
22	7	15.5	...	
23	5	20.5	...	
24	9	25.5	...	
25	6	25.5	1	
26	5	26	...	
27	7	33	...	
28	11	38.5	...	6.174
29	7	45	...	
30	10	52	...	
31	8	57	...	
32	13	63	1	
33	5	62.5	...	
34	6	70	2	
35	14	74	1	
36	6	76.5	1	
37	11	79.5	1	
38	5	79.5	2	
39	8	81.5	3	
40	10	78	...	9.217
41	3	78.5	2	
42	5	80.5	...	
43	4	76.5	...	
44	5	75	1	
45	2	73.5	2	
46	8	75	...	
47	3	75.5	3	
48	1	71	...	
49	1	71	1	
50	2	68	2	
51	4	66	1	
52	2	65.5	4	
53	3	61.5	1	11.999
54	1	60	3	
55	2	57	2	
56	1	52.5	...	
57	3	52.5	1	
58	2	50.5	...	
59	1	51	3	
60	2	47	1	
61	...	44.5	2	
62	1	38.5	2	
63	...	36.5	1	
64	1	34	...	14.171
65	2	35	3	
66	...	30	1	
67	...	28	2	
68	...	26.5	...	
69	3	27	4	
70	...	22	3	
71	...	19	...	
72	...	19	3	
73	...	15	3	
74	...	10	...	
75	...	9	1	9.749
76	...	7	1	
77	...	5	...	
78	...	5	2	
79	...	3	1	
80	...	2	...	
Totals	226	2,759.5	68	53.086

Age	Entered	Exposed to Risk	Died	Expected by H ^a
21064
22	...	5	...	
23	1	1	...	
24	...	1	...	
25	...	1	...	
26	...	1	...	
27	...	1	...	
28	...	1.5	...	
29	1	2	...	
30	...	2.5	...	
31	1	2.5	...	
32	1	3.5	...	
33	1	4.5	...	
34	1	5369
35	...	5	...	
36	...	5	1	
37	...	4	...	
38	...	4.5	...	
39	1	5	...	
40	...	4	...	
41	...	4	1	
42	...	3	...	
43	...	3	...	
44	...	3404
45	...	3	...	
46	...	3	...	
47	...	3	...	
48	...	3.5	...	
49	1	3	...	
50	...	3	...	
51	...	3	...	
52	...	3	...	
53	...	3	...	
54	...	3.5558
55	1	4	...	
56	...	2	...	
57	...	2	...	
58	...	2	...	
59	...	2	...	
60	...	2	...	
61	...	2	...	
62	...	2	...	
63	...	2	...	
64	...	2	...	
65	...	2849
66	...	2	...	
67	...	2	...	
68	...	2	...	
69	...	2	...	
70	...	2	...	
71	...	2	...	
72	...	2	...	
73	...	2	...	
74	...	2	...	
75	...	2	...	1.577
76	...	1	...	
77	...	1	...	
78	...	1	...	
79	...	1	...	
80	...	1	1	
Totals	9	117.5	3	3.821

TABLE OF OBSERVATION—V.
*Father and Brother or Sister—
 Consumption.*

Age	Entered	Exposed to Risk	Died	Expected by H ^u
24	...	5080
25	1	1.5	...	
26	1	2	...	
27	2	2	...	
28	...	2.5	...	
29	1	3487
30	...	3	...	
31	...	3.5	...	
32	1	3.5	...	
33	1	4.5	...	
34	1	5.5916
35	1	7	...	
36	3	7	...	
37	...	6.5	...	
38	...	6.5	...	
39	1	7.5733
40	1	8	...	
41	...	6.5	...	
42	1	7	...	
43	...	7.5	...	
44	1	8.5264
45	1	8.5	1	
46	...	7	...	
47	...	8	...	
48	2	7	...	
49	1	7264
50	...	6	...	
51	...	5	...	
52	...	4.5	...	
53	...	3	...	
54	...	3264
55	...	3	...	
56	...	3	...	
57	...	3	...	
58	...	3	1	
59	...	3264
60	...	1	...	
61	...	1	...	
62	...	1	...	
63	...	1	...	
64	...	1264
65	...	1	...	
66	...	1	...	
Totals	20	84.5	2	2.480

TABLE OF OBSERVATION—VI.
*Mother and Brother or Sister—
 Consumption.*

Age	Entered	Exposed to Risk	Died	Expected by H ^u
22	...	1104
23	2	1.5	...	
24	1	2.5	...	
25	1	3.5	...	
26	1	3.5	...	
27	...	1.5	...	1.078
28	...	1	...	
29	...	1	...	
30	...	2.5	...	
31	3	4.5	...	
32	1	6	...	1.930
33	2	10	1	
34	6	12.5	...	
35	3	14.5	...	
36	3	16.5	...	
37	1	16	...	1.930
38	...	18	...	
39	4	18	1	
40	1	15.5	...	
41	1	16	...	
42	...	16.5	...	2.223
43	1	17	...	
44	...	17.5	...	
45	1	16	...	
46	1	15	...	
47	...	14.5	...	3.105
48	1	15	...	
49	...	15	...	
50	...	14	...	
51	2	14.5	...	
52	...	12.5	1	3.105
53	...	11	...	
54	...	11	...	
55	...	11	1	
56	...	10	...	
57	...	9	...	1.479
58	...	9	1	
59	...	8.5	...	
60	1	9	...	
61	...	9	...	
62	...	9	...	1.479
63	...	9	...	
64	...	8	...	
65	...	8	1	
66	...	7	...	
67	...	6	...	1.479
68	...	6	1	
69	...	5	...	
70	...	5	...	
71	...	5	...	
72	...	4	...	1.479
73	...	3	1	
74	...	2	...	
75	...	1	1	
Totals	37	498.5	9	9.919

TABLE OF OBSERVATION—VII.

Mother—Childbirth ;
Sister—Childbirth.

Age	Entered	Exposed to Risk	Died	Expected by H ^m
27	...	·5	...	·011
28	1	·5	...	
29	...	·5	...	
30	1	·5	...	
31	
32	·110
33	
34	
35	...	·5	...	
36	1	1·5	...	
37	1	2	...	
38	...	3	...	
39	2	4	...	
40	...	4	...	
41	...	4	1	
42	...	3	...	·345
43	...	3	...	
44	1	3	...	
45	...	3	...	
46	...	3	...	
47	...	2	...	
48	...	2	...	
49	...	2	...	
50	...	2	...	
51	...	2	...	
52	...	2	...	·483
53	...	2	...	
54	...	2	...	
55	...	2	...	
56	...	2	...	
57	...	2·5	...	
58	1	3	...	
59	...	3	...	
60	...	3	...	
61	...	3·5	...	
62	1	4	...	·867
63	...	3	...	
64	...	3	...	
65	...	2	...	
66	...	1	...	
67	...	1	...	
68	...	1	...	
69	...	1	...	
70	...	1	...	
71	...	1	...	
72	...	1	...	·530
73	...	1	...	
74	...	1	...	
75	...	1	...	
76	...	·5	...	
Totals	9	93·5	1	2·346

TABLE OF OBSERVATION—VIII.

Mother—Childbirth ;
Sister—Consumption.

Age	Entered	Exposed to Risk	Died	Expected by H ^m
23	...	·5	...	
24	1	·5	...	
25	
26	
27	
28	...	·5	...	·017
29	1	1	...	
30	1	·5	...	
31	...	·5	...	
32	1	1	...	
33	...	1	...	
34	...	1	...	
35	...	1	...	
36	...	1	...	
37	...	1	...	
38	...	1	...	·080
39	...	1	...	
40	...	1	...	
41	...	1	...	
42	...	2	...	
43	2	3	...	
44	...	3	...	
45	...	3	...	
46	...	3·5	...	
47	1	4	...	
48	...	3·5	...	·342
49	...	3	...	
50	...	3	1	
51	...	2	...	
52	...	2	...	
53	...	2	...	
54	...	2	...	
55	...	2	...	
56	...	2	...	
57	...	2	...	
58	...	2	...	·433
59	...	2	...	
60	...	2	...	
61	...	2	...	
62	...	2	...	
63	...	2	...	
64	...	2	...	
65	...	2	...	
66	...	2	...	
67	...	2	...	
68	...	2	...	·849
69	...	2	...	
70	...	2	...	
71	...	2	1	
72	...	1	...	
73	·335
74	
Totals	7	84·5	2	2·056

TABLE OF OBSERVATION—IX.
*Father and Mother, and Sister or
 Brother—Consumption.*

Age	Entered	Exposed to Risk	Died	Expected by H ^m
26	...	5024
27	1	1	...	
28	...	1	...	
29	...	1	...	
30	...	1	...	
31	...	1050
32	...	1	...	
33	...	1	...	
34	...	1	...	
35	...	1	...	
36117
37	
38	
39	
40	...	5	...	
41	1	1051
42	...	1	...	
43	...	1	...	
44	...	1	...	
45	...	1	...	
46	...	1051
47	...	1	...	
48	...	1	...	
49	...	1	...	
50	...	1	...	
51	...	1051
52	...	1	...	
Totals	2	22242

TABLE OF OBSERVATION—X.
Sister only—Childbirth.

Age	Entered	Exposed to Risk	Died	Expected by H ^m
21	...	5316
22	1	1	...	
23	...	2	...	
24	2	3	...	
25	...	4	...	
26	2	6	...	2.468
27	2	8.5	...	
28	6	9.5	1	
29	1	10.5	...	
30	3	11.5	...	
31	6	16	...	4.891
32	7	19	...	
33	2	22	...	
34	6	29.5	...	
35	10	33.5	...	
36	2	33.5	...	8.049
37	2	35.5	...	
38	6	36.5	...	
39	2	37	1	
40	3	37.5	...	
41	3	38	...	8.925
42	3	40	...	
43	5	40.5	...	
44	2	40.5	...	
45	5	40.5	...	
46	1	40	...	4.260
47	5	42	1	
48	2	39.5	1	
49	1	39.5	2	
50	5	40	...	
51	3	42.5	1	8.925
52	4	41	2	
53	1	39	1	
54	2	38	1	
55	...	37.5	2	
56	3	38.5	...	4.260
57	4	40.5	1	
58	3	37.5	1	
59	...	33.5	...	
60	2	32	...	
61	...	30	1	.660
62	1	28.5	...	
63	1	27.5	1	
64	...	24	3	
65	...	19.5	1	
66	1	18.5	2	.660
67	...	15.5	...	
68	...	15	1	
69	...	13	1	
70	...	11	2	
71	...	8660
72	...	7.5	1	
73	...	6	1	
74	...	5	...	
75	...	5	1	
76	...	4	1	.660
77	...	3	1	
78	...	1	...	
79	...	1	...	
80	...	1	...	
81	...	1660
82	...	1	...	
83	...	1	1	
Totals	120	1,384	33	29.569

TABLE OF OBSERVATION—XI.

*One Brother or Sister—
Consumption.*

Age	Entered	Exposed to Risk	Died	Expected by H ^M
16	...	1060
17	2	2	...	
18	...	3.5	...	
19	3	5.5	...	
20	1	8	...	
21	4	12	...	2.509
22	5	18	...	
23	8	24	...	
24	6	28	...	
25	6	35.5	...	
26	11	46	...	9.116
27	12	54.5	...	
28	10	63.5	...	
29	13	71.5	2	
30	9	73.5	...	
31	8	80.5	1	15.109
32	14	88.5	1	
33	14	98	...	
34	13	103	...	
35	8	109	3	
36	14	114	3	15.109
37	11	116	...	
38	11	120	...	
39	8	123	...	
40	8	124	1	
41	7	122.5	1	19.824
42	2	121	2	
43	7	118.5	...	
44	6	122.5	...	
45	8	124.5	1	
46	6	125.5	1	16.935
47	6	125.5	1	
48	3	126.5	1	
49	4	120	...	
50	2	117.5	3	
51	6	113	2	16.935
52	...	107.5	2	
53	1	102.5	1	
54	5	98.5	...	
55	3	97	4	
56	3	91	1	7.957
57	3	86	3	
58	2	81	2	
59	1	76.5	...	
60	1	73.5	7	
61	2	61.5	3	7.957
62	...	53	1	
63	...	47.5	2	
64	1	45	...	
65	...	38.5	2	
66	1	34.5	...	7.957
67	...	33	1	
68	...	32	2	
69	2	30	1	
70	...	25	5	
71	...	18	1	7.957
72	...	16	2	
73	...	13	2	
74	...	11	3	
75	...	7	...	
76	...	4	1	7.957
77	...	3.5	2	
78	1	2	1	
79	...	1	...	
80	...	1	1	
Totals	282	4,149.5	73	71.510

TABLE OF OBSERVATION—XII.

*Two or more Brothers or Sisters—
Consumption.*

Age	Entered	Exposed to Risk	Died	Expected by H ^M
185394
19	1	1	...	
205	...	
21	...	1	...	
22	2	1	...	
23	...	1.5394
24	1	3	...	
25	2	5	...	
26	2	6.5	...	
27	1	9	...	
28	4	12	...	2.526
29	3	15.5	...	
30	7	19.5	...	
31	5	21.5	...	
32	1	22.5	...	
33	3	26	...	2.526
34	5	28	1	
35	3	27	...	
36	...	31.5	...	
37	9	35.5	...	
38	1	36	1	4.492
39	...	35.5	1	
40	4	36	...	
41	...	35	...	
42	2	37	...	
43	4	37.5	...	4.492
44	1	36	...	
45	...	36.5	1	
46	1	36	...	
47	1	36	...	
48	2	38	1	5.987
49	2	37.5	...	
50	5	37	3	
51	...	33	...	
52	2	33	...	
53	...	33	2	5.987
54	2	31	...	
55	...	31	...	
56	1	28	...	
57	1	26.5	1	
58	...	23.5	1	5.540
59	...	19	1	
60	...	17	...	
61	...	17	...	
62	...	17	...	
63	...	17	1	5.540
64	...	14	2	
65	2	12	...	
66	...	12	...	
67	...	11.5	1	
68	1	11	2	3.681
69	...	8	...	
70	...	8	...	
71	...	8	...	
72	...	6	...	
73	...	5	...	3.681
74	...	4	...	
75	...	4	...	
76	...	4	1	
77	...	3	...	
78	...	2	2	
Totals	81	1,180.5	22	22.620

As the total numbers under observation, comprising both male and female, only amounted to 945, and the females were only 82, or 8·7 per-cent of the whole, it was not considered necessary to separate the sexes. That this has had no injurious effect on the mortality will be apparent from the ages at death of the 22 females who died. The ages are 30, 33, 37, 45, 54, 56 (2), 59, 60, 61, 64, 65 (2), 68, 71 (3), 73, 75, 79 (2), 82, so that it may be fairly assumed that, if anything, the inclusion of the females has had the effect of improving the rate of mortality.

At the first glance it might be considered that the material was not large enough to form a basis for satisfactory deductions; but as the whole of the observations comprise 12,597 years of life and 255 deaths, it would, I think, have proved sufficient if it were possible to use the whole; but no useful purpose would be served thereby, as it would be no guide whatever to determining the rate to be charged for any particular case that might hereafter be submitted. When we examine, however, each separate class, the materials are certainly meagre, but we can see how far the actual experience differed from the expected, and whether any combinations would produce useful results.

EXAMINATION OF THE TABLES OF OBSERVATION.

TABLE I—*Fathers only*.—The mortality is 24·3 per-cent in excess of the expected, and this increase occurs between the ages 39 and 50 and after 65.

TABLE II—*Mothers only—Consumption*.—The mortality is 25·2 per-cent in excess of the expected, and the great increase occurs between the ages 50 and 60 and after 70.

TABLE III—*Mothers only—Childbirth*.—The mortality is 28·1 per-cent in excess of the expected, and the excess occurs between 35 and 10, and from 50 to the end.

TABLE IV—*Father and Mother—Consumption.*

TABLE V—*Father and Brother or Sister—Consumption.*

TABLE VI—*Mother and Brother or Sister—Consumption.*

TABLE VII—*Mother—Childbirth; Sister—Childbirth.*

TABLE VIII—*Mother—Childbirth: Sister—Consumption.*

TABLE IX—*Father and Mother, and Sister or Brother—Consumption.*

TABLE X—*Sister—Childbirth.*—The mortality is only 11·6 per-cent in excess of the expected, and the whole of the increase occurs after age 65, where the numbers exposed to risk are very small.

TABLE XI—*One Brother or Sister—Consumption.*—In this table we have the largest number under observation, comprising 4,149·5 years of life and 73 deaths, and as the average duration of the lives is 14·7 years, the results deduced should be considered of some value. Up to age 69 the expected deaths were 63·553, and the actual 55, showing an exceedingly light mortality, and if it had not been for two unfortunate cases of persons entering at age 69 and both dying within two years, the total deaths would not have exceeded the total expected.

TABLE XII—*Two or more Brothers or Sisters—Consumption.*—Here the numbers are not very large, but as the results are deduced from the observation of 81 lives over an average of $14\frac{1}{2}$ years, some reasonable value should be attached to them. The experience on the whole has been favourable to the office, and there is no point where the actual and expected differ materially.

The numbers in these Tables are too small to make the observations of any value, but the experience in each has been favourable to the office.

A general survey of these observations leads, I think, to the following conclusions:

1st.—That the cases where there has been evidence of consumption in a parent and another member of the family are too few to make the observations of any value, but as the experience has been lighter than the expected, we are bound to come to the conclusion that they were selected with extreme care, and that great credit is due to the medical advisers.

2nd.—That, so far as these observations have any weight, cases of consumption in the family, other than in a parent, may be disregarded where the applicant is perfectly sound. As the total observations under the head of “Collaterals” embrace 6,714 years of life and 128 deaths, I think it will be considered that some weight does attach to them.

There are only left for consideration the cases where one parent alone has died of consumption, and where the mother has died of childbirth. As regards the total mortality, it appears that the effect is exactly the same whether the father or the mother died of consumption; but the greatest excess has occurred at different ages, owing probably to the paucity of the numbers. By combining the two, the mortality will run more smoothly, and we shall obtain a very fair estimate of the mortality prevailing where *one parent* died of consumption, deduced from 2,193 years of life and 42 deaths, the observations extending over an average period of $14\frac{1}{2}$ years.

With respect to those cases where the mother died in childbirth, the mortality has been slightly heavier than where one parent died of consumption; but the greatest excess has occurred in a different place. As we may safely assume that “Collaterals”, especially in childbirth cases, do not count, I have combined Table III—mother of childbirth, and Table VII—mother, childbirth; sister, childbirth—thus giving a total of 2,853 years of life and 69 deaths, the observations extending over an average of 12 years.

In proceeding to graduate a table by the graphic method where the number of deaths are few and scattered, it is necessary to take rather large groupings in order to obtain a certain amount of regularity in the probabilities of dying, and also to draw the curve with a free and bold hand. The groupings, as Mr. Sprague

has explained, must be determined by trial. The following are the groupings finally fixed upon:

One Parent—Consumption.

Groups of Ages	Sum of Exposed to Risk	Σd_x	q_x
11-19	32
20-32	467.5	3	.0064
33-40	521.5	6	.0115
41-53	716.5	11	.0154
54-61	285	8	.0281
62-69	135.5	8	.0590
70-78	35	6	.1714
	2,193	42	...

Mother—Childbirth.

Groups of Ages	Sum of Exposed to Risk	Σd_x	q_x
14-26	146	1	.0066
27-37	656.5	6	.0091
38-44	573.5	9	.0157
45-51	516	9	.0174
52-58	415	11	.0265
59-63	234	9	.0385
64-70	212.5	13	.0612
71-74	67	6	.0895
75-80	32.5	5	.1538
	2,853	69	...

The q_x for the groups of ages were marked out in thin red lines on diagram paper as shown by the dotted lines in the following two reduced charts. I always use every other line for the ages, so as to obtain an extended curve. A curve line was then drawn boldly through these red lines—not necessarily through the middle—for all the positive ordinates of the red line must equal the negative ordinates, and the point where the curve cuts the straight line must be determined by the form of the curve. The curve in Diagram II (childbirth) was drawn first, the q_x by the H^M Table being adopted up to age 30. The first trial was fairly successful, but it required some slight modification where the bends are somewhat sharp. With the experience gained in drawing the first curve, I drew the second (Diagram I)

DIAGRAM I—ONE PARENT OF CONSUMPTION.—*Showing how the Graduated Table of q_x was obtained by means of the Graphic Method.*

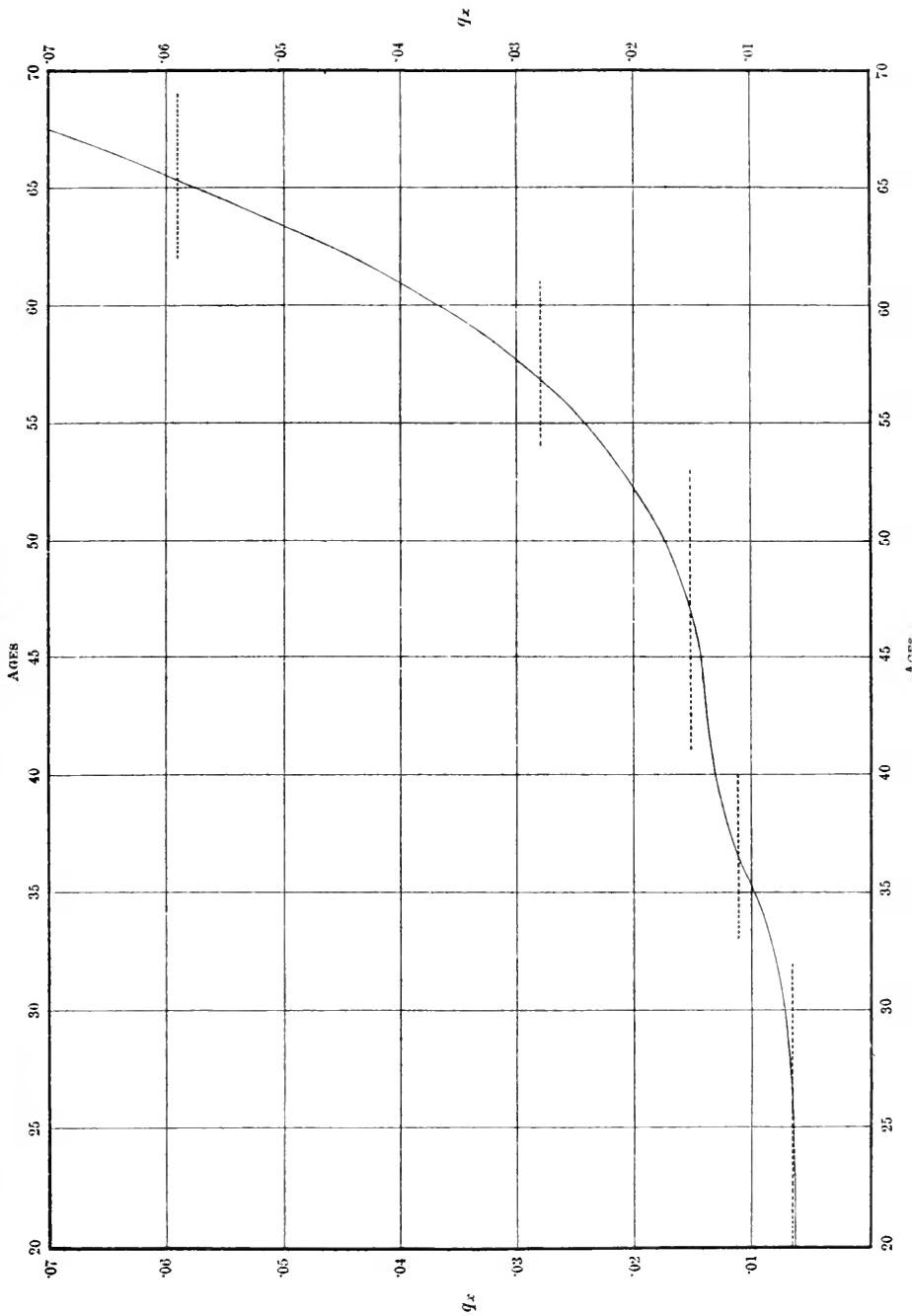


DIAGRAM II—CHILD BIRTH.—Showing how the Graduated Table of q_x was obtained by means of the Graphic Method.

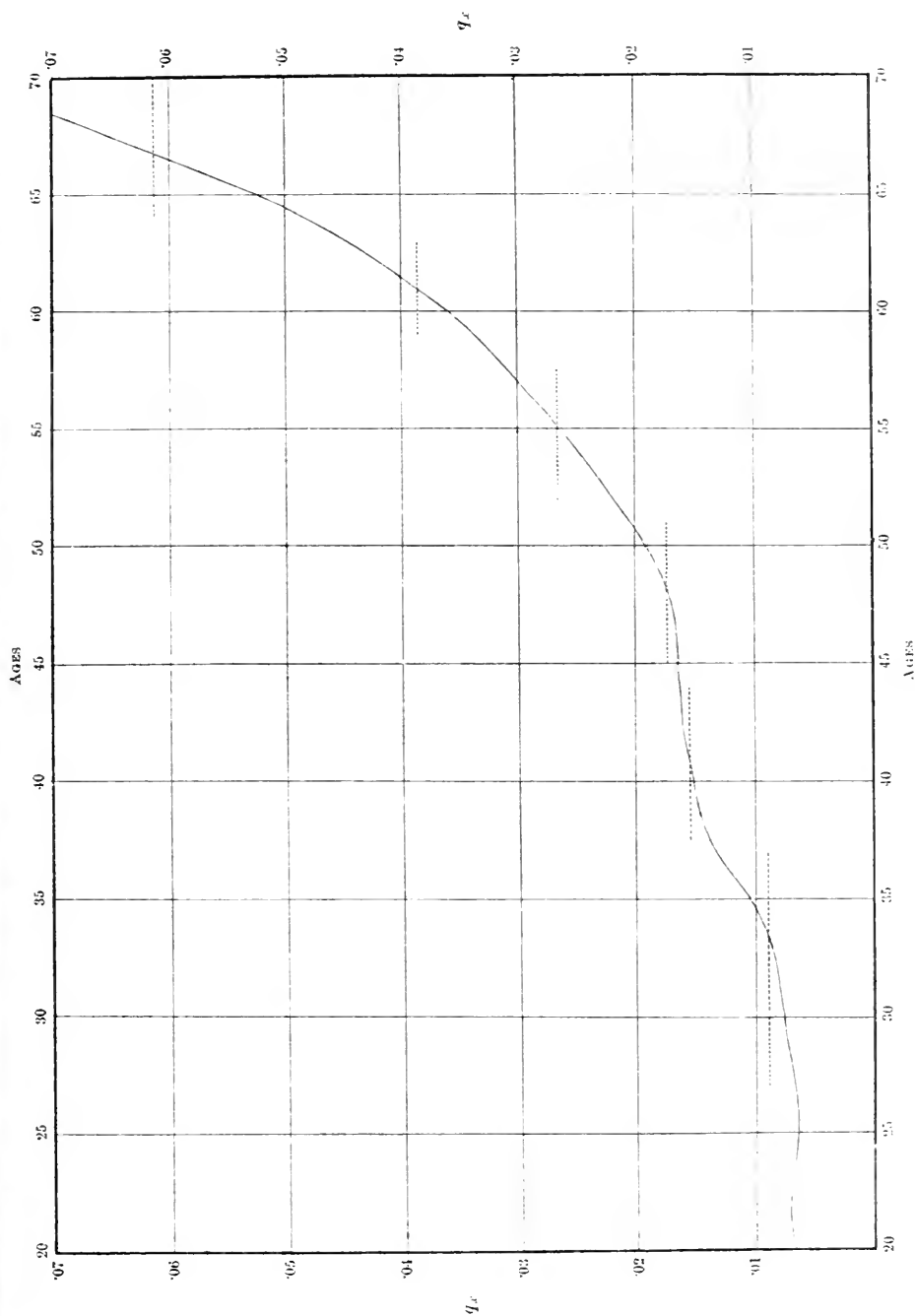
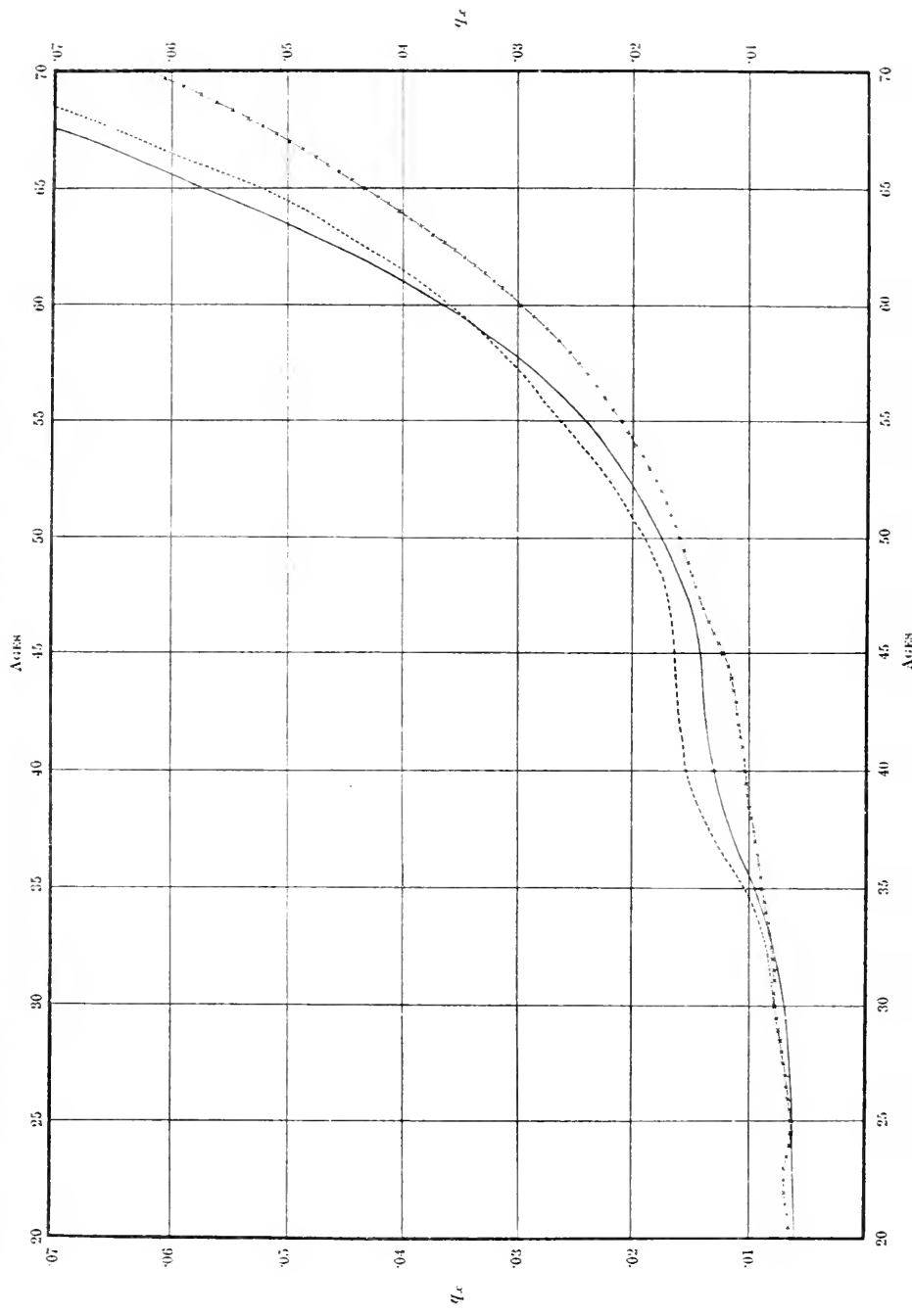


DIAGRAM III.—Showing the Curves of the Probability of Dying in a Year, according to the H^M Table (x-x-x-x-x-x); the “Consumptive Parent” Table (—); and the “Childbirth” Table (.....).





with such precision the first time that I resolved to make no alteration whatever in it, so as to show with what accuracy a curve can be drawn after a little practice. I may add that not a single reading was taken off until the curve was completed to age 70. The paucity of the facts after 70 years of age necessitated in each table an artificial termination, and in both cases the values of q_x were extended by second differences.

A diagram is added showing the above curves, and that of the probabilities of dying by the H^M Table, on the same plane.

[See DIAGRAMS.]

In the following Tables of Values I have given the original observations, the graduated probability of dying in a year, the expected deaths according to the graduated tables, and a continuous summation of the actual and expected deaths.

MORTALITY TABLE NO. 1.

Consumption—One Parent.—Table of Unadjusted and Adjusted Values.

Age x	Exposed to Risk	Deaths d_x	Adjusted q_x	Expected Deaths	Sum of		Δ
					Actual Deaths	Expected Deaths	
11	5	...	·0037	·002
12	1	...	·0037	·004
13	1·5	...	·0038	·006
14	2	...	·0040	·008
15	2·5	...	·0043	·011
16	3·5	...	·0047	·016
17	5·5	...	·0052	·029
18	7·5	...	·0056	·042
19	8	...	·0059	·047
20	10·5	...	·0061	·064
21	16·5	...	·0062	·102
22	21·5	...	·0063	·135
23	26·5	...	·0063	·167
24	33	...	·0064	·211
25	37	...	·0064	·237	...	1·08	-1·08
26	40	...	·0065	·260	...	1·34	-1·34
27	42	...	·0066	·277	...	1·62	-1·62
28	44	...	·0067	·295	...	1·91	-1·91
29	47	2	·0069	·324	2	2·21	-·24
30	48	1	·0071	·341	3	2·58	+·42
31	49	...	·0073	·358	3	2·94	+·06
32	52·5	...	·0077	·404	3	3·34	-·34
33	57	...	·0083	·473	3	3·81	-·81
34	61·5	...	·0090	·553	3	4·37	-1·37
35	64	2	·0097	·621	5	4·99	+·01
36	66	...	·0105	·693	5	5·68	-·68

MORTALITY TABLE NO. 1—(continued).

Age x	Exposed to Risk	Deaths d_x	Adjusted q_x	Expected Deaths	Sum of		Δ
					Actual Deaths	Expected Deaths	
37	69	1	.0113	.780	6	6.46	— .46
38	69	1	.0120	.828	7	7.28	— .28
39	68.5	2	.0126	.863	9	8.15	+ .85
40	66.50130	.865	9	9.02	— .02
41	67	1	.0134	.898	10	9.91	+ .09
42	66.5	1	.0137	.911	11	10.83	+ .17
43	65.5	2	.0139	.910	13	11.74	+ 1.26
44	63.5	4	.0144	.895	17	12.63	+ 1.37
45	610142	.866	17	13.50	+ 3.50
46	59.5	1	.0146	.869	18	14.37	+ 3.63
47	56.5	1	.0150	.848	19	15.21	+ 3.79
48	500157	.785	19	16.00	+ 3.00
49	48.50165	.800	19	16.80	+ 2.20
50	45	1	.0171	.783	20	17.58	+ 2.42
51	44.50185	.823	20	18.40	+ 1.60
52	440197	.867	20	19.27	+ .73
53	450209	.941	20	20.21	— .21
54	44	1	.0224	.986	21	21.20	— .20
55	43	2	.0240	1.032	23	22.23	+ .77
56	40	1	.0260	1.040	24	23.27	+ .73
57	37.5	1	.0282	1.058	25	24.33	+ .67
58	340307	1.044	25	25.37	— .37
59	30	1	.0334	1.002	26	26.37	— .37
60	29.5	2	.0366	1.080	28	27.45	+ .55
61	270400	1.080	28	28.53	— .53
62	260438	1.139	28	29.67	— 1.67
63	23	2	.0480	1.104	30	30.78	— .78
64	20	1	.0526	1.052	31	31.83	— .83
65	16	1	.0573	.917	32	32.75	— .75
66	15.50620	.961	32	33.71	— 1.71
67	15	2	.0670	1.005	34	34.71	— .71
68	11	1	.0726	.799	35	35.51	— .51
69	9	1	.0791	.712	36	36.22	— .22
70	80868	.710	36	36.93	— .93
71	70960	.672	36	37.61	— 1.61
72	6	2	.1070	.642	38	38.25	— .25
73	41201	.480	38	38.73	— .73
74	4	1	.1356	.542	39	39.27	— .27
75	3	2	.1538	.461	41	39.73	+ 1.27
76	11750	.175
77	11995	.200
78	1	1	.2276	.228
792496
802758
813165
823620
834126
844686
855303
865980
876720
887526
898401
90	1.

MORTALITY TABLE NO. 2.

Childbirth.—Table of Unadjusted and Adjusted Values.

Age x	Exposed to Risk	Deaths d_x	Adjusted q_x	Expected Deaths	Sum of		Δ
					Actual Deaths	Expected Deaths	
14	1	...	·0028	·003
15	2·5	...	·0029	·007
16	2	...	·0032	·006
17	1·5	...	·0039	·006
18	2·5	...	·0048	·012
19	4·5	...	·0057	·026
20	7	...	·0063	·044
21	11·5	...	·0067	·077
22	15·5	...	·0068	·106
23	20·5	...	·0068	·139
24	25·5	...	·0066	·169
25	25·5	1	·0066	·169	1	·76	+ ·24
26	26·5	...	·0067	·177	1	·91	+ ·06
27	33·5	...	·0069	·231	1	1·17	- ·17
28	39	...	·0072	·281	1	1·45	- ·45
29	45·5	...	·0075	·311	1	1·79	- ·79
30	52	...	·0078	·406	1	2·20	-1·20
31	57	...	·0081	·462	1	2·66	-1·66
32	63	1	·0083	·523	2	3·19	-1·19
33	62·5	...	·0088	·550	2	3·73	-1·73
34	70	2	·0094	·658	4	4·39	- ·39
35	74·5	1	·0101	·775	5	5·16	- ·16
36	78	1	·0117	·913	6	6·08	- ·08
37	81·5	1	·0131	1·068	7	7·15	- ·15
38	82·5	2	·0141	1·163	9	8·31	+ ·69
39	85·5	3	·0148	1·265	12	9·58	+2·42
40	82	...	·0152	1·246	12	10·82	+1·18
41	82·5	3	·0156	1·287	15	12·11	+2·89
42	83·5	...	·0160	1·336	15	13·45	+1·55
43	79·5	...	·0162	1·288	15	14·73	+ ·27
44	78	1	·0163	1·271	16	16·01	- ·01
45	76·5	2	·0165	1·262	18	17·27	+ ·73
46	78	...	·0166	1·295	18	18·56	- ·56
47	77·5	3	·0169	1·310	21	19·87	+1·13
48	73	...	·0172	1·256	21	21·13	- ·13
49	73	1	·0182	1·329	22	22·46	- ·46
50	70	2	·0191	1·337	24	23·79	+ ·21
51	68	1	·0202	1·374	25	25·17	- ·17
52	67·5	4	·0216	1·458	29	26·63	+2·37
53	63·5	1	·0231	1·467	30	28·09	+1·91
54	62	3	·0246	1·525	33	29·62	+3·38
55	59	2	·0263	1·552	35	31·17	+3·83
56	54·5	...	·0280	1·526	35	32·70	+2·30
57	55	1	·0297	1·634	36	34·33	+1·67
58	53·5	...	·0315	1·685	36	36·02	- ·02
59	54	3	·0335	1·809	39	37·82	+1·18
60	50	1	·0360	1·800	40	39·62	+ ·38
61	48	2	·0387	1·858	42	41·48	+ ·52
62	42·5	2	·0417	1·772	44	43·25	+ ·75
63	39·5	1	·0449	1·774	45	45·03	- ·03
64	37	...	·0480	1·776	45	46·80	-1·80
65	37	3	·0525	1·943	48	48·75	- ·75

MORTALITY TABLE NO. 2—(continued).

Age x	Exposed to Risk	Deaths d_x	Adjusted q_x	Expected Deaths	Sum of		Δ
					Actual Deaths	Expected Deaths	
66	31	1	·0575	1·783	49	50·53	-1·53
67	29	2	·0625	1·813	51	52·34	-1·34
68	27·5	...	·0675	1·856	51	54·20	-3·20
69	28	4	·0725	2·030	55	56·23	-1·23
70	23	3	·0775	1·783	58	58·01	-·01
71	20	...	·0820	1·640	58	59·65	-1·65
72	20	3	·0860	1·720	61	61·37	-·37
73	16	3	·0940	1·504	64	62·87	+1·13
74	11	...	·1040	1·144	64	64·02	-·02
75	10	1	·1150	1·150	65	65·17	-·17
76	7·5	1	·1280	·960	66	66·13	-·13
77	5	...	·1450	·725	66	66·86	-·86
78	5	2	·1640	·820	68	67·68	+·32
79	3	1	·1890	·567	69	68·24	+·76
80	2	...	·2200	·440
81	·2580
82	·3040
83	·3590
84	·4240
85	·5000
86	·5880
87	·6890
88	·8040
89	·9340
90	1·

It is not easy to compare the graduated results with the original facts where the deaths are so few, but perhaps the best and severest test is to compare a continuous summation of the actual deaths with a like summation of the expected deaths by the graduated table.

In the table for "Consumption—One Parent", the total deaths by the graduated table are within ·5 of the total actual deaths at 17 ages, namely, 29, 30, 31, 32, 35, 37, 38, 40, 41, 42, 53, 54, 58, 59, 69, 72, 74; and in the table for "Childbirth" at 23 ages, namely, 25, 26, 27, 28, 34, 35, 36, 37, 43, 44, 48, 49, 50, 51, 58, 60, 63, 70, 72, 74, 75, 76, 78.

I have calculated annuities at $3\frac{1}{2}$ per-cent by four-figure logarithms, and the values will be found in the following table, to which I have added the values by the H^M Table. It will be noticed that the annuities by the "Childbirth" table are less than those by the "Consumption—One Parent" table under age 46, but better afterwards.

Annuity Tables, 3½ per-cent Interest.

Ages	Consump- tion— One Parent	Childbirth	H ^M	Ages	Consump- tion— One Parent	Childbirth	H ^M
25	18.86	18.63	19.10	53	11.06	11.25	12.22
26	18.65	18.42	19.21	54	10.69	10.92	11.89
27	18.43	18.19	19.02				
28	18.20	17.96	18.82	55	10.31	10.59	11.55
29	17.97	17.72	18.62	56	9.94	10.25	11.21
				57	9.56	9.92	10.87
30	17.73	17.48	18.42	58	9.18	9.58	10.53
31	17.48	17.23	18.21	59	8.80	9.24	10.18
32	17.22	16.98	18.00				
33	16.96	16.72	17.78	60	8.43	8.89	9.84
34	16.71	16.46	17.55	61	8.06	8.55	9.49
				62	7.68	8.20	9.15
35	16.45	16.20	17.33	63	7.32	7.86	8.81
36	16.19	15.94	17.09	64	6.96	7.52	8.47
37	15.94	15.69	16.85				
38	15.69	15.46	16.61	65	6.60	7.17	8.14
39	15.43	15.23	16.36	66	6.25	6.83	7.89
				67	5.89	6.50	7.47
40	15.17	15.00	16.10	68	5.54	6.18	7.14
41	14.91	14.77	15.84	69	5.18	5.86	6.80
42	14.64	14.52	15.57				
43	14.37	14.28	15.29	70	4.82	5.51	6.47
44	14.08	14.02	15.00	71	4.46	5.22	6.14
				72	4.11	4.88	5.82
45	13.78	13.75	14.71	73	3.76	4.53	5.51
46	13.47	13.47	14.41	74	3.43	4.17	5.22
47	13.15	13.18	14.11				
48	12.82	12.87	13.81	75	3.10	3.82	4.95
49	12.48	12.55	13.50	76	2.79	3.47	4.68
				77	2.50	3.12	4.42
50	12.13	12.23	13.19	78	2.23	2.77	4.16
51	11.78	11.91	12.87	79	1.99	2.43	3.91
52	11.42	11.58	12.55				

These tables show that in the case of first-class lives, where one parent has died of consumption, an addition of 3 to 4 years of age will cover the extra risk; but where the mother has died in childbirth an addition of 5 years under age 40 should be required; from 40 to 45, 4 years; and after 45, 3 years.

No great value, however, must be attached to these tables where the lives are under 30 years of age, on account of the paucity of the observations.

It is a practice, I believe, with some offices to take these lives at ordinary rates for endowment assurances, and I have therefore calculated the premiums for such assurances and placed them side by side with the H^M rates. Roughly, it may be said that the extra risk would be covered by adding

Annual Premiums for Endowment Assurances, $3\frac{1}{2}$ per-cent.

Age	At 65 or DEATH			At 60 or DEATH			At 55 or DEATH			At 50 or DEATH			At 45 or DEATH		
	H ^m	Table 1	Table 2	H ^m	Table 1	Table 2	H ^m	Table 1	Table 2	H ^m	Table 1	Table 2	H ^m	Table 1	Table 2
25	1.83	1.90	1.97	2.07	2.12	2.19	2.43	2.47	2.53	2.99	3.01	3.06	3.87	3.88	3.92
30	2.20	2.30	2.39	2.54	2.62	2.71	3.08	3.15	3.22	3.95	4.00	4.07	5.48	5.51	5.56
35	2.69	2.84	2.96	3.20	3.33	3.45	4.05	4.17	4.28	5.56	5.68	5.77	8.71	8.82	8.88
40	3.38	3.55	3.67	4.19	4.33	4.46	5.67	5.81	5.92	8.79	8.93	9.03
45	4.43	4.57	4.68	5.86	5.97	6.07	8.91	9.03	9.12
50	6.15	6.33	6.43	9.17	9.29	9.39
55	9.53	9.79	9.83

2s. to the premium for an assurance at 45 or death; 3s. at 50 or death; 3s. 6d. at 55 or death; 4s. at 60 or death; and 5s. at 65 or death.

The figures which I have laid before you are very interesting in themselves, and, so far as they go, the statistics are valuable, but I must confess that the numbers are not sufficiently large to justify any hard and fast rules being based upon the results. Moreover, I would strongly deprecate any attempt to justify, by these figures, any relaxation in the severity of the medical examination, or to over-ride the opinions of the medical advisers. I must repeat here that these figures apply only to first-class lives who have been subject to strict examination, and who, in the judgment of the examiners, were practically free from any extra risk of consumption. It may be that our medical advisers, if they read this paper, will be assisted in forming a judgment as to the value of the extra risk in future cases, and if they should find my figures valuable in even a slight degree, my labour will not have been in vain.

There have been many pet theories respecting the degree of virulence of the consumptive taint, according to various conditions and circumstances, which, with the few figures at

command, I have been unable to investigate, but which it would be interesting to consider if we should happen to get a much larger number of statistics. For instance, I believe it is a rule amongst some of the American offices to absolutely decline all cases where there is evidence of two cases of consumption in the family. The cases in my statistics embrace 140 lives and 33 deaths, omitting childbirth (Tables of Observation V, VI, IX, and XII), and the facts certainly do not justify the theory, for the mortality in these tables is as good as any under observation. It appears to me that the rule is too drastic, and all that is required is increased caution on the part of the medical officer.

Then, again, there is a little pet theory that the risk is greater where the age of the applicant is less than the age at death of the relative who died of consumption. The cases are not numerous enough to justify any conclusion being drawn from them, but I cannot find that that had any weight with the medical examiners in selecting these lives, nor have I any reason to suppose that the mortality would be different from the rest of the class.

Another theory is that a person who is born within 7 years of the parent's death is a practically uninsurable life. Of that I cannot speak at all, because the date of death of the parents is not recorded on the papers.

There is still another theory, that the time of the greatest risk of the hereditary taint being developed is in early manhood; and that when that time is past and the applicant is of sound constitution, and in every respect, but for the family history, a first-class life, then the risk, as is actually shown by these figures, is small.

Personally, I have a great respect for the last theory; and I am inclined to think that all persons under 26 years of age who had a parent die of consumption or childbirth should be charged the premium for age 30, but that after the age of 25 an addition of from 5 to 3 years is quite sufficient to cover the risk.

In conclusion, I would desire to express a strong hope that others will be induced by this paper to extract the experience *with regard to the history of consumption* from the records of their own offices. I have purposely recorded the observations in such a way that they could be combined with other observations, and if other offices undertook a similar task, we should obtain in time a sufficient mass of statistics to make our conclusions really valuable.

Remarks on Consumption in relation to Life Assurance. By
T. GLOVER LYON, M.A., M.D., *Assistant Physician to the*
Victoria Park Chest Hospital, one of the Medical Officers
of the Mutual Life Assurance Society.

[Read before the Institute, 29 February 1892.]

THE subject of consumption in relation to insurance is, perhaps, the one of all others which calls for co-operation between actuary and medical officer, yet up to the present time it has been left in the hands of the medical officer alone. The result has been that when the question of rating-up on account of a family taint of consumption has arisen, it has been decided by certain arbitrary rules founded upon the personal impressions of the medical officer. Such a condition of things, at one time unavoidable, has no justification now, for there is ample material, I am told, in London, if made use of, to settle the question on equitable lines, according to strict actuarial principles.

The President of the Institute of Actuaries some months ago, with much wit and some truth, said, "doctors step in where actuaries fear to tread." But as far as consumption is concerned, if doctors have ventured into the province of the actuary, it has been with great reluctance and under pressure. The actuaries, knowing the doctors could not swim, pushed them into deep water. They have laughed long enough at their clumsy aquatic struggles—it is time they came to the rescue.

A short glance over the general subject of phthisis, so far as it concerns life assurance, may not be amiss here.

Consumption is now clearly made out to be caused by the entrance into the body, and growth there, of a specific germ—a microscopic rod called the tubercle bacillus. The disease germ is frequently found in the expectoration of consumptive patients, and there is little doubt that the disease is disseminated chiefly by means of dust formed by the drying of the expectoration of phthisical patients.

The degree to which persons are subjected to the infection varies according to the circumstances in which they live.

The susceptibility to the disease varies in different persons, and in the same person according to the condition of his health. It would also seem to vary according to his age.

Though disputed by some, there is little doubt that persons specially susceptible to the disease are, as a rule, of delicate constitutions.

The susceptibility to phthisis, as in all zymotic diseases, is largely hereditary.

There is no reason to suppose that the actual disease is transmitted from parent to offspring, and the fact that phthisis is rare in childhood would point to the conclusion that if such transmission does take place, it is only in exceptional cases.

With these considerations in mind, the following method for guarding insurance offices from loss is indicated:

1. Let special classes be formed, according to the different varieties of consumptive family history, and let the extra risk corresponding to these classes be ascertained and charged.
2. In the examination of applicants having a consumptive family history, let great care be taken to accept only those who are robust, and whose occupations do not specially expose them to the infection of the disease.

The first is actuarial, the second medical work.

How far the offices have been protected from loss by consumption generally will be seen by the following comparison of the death-rate from consumption, deduced from the experience of three insurance offices with those recorded of males in the *Registrar-General's Report (Supplement, 1885)*:

Age at Death	INSURANCE OFFICES			REGISTRAR- GENERAL
	Policies at Risk	Deaths from Phthisis	Deaths per Mille per Annum	Deaths per Mille per Annum Males
20-24	10,253	21	2.05	3.09
25-34	88,294	203	2.29	3.70
35-44	130,721	299	2.28	4.12
45-54	106,988	186	1.93	3.86
55-64	57,412	103	1.79	3.19
20-64	392,668	802	2.06	3.76

From the same supplement, it appears that the percentage of children dying of consumption is very small. The effect of age upon the frequency to contract phthisis is thus referred to in a standard medical text-book published before 1885:—"No age is free from liability to tuberculosis; it is extremely common in young children; but, putting these on one side, the age of greatest liability is from 20 to 30 or 35." I have quoted this to show how erroneous results are apt to be when founded upon medical experience alone.

The following tables give the deaths which have occurred from various causes amongst members of the Mutual Life

Table showing the Causes of Death of Members of the "Mutual" who had a Consumptive Family History.

Relations who died of Consumption (or Childbirth)	1. Fathers only	2. Mothers only, Consumption	3. Mothers, Childbirth	4. Father and Mother, Consumption	5. Father and Mother or Sister	6. Mother and Brother or Sister	7. Mother, Childbirth, Sister, Childbirth	8. Mother, Childbirth, Brother or Sister, Consumption	10. Sister, Childbirth	11. One Brother or Sister, Consumption	12. Two or more Brothers or Sisters	Totals	Ratio to 100 Deaths
CLASS I—ZYMOTIC DISEASES.													
Miasmatic	2	5	1	3	2	1	14	5.49
Dietic	1	1	.39
	...	2	5	1	3	2	2	15	5.88
CLASS II—CONSTITUTIONAL.													
Dropsy	1	1	1	2	.78
Cancer	1	1	...	1	1	1	1	2	11	4.31
Tumours, &c. . . .	1	1	1	...	1	...	1	1	1	6	2.36
Tubercular	4	4	11	1	1	1	2	9	3	36	14.12
	6	7	13	1	2	2	1	...	3	14	6	55	21.57
CLASS III—LOCAL.													
Nervous System	4	3	9	2	1	5	18	2	44	17.25
Organs of Circulation	2	2	9	1	4	16	1	35	13.73
Respiratory Organs	3	5	9	1	5	8	4	35	13.73
Digestive Organs	2	2	10	1	4	7	2	31	12.16
Urinary Organs	5	6	2	3	16	6.27
Organs of Locomotion	1	1	.39
	12	12	42	2	...	5	24	51	12	162	63.53
CLASS IV—DEVELOPMENT.													
Old Age	1	1	1	2	3	...	8	3.14
CLASS V & VI.													
Violent and Sudden Deaths	1	...	6	1	1	3	2	14	5.49
CLASS VII.													
Causes not classified	1	1	.39
TOTALS	20	22	68	3	2	9	1	2	33	73	22	255	100.00

Table of the Comparative Ratio of Deaths from Various Causes.

	Mutual, One Parent, of Con- sumption	Mutual, Mother, of Child- birth	Mutual, Colla- terals, of Con- sumption	Scottish Widows' Fund, 1815-1873, General	British Empire Mutual, 1847-1878, General
CLASS I—ZYMOTIC.					
Miasmatic	4.76	7.35	3.09	11.89	...
Dietic	1.03	.78	...
Others02	...
	4.76	7.35	4.12	12.69	6.95
CLASS II—CONSTITUTIONAL.					
Dropsy	4.76	} 5.42	17.75
Cancer	2.38	1.47	6.19		
Tumours, &c.	4.76	1.47	2.06		
Tubercular	19.05	16.18	12.37	7.58	17.75
	30.95	19.12	20.62	13.00	17.75
CLASS III—LOCAL.					
Nervous System	16.67	13.24	21.65	21.00	16.24
Organs of Circulation	9.53	13.23	18.56	14.20	11.04
Respiratory Organs	19.05	13.24	12.37	12.91	14.66
Digestive Organs	9.52	14.71	9.28	11.50	11.93
Urinary Organs	7.35	5.15	5.86	4.75
Organs of Locomotion	2.3843	1.13
Integumentary System18	.44
Organs of Generation14	...
	57.15	61.77	67.01	66.22	60.19
CLASS IV—DEVELOPMENT.					
Old Age	1.76	1.47	3.09	4.30	2.89
Childbirth, &c.98
	3.87
CLASSES V AND VI.					
Violent and Sudden Deaths	2.38	8.82	5.16	3.15	4.00
CLASS VII.					
Causes not classified	1.4764	7.24
	100.	100.	100.	100.	100.

Assurance Society with consumptive family history, arranged in the classes which it is usually considered advisable to adopt. In the second of these, comparison is made with the causes of deaths occurring amongst the general body of some other life offices.

The excess of mortality from consumption in the class where one parent died is in accordance with the conclusions come to by medical examiners.

I may add that the percentage of deaths from consumption compared with other causes has been, for the period 1836-1890, 8·5 per-cent.

The question of the value of age in selecting lives with consumptive family history can be settled only by actuarial investigations. There are no data in the hands of medical men to disprove or prove any theory that may be formulated. There is, however, an impression amongst medical examiners that lives which have lived longer than the relative who died of consumption, are better than those that have not done so.

Again, the question as to how far the age of the applicant alone should influence his selection remains unsettled, but could no doubt be put at rest by collecting a larger number of statistics from the insurance offices. From a medical point of view, one is tempted to defer the examination, in the case of phthisical family history, on the principle that the older a man is the easier it is to judge of his constitution.

The rule by which all applicants born within seven years of the death of their phthisical parent are refused does not hold at the Mutual, and is, I imagine, becoming obsolete—it is a relic of the idea that consumption was entirely hereditary or developmental, and was not dependent upon the entry of a germ from without.

The fact brought out by the Mutual figures that childbirth in the mother may be regarded as equivalent to consumption is very interesting. No doubt in many of the cases when childbirth is given as the cause of death, the real one is consumption; but apart from this, the higher mortality in this class is not surprising when we remember that the inability to perform a natural function denotes a feeble vitality—a condition pre-eminently hereditary.

DISCUSSION ON THE PAPERS OF MR. MANLY AND DR. LYON.

The PRESIDENT (Mr. B. Newbatt) was glad to know that they had present several medical men, who were not only skilled examiners in connection with insurance companies, but who also had a special knowledge of the terrible disease consumption; and he hoped they would take part in the discussion. He should like to say that he was not that "President of the Institute of Actuaries"

who enunciated the startling doctrine that "doctors step in where actuaries fear to tread." He had been, during the whole course of his business life, so closely associated with medical men of eminence, who brought their stores of knowledge and acuteness to the consideration of questions of this kind, that he should be the very last man to wish, in the slightest degree, to detract from the great value which medical science had conferred upon the cause of life assurance. He called upon the two referees to open the discussion.

MR. JAMES CHISHOLM said that consumption was a disease which pre-eminently attacked and developed itself in lives at the assuring ages. Seeing that that was the case, it was perhaps surprising that so few statistics had been brought forward. Many reports had been made upon the mortality experience of different offices, but there was no published record dealing with the one disease consumption, in such full detail as in Mr. Manly's paper. They must not expect to attain finality at the outset, and Mr. Manly had shown the spirit in which he submitted his paper by giving the full facts so as to enable others to contribute their experience for comparison. In Mr. Manly's "Summary of Observations", there were given 67 cases in which the father only was afflicted with consumption, 71 in which the mother only was so afflicted, and 208 in which the mother died in childbirth. He seemed to imply that in those cases where the mother died in childbirth the cause of death ought to have been truly stated as consumption. He (Mr. Chisholm) agreed that in the great majority of cases submitted to insurance offices in which the mothers were stated to have died in childbirth, it was found that the cause of death was really consumption. He did not understand how Mr. Manly had been able, from internal evidence, to decide absolutely that the mothers in those cases did die of consumption; but, assuming the deduction to be correct, they found there were 67 cases in which the father died of consumption, and 279 cases in which the mother died of consumption or in childbirth. He had never heard that fathers were less subject to consumption than mothers, or males than females, and he could not understand at first sight how this disproportion between the two sets of figures arose. He could only suppose that, as in the case of the mother the real cause of death was covered up by the statement that it happened in childbirth, so when fathers died of consumption the cause of death was stated to be bronchitis, congestion of the lungs, pneumonia, or some such disease. He did not say that the offices were fraudulently deceived. The word "consumption" was a very ugly one, and people did not like it to be known that it had occurred in the family. As regards the conditions under which the observations were made, he asked whether the mode of selection was the same now as it was fifty years ago. Mr. Manly said they were all first-class lives. So they were, in the sense that they had passed the scrutiny of a medical man who had brought to bear his best judgment, but he must have been governed by the theories in vogue at the time. If they found there was an idea in the medical world at the time of selection that physique, conformation, robustness, had nothing to do with a tendency to develop consumption, they could imagine that the doctor would admit lives irrespective of the fact that father or mother

had died of consumption, so long as there was no positive disease or malformation to be found in the applicant. In many cases it was undoubted that lives passed 30 or 40 years ago would not have been passed at the present time, or would only have been accepted at a considerably increased rate of premium. In his own office, he found that for the period from 25 to 50 years ago they were in the habit of disregarding family history altogether. That was found in practice to lead to an increased rate of mortality. The practice was changed, and those whose parents had died from consumption were put in a class by themselves, and charged with a premium considered adequate to the risk—generally representing five years' addition to the real age. The result was found still more unsatisfactory, not on theoretical but on practical grounds. If the applicant could get insured elsewhere at the ordinary rates he would do so, and the result was that the special class contained those lives that could not get insured elsewhere, and did not retain those personally robust and healthy. Following on this further experience, the practice was again changed, and that one adopted which he believed was now followed by a great many other offices, of differentiating between one life and another, and if a life had attained the age of 35 without any indication of disease, and had a good physique, conformation, weight, and occupation, and if the age at which the father or mother had died were passed, it was probably admitted at ordinary rates. Below 35 it would be looked upon with very great caution, and at the younger ages especially it would be rated up heavily or declined. From the other tables he thought it would be legitimate to draw the conclusion that, if they omitted those lives where the father or mother died of consumption under age 25, they might exclude a great deal of the extra risk of consumption, and then the results shown by Tables I, II, and III, would be very similar to those of Tables X, XI, and XII.

Mr. GEORGE KING said that, as illustrating the danger of trusting too much to statistics without giving due weight to the number of observations, he might point out that where both father and mother and some collaterals had died of consumption they did not have a single death. They must be careful how they judged of percentages only, and the actual totals must be borne in mind. The first interesting fact that came out from Mr. Manly's enquiry, was that it did not make very much difference whether it was the father or mother who died of consumption. He had believed that the death of the mother was a very much more serious blot in a family history than that of the father, but from these statistics that did not appear to be the case. Another peculiarity was the fatality when mothers had died in childbirth, not of consumption. The fact that there were 69 deaths of the father from consumption and 83 of the mother where it was certainly consumption, would go to show that probably consumption was not the cause of death in any considerable number of those 208 cases of deaths in childbirth. It would be useful to have statistics of deaths from consumption where there was no consumptive family history. The Scottish Widows' Fund at each septennial valuation gave a table of causes of death, the numbers at risk at each interval of ages, and the deaths from each cause, and from that one could work out results of the most valuable kind. The figures of the

Scottish Widows were much more favourable than those quoted by Dr. Lyon, and in many cases were more than 50 per-cent below the death-rates from consumption in the general population. That was in large part due to medical selection. Consumption usually took some years to produce a fatal result, and during that period the life was ineligible for insurance. As to the matter of graduation, Mr. Manly explained very clearly how he required to take certain large groups in order to get a smooth sequence of value, and there was some danger in such arbitrary methods of grouping. But in Mr Manly's hands the results might be trusted, because they were afterwards fully tested by putting side by side for each year of life the expected and the actual deaths. The question of heredity in consumption was a most interesting one, and if one might judge from what Dr. Lyon and other medical men had said, it seemed to be thought that heredity was a susceptibility to disease but not the disease itself, and that the disease had to be introduced by an outside agent altogether. As scarlet fever was infectious and not hereditary, so possibly it might be found that consumption was infectious more than hereditary. He joined in Mr. Manly's hope that there might some day be a union among companies for the purpose of making a collective investigation of the subject.

Dr. LYON, referring to Mr. Chisholm's remark as to the change in regard to medical examination, said that when looking over a number of old declined proposal papers of the Mutual, he came to the conclusion that Dr. Brinton, who was the medical examiner of most of the town cases, was quite as well able to find out whether a man was robust or not as anyone at the present day. He noticed that occasionally, where there was a family history of consumption, Dr. Brinton refused a life on apparently somewhat frivolous grounds. But it could be seen perfectly well that Dr. Brinton thought it was not a robust life, and refused it really on that account. He believed the lives accepted by the office, say 20 years ago, as robust lives, were as likely to have been robust lives as those passed yesterday. The cause of death, childbirth, simply meant that and nothing else, and the cases were put in a class alone. With respect to consumption, they only included those cases in which they were practically certain that death was due to that disease. He hoped that as the result of the discussion other offices would make investigations, so that the medical officers might be relieved of some responsibility.

Dr. POLLOCK said that there could be no doubt that the influence of the parent was supreme in governing the heredity of consumption, and he believed the condition of collateral relatives had not much to say in regard to it. Those two points had been clearly laid down in the paper, which was the result of a compilation of facts extending over 50 years. He believed with Mr. Chisholm that childbirth commonly included consumption, but that it must be held also to include the ordinary incidents of childbirth, such as hemorrhage and puerperal fever. They were constantly being deceived by the public on that point, and medical officers could not be too minute in their cross-examination or too careful to get certificates from Somerset House to prove what was the real cause of death when childbirth had been put forward. He

was personally gratified to find that the paper was a large contribution of facts to the truth of heredity. The office, the Mutual, did not apparently start with the idea that heredity was to be much considered. They were taking first-class lives only, and at that time, as Mr. Chisholm had correctly stated, it was not the habit for medical officers of companies to lay much stress on family history. The fact now came out, not only about phthisis but about several diseases, that heredity was a main factor. If they found that the father or mother had influence in transmitting consumption, and that the collateral relatives of the same generation did not influence the life in question, they must see that something had been transmitted by the parent. What was "transmitted"? Was it a germ, a bacillus, something tangible that could be weighed in the balance, that could be analyzed chemically or could be placed under a microscope? Perhaps not. But it might be said that it was a susceptibility, a liability. How could a susceptibility be transmitted? What could be transmitted except something tangible? That a mere susceptibility was transmitted he could not believe. The transmitted disease or susceptibility manifested itself very often at a certain age in the same family. He knew of families many of whom had died at the ages of 25 or 27 of phthisis. He knew, also, that forms of disease were transmitted. Some varieties ran on for years and did not eventually kill, such as were called fibroid phthisis; others came rapidly to a conclusion. He had noticed in families that the forms of disease were transmitted, that the father transmitted to the child a rapid form which killed him in six months at the age of 25; that a brother grew up to the same age and died, and a sister, and so on. Then there must be something more tangible than a susceptibility, something more than a prepared bed well manured in which the seed was to grow. He was glad to hear Mr. Manly state that medical investigation should be most strictly carried out, in addition to enquiry into family history; and if the family history was fairly given, and medical investigation rejected those who were not robust, or had had a consumptive parent, they would arrive at the greatest safety in insuring life. It was suggested that they should have uniformity of rating. The offices saw through the same spectacles, they had the same interest at heart, the same knowledge to go upon, and therefore should, he thought, arrive at a greater uniformity. He thought Mr. Manly did not add sufficient rating for consumptive parents. The father transmitted that which he had proved increased the mortality 24 per-cent and the mother about 25 per-cent. As physician to the Consumption Hospital, he could say that the figures quoted were not brought forward to illustrate a theory, but as facts. They were bricks which must be made use of, if it was desired to build up a history of consumption.

Dr. MACKENZIE said that he was much struck with the second of the general conclusions arrived at by the author, that cases of consumption in a family other than in a parent might be disregarded where the applicant was perfectly sound. Mr. Chisholm had truly observed that there was the greatest difficulty in getting an accurate account of the deaths of parents. It would be admitted by all who had seen much of consumption, that a very strong factor in the

causation of the disease was heredity, and if a brother or sister had died from consumption it was very strong evidence of a tendency in the family to consumption. Having analyzed to some extent the evidence upon which Mr. Manly had founded his second conclusion, he had noticed what Mr. Chisholm had also pointed out, namely, that if they combined Tables I, II, and III, and in the same way Tables X, XI, and XII, they found a much larger proportion of young lives in the first three. That, no doubt, had a good deal to do with the results on which the second conclusion was founded.

Mr. H. R. HARDING referred to the statement made by Mr. Manly, that some offices accepted lives with a consumptive tendency at the ordinary rate under the endowment assurance table. He hoped that was not the usual practice, as it was undoubtedly the fact that with such lives the heaviest mortality was in the earliest years of the assurance. The Washington Life Office in 1890 issued a report in reference to the deaths of its lives assured. The report gave the causes of death of 2,000 lives, of whom 353 died of consumption. Of these latter, dividing into two groups only—

73.09 per-cent	entered under 40 years of age, and
26.91	over that age.

Then, in regard to their ages at death:

76.77 per-cent	were under 50 years of age.
23.23	50 years of age or over.

Then, in regard to the effect of medical examination, the report showed, that of the 353 deaths, there occurred:

In the first year of insurance	10.90 per-cent.
„ 2nd „ ..	22.47 ..
„ 3rd to 5th (inclusive)	24.18 ..
„ 6th to 10th ..	17.36 ..
Above the 10th year	13.94 ..

These figures seemed to show, that in the case of lives with a consumptive tendency, medical examination was of little or no avail beyond the first year. The report confirmed Mr. Manly's conclusion that consumption in the parents was of greater importance than amongst the brothers and sisters—the relative mortality in the former case being double the latter. It was also shown, that in the case of a life with a tendency to consumption, the percentage of deaths was only 6.13 when the weight of the assured was above the standard; that it was 27.27 when the weight was at the standard; but 48.39 when the weight was under the standard. That was an important conclusion, showing that in any future investigation weight, as compared with height, must be a material factor, in addition, of course, to occupation.

The PRESIDENT said that the one thing that had most impressed itself on his mind, in reading the statistics, was the necessity of the caution which had been given by Mr. Manly in respect of the smallness of the number of facts. Mr. Manly had said that, “so far

as these observations have any weight, cases of consumption in the family, other than in a parent, may be disregarded where the applicant is perfectly sound." He (the President) was not prepared, even on the strength of those figures, to accept that as a general statement which might be relied upon. They had heard from Dr. Pollock—as, indeed, they would all expect on *a priori* grounds—that if consumption was a transmissible disease, they had to deal with a very much more serious fact when it was the parent who had manifested the disease rather than a collateral. Dr. Mackenzie, differing somewhat from Dr. Pollock, thought that death amongst collaterals was serious evidence of taint of heredity, even though the disease had not manifested itself in either parent. There was a popular idea that those diseases often skipped a generation, and if that were so, the presence of consumption in a collateral might be evidence that a grandfather or grandmother, even though the father and mother had themselves escaped, might have transmitted the disease. Mr. King had already pointed out an anomalous result, due to the smallness of the observations, and others might be mentioned. That led him to the suggestion which Mr. Manly had thrown out, that the offices should combine their experience. As a general proposition, no one was more willing than himself to give assent to that aspiration, but he was not sure that the times were propitious for any such proceeding. They were living in days when they could not afford to be frank. The meannesses and sinuosities of modern competition were such, that he doubted whether in many cases they would be well advised to lay before the world at large many of the facts which were in the keeping of some of them. Reference had been made to the desirableness of uniformity of rating. That was only another phase of the same problem. Dr. Pollock had been bold enough even to suggest uniformity of rating with regard to invalid lives. He would remind Dr. Pollock, in the first place, that there was no such thing as uniformity of rating, even with regard to healthy lives. Practically, there were no two tables of rates for healthy lives exactly the same. But that was not the most important point of the case. There was, further, that most honourable reason, that judgment necessarily differed with regard to difficult cases. That in itself was a factor which could not possibly be eliminated, and he was not quite sure that it was one they would desire to eliminate. And, in the third place, they had to deal with that terrible thing called competition. It had been said that when they rated up lives they did not secure, probably, more than one-half, and the worse half—those who either accepted their dictum because they knew they had got the best of it, or had failed to get better terms elsewhere. He therefore feared that the idea of uniformity of rating was one of those things which must be classed amongst chimeras. Of one thing, however, they would all be certain, that the labour of Mr. Manly was not in vain. He had, if not by his actual deductions, at least by his methods and processes, set them an example which they would be, he hoped, in many cases only too glad to follow.

Mr. MANLY, in reply, said that when he commenced to collect his facts, he feared the subject was almost too difficult to deal with, as

there were so many combinations, and that when they were unravelled he would obtain so few facts in each class that they would be of very little use. As the matter grew, however, it really appeared that the facts might be made of some value, and hence their appearance in their present form. With reference to the peculiarity pointed out by Mr. Chisholm that in the class where lineals had died of consumption they found younger lives than where there were deaths of collaterals only, he pointed out that such must necessarily be the case. What age could a collateral be at death who was a brother of the person who presented himself, say at the age of 20? On the other hand they might have a great many young lives whose fathers or mothers had died of consumption between the ages of 35 and 55. It would, of course, be desirable if they could get a sufficient number of facts, to separate them into classes according to the age at entrance, say to place all those who entered under 30 or 35 into one class and those who entered over that age into another class, but he doubted whether they would obtain any different results.

On a system of Bonus Distribution considered in relation to the Office Premiums and modes of Valuation. By HOWARD J. ROTHERY, Fellow of the Institute of Actuaries, Actuary and Secretary of The British Empire Mutual Life Assurance Company.

[Read before the Institute, 28 March 1892.]

IT has frequently been insisted upon that in the financial management of a life assurance company the scales of premiums, the method of valuation, and the mode of distribution of surplus should be considered together, and not independently of one another. Many instances could be given of unfairness in the treatment of the members of different classes and ages due to a want of recognition of this principle. It is, of course, impossible, in the practical management of a life office, that abstract justice should be administered as between the various policyholders; but as the system of charging uniform premiums for all ages at entry has given way to that of graduating the premiums according to the age of the life assured, so it may be possible and politic from time to time to find means of making the payments of the assured still more accurately conform to the exact value of the benefits they are to receive.

In considering a system of bonus apportionment, therefore, the premiums payable and the method of valuation should be taken into account. It is also very desirable that the system adopted should be one easily applied, and should give results

which meet the popular requirements. As regards the last-named condition, I think it will be admitted that the majority of the assuring public desire a system which will give reversionary bonuses which increase in amount with the duration of the policy. In proof of this I may mention that in nearly all recent cases of alteration of bonus system, the plan known as the "compound reversionary bonus system" has been adopted in place of some other method. There are many different systems in vogue at the present time, and it is probable that by suitably choosing the rates of premium and the valuation basis they could nearly all be made to give fairly equitable results. To analyze these different methods and to discuss the most suitable modes of applying them would, however, require greater time and attention than I am able to bestow upon this paper, and it is necessary to confine myself on the present occasion to the principal methods which meet the popular demand for increasing bonuses. And of these I will deal with three plans only, namely, (1) the compound reversionary bonus system, (2) the "contribution" plan, which in its simplest form divides the surplus into "interest" and "loading" profit, and (3) a modification of the contribution plan which I shall describe.

COMPOUND REVERSIONARY BONUS SYSTEM.

A large number of offices have adopted this well-known system (under which the reversionary bonus is calculated as a percentage upon the sum assured and previous bonuses), and it possesses the very great advantage of simplicity and of being easily understood by the public. But undoubtedly the scale of premiums should be carefully considered before adopting this plan. It is frequently said that competition entirely fixes the rates of premium to be charged for life assurance, but this statement is not strictly true regarding non-profit policies, and certainly requires qualification when dealing with participating assurances. To show that this point is sometimes overlooked in practice, I give the rates for whole-life with-profit assurances at ages 20, 40, and 60 of two offices, both of which adopt the system of compound reversionary bonuses:

Age	Office A				Office B			
20	£2	2	1	per-cent	£2	2	1	per-cent
40		3	6	3		3	5	5
60		7	4	9		6	15	5

and many similar discrepancies could easily be found. It is true

that few policies are effected at so advanced an age as 60, but a table of premiums should certainly be consistent; and sometimes lives of a still greater age are assured. A case recently arose where the with- and without-profit rates were calculated (consistently with the published tables for younger ages) for age 79, the difference between the two being £3 per-cent, while the office was declaring a compound reversionary bonus of about 30s. per-cent only. In fact, it is clear that, under the system in question, the older lives can never live long enough to obtain very large bonuses, and therefore the rates of premium should be low compared with those suited to other bonus systems. The rates of Office B are calculated (upon the "select" mortality tables) to provide for compound reversionary bonuses of £1 per-cent allotted quinquennially, and are loaded accordingly; those of Office A are probably calculated by adding a percentage loading to the net premiums.

It seems to me that some such principle as that employed in the calculation of the premiums in the case of Office B would be correct in the case of an office adopting the compound bonus system. If, however, an office charges premiums which are proportionately high at the older ages, it should be considered whether some other method is not more suitable, unless the office is prepared to entirely remodel its rates— a course which has many disadvantages, and which, unless a new section is commenced, produces unfairness as between the new and old members.

An objection which may be urged against the compound bonus system is its inelasticity. For example, it is pretty evident that if an office reduces its valuation rate of interest from $3\frac{1}{2}$ per-cent to $2\frac{1}{2}$ per-cent during a certain period, some adjustment of the apportionment of bonus between the older and the newer policyholders ought to be made, but under the system in question this cannot be done. The system is, however, deservedly popular, and with valuations made at a fairly low rate of interest, to provide for future increases in the rate of bonus, has been found to work well in practice.

"CONTRIBUTION" METHOD.

This method has been adopted in various forms, and in theory distributes the profits with strict equity. The profit or loss from interest, mortality, loading, and other sources, is ascertained and apportioned, and the result is usually to produce bonuses which increase with the age at entry and the duration of the policy. I

cannot help thinking that this attempt at refinement is unnecessary, and involves a needless amount of labour. It is also likely to produce irregularities in the results, owing to some accidental cause operating during the valuation period. The method in its simplest form divides the profit into two portions only, namely, "interest" and "loading" profit, and it seems to me that it is undesirable to attempt to carry the apportionment further. The time has not arrived when doctors and grocers, butchers and actuaries, are to be placed in separate sections, although it is quite evident that these classes are not subject to the same rate of mortality. In fact, as some offices are willing to assure mariners and persons proceeding to tropical climates at the ordinary rate of premium, the tendency appears to be quite in the other direction. In devising the modification of the contribution method which I am about to explain, I have therefore kept in view the desirability of dealing with the question on broad and general grounds.

MODIFIED "CONTRIBUTION" SYSTEM.

To fully explain the suggested modification of the "contribution" system, I have thought it desirable, by way of example, to set out in some detail the conditions and results regarding a particular office to which the method has been applied, and this may indicate the manner in which the general question should, in my opinion, be approached.

In the office referred to, the rates of premium for different classes of assurance had been calculated from time to time upon various bases, and were in some respects inconsistent with one another. But these faults were not sufficiently pronounced as to render it desirable to close the existing series of policies, and to commence another with new rates of premium and a new bonus system—a plan which, for many reasons, should not, I think, be adopted, unless the circumstances are very exceptional. And, of course, the tables of premiums, upon whatever basis calculated, had been framed, to some extent, with a view to competition with other offices, and they were thus (so far as the principal tables are concerned) fair average rates.

Dealing first with the non-profit premiums, in regard to which the bonus system did not require consideration (but an examination of which will simplify the discussion of the with-profit rates), and remembering that the primary object was to see that the actual risk was covered, and that the rates contained a fair margin

for expenses and contingencies, a few general principles were easily fixed upon.

The assumed rate of interest was 4 per-cent—certainly the highest that could be adopted—perhaps some may think it too high. As regards mortality, Mr. T. B. Sprague's extremely valuable "select" tables, founded upon the experience of lives of the exact age at entry, appeared to me to give exactly what was required for the calculation of office premiums. The premiums based upon the ordinary H^m Table are undoubtedly too low at the younger and too high at the older ages.

Some of the "select" values are given in the pages of the *Journal* (*J.I.A.*, xxi, 229; xxii, 391), while others appear in a pamphlet printed by Mr. Sprague for the use of his own office. There is some reason to hope that these and other tables based upon "select" mortality may be published in separate form, and thus be made more accessible to members of the Institute. When the preparation of a new experience of life offices is undertaken by the Council, an analysis of the effects of selection upon the lines laid down by Mr. Sprague will probably form one of the most interesting and instructive portions of the work.

Having decided upon the rate of interest and the mortality table, the next step was to provide for the expenditure and the margin for contingencies. The commission payable and some of the other charges are now generally calculated upon the sum assured, rather than upon the amount of premium payable. The commission is frequently £1 per-cent upon the sum assured, the stamp duty and the medical fee partially depend upon the amount of the policy, and the branch office expenses are implicitly if not explicitly regulated in the same manner. On the other hand, the amount of the first premium does have some influence upon the expenditure. The following loadings were therefore decided upon:

1. In respect of the first premium, £2 per-cent upon the sum assured, and 5 per-cent upon the gross premium.
2. In respect of each premium, including the first, 8 per-cent upon the gross premium.

The total loading therefore amounted to £2 per-cent on the sum assured, 13 per-cent upon the first premium, and 8 per-cent upon the renewals. (It may be incidentally mentioned that upon an average premium of £3 per-cent, this amounts to 80 per-cent of the first premium and 8 per-cent on renewals.)

Writing P^{NP} for the office non-profit premium, the formula is

$$\begin{aligned} P^{NP} &= \frac{1}{1 + a_{[x]}(4^{\circ}_{\circ})} \cdot 02 + \frac{05 P^{NP}}{1 + a_{[x]}(4^{\circ}_{\circ})} \cdot 1 \\ &= \frac{P_{[x]}(4^{\circ}_{\circ}) (1 + a_{[x]}(4^{\circ}_{\circ})) + 02}{092 \{1 + a_{[x]}(4^{\circ}_{\circ})\}} - 05 \\ &= \frac{\Lambda_{[x]}(4^{\circ}_{\circ}) + 02}{092 \{1 + a_{[x]}(4^{\circ}_{\circ})\}} - 05 \end{aligned}$$

This formula was used in the calculation of the whole-life non-profit table, and (with the proper substitutes for $\Lambda_{[x]}$ and $a_{[x]}$) for endowment-assurance premiums. The limited-payment rates were derived from the formula

$${}_nP^{NP} = P^{NP} \cdot \frac{1 + a_{[x]}(4^{\circ}_{\circ})}{1 + {}_{n-1}a_{[x]}(4^{\circ}_{\circ})},$$

and, for the single premiums,

$$A^{NP} = P^{NP} \{1 + a_{[x]}(4^{\circ}_{\circ})\}.$$

The results are given in the following table:

TABLE A.
Office Premiums for Non-Profit Assurances.

Age	WHOLE LIFE					ENDOWMENT ASSURANCES				Age
	Annual	Single	15 Payments	25 Payments		Death or 50	Death or 60			
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.			
20	1 12 8	31 2 8	2 17 1	2 2 1	2 14 2	2 0 10				20
30	1 19 10	35 15 11	3 5 5	2 8 8	1 6 0	2 15 9				30
40	2 14 3	43 13 5	4 1 1	3 1 9	9 11 8	4 10 9				40
50	3 19 4	54 1 9	5 5 0	4 4 1	...	9 18 4				50
60	6 5 3	66 13 6	7 4 7					60

These rates appeared to be suitable for adoption, and there are undoubted advantages in a formula which applies equally to several classes of assurance.

Having decided upon the non-profit rates, the premiums for participating assurances had to be considered. The system of bonus distribution in the office in question was that of an uniform cash percentage upon the premiums paid during the intervaluation period. This plan, like most others, can no doubt be made fairly equitable if the premiums and valuation basis be suitably chosen; but it is an unpopular system at the best, and it had been decided that it should be superseded by one which

provided bonuses which increased with the duration of the policy. It was found by experiment that the existing tables of premiums were not suited to the compound reversionary bonus system, and there were other reasons which made it desirable to look at the question from a cash bonus rather than a reversionary bonus point of view. Taking the non-profit rates as a standard, it was easy to see that a further cash percentage loading would provide an uniform rate of cash bonus, while a reduction in the rate of interest assumed would allow for an increment in the rate. I found that an increase of 10 per-cent in the loading, combined with a reduction in the rate of interest to $3\frac{1}{2}$ per-cent, would produce premium rates which differed but slightly from the existing rates, both as regards whole-life policies and endowment assurances. The with-profit premiums (P^{WP}) were accordingly calculated from the formula

$$\begin{aligned} P^{WP} &= \left\{ P_{[r]}(3\frac{1}{2}\%_o) + \frac{\cdot 02 + \cdot 05 P^{WP}}{1 + a_{[r]}(4\%_o)} \right\} \cdot 82 \\ &= \frac{P_{[r]}(3\frac{1}{2}\%_o) \{1 + a_{[r]}(4\%_o)\} + \cdot 02}{\cdot 82 \{1 + a_{[r]}(4\%_o)\} - \cdot 05} \end{aligned}$$

The following are the resulting values; the single and limited premiums being obtained from P^{WP} by commutation, at 4 per-cent interest:

TABLE B.
Office Premiums for With-Profit Assurances.

Age	WHOLE LIFE												ENDOWMENT ASSURANCES						Age
	Annual			Single			15 Payments			25 Payments			At 50			At 60			
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
20	1	18	7	36	16	4	3	7	5	2	9	9	3	4	3	2	8	7	20
30	2	7	2	42	7	6	3	17	5	2	17	7	5	0	11	3	6	1	30
40	3	3	9	51	6	2	4	15	3	3	12	6	11	0	8	5	6	2	40
50	4	12	3	62	17	6	6	2	1	4	17	9	11	8	2	50
60	7	4	0	76	13	8	8	6	3	60

For deferred-profit policies (*i.e.*, policies at reduced rates of premium, the bonuses being contingent upon the attainment of a specified age) the non-profit loadings were retained, but $P_{[r]}$ was taken at $3\frac{1}{2}$ per-cent. Thus

$$\begin{aligned} P^{DP} &= \left\{ P_{[r]}(3\frac{1}{2}\%_o) + \frac{\cdot 02 + \cdot 05 P^{DP}}{1 + a_{[r]}(4\%_o)} \right\} \cdot 92 \\ &= \frac{P_{[r]}(3\frac{1}{2}\%_o) \{1 + a_{[r]}(4\%_o)\} + \cdot 02}{\cdot 92 \{1 + a_{[r]}(4\%_o)\} - \cdot 05} \end{aligned}$$

The formulas for with-, without-, and deferred-profit policies, are thus all identical in principle, and they are very easy of application. The rates can be calculated for every fifth age, and the other values obtained by interpolation.

As has been seen, the rates of premium for with-profit policies contained a margin for profits as compared with the non-profit rates which was partly a percentage upon the premiums themselves, and partly in the lower rate of interest assumed. Leaving for a moment the question of the minor sources of profit, such as mortality lighter than the estimate, lapsed and surrendered policies, &c., the surplus, at a valuation, might be apportioned between interest profit and loading profit. The former could fairly be divided in proportion to the reserve value of the policies at the previous valuation, and the latter as a percentage upon the premiums paid in the valuation period. And, for the sake of simplicity, every other profit might be thrown into one or other of these two divisions.

In the case under notice, the valuations were made triennially, at $3\frac{1}{2}$ per-cent interest, and the actual rate of interest realized was about $4\frac{1}{4}$ per-cent. Accumulating the three years' profit from interest, and allowing something for the miscellaneous profits that might be considered to be appropriately divisible with regard to the duration of the policy, the circumstances seemed to justify an allotment of 3 per-cent upon the reserve value at the beginning of the period as the first portion of the bonus distribution, the remaining surplus being apportionable in respect of the three years' premiums. The percentage of the reserve value is difficult to calculate in practice, but, by means of a very simple approximate proportion which the amount of the reserve value, by any ordinary method, bears to the amount of the premiums paid from the commencement, a plan was devised which, while easy of application, is substantially correct. So long ago as 1857, Mr. Sprague pointed out (*J.I.A.*, vii, 60) that the reserve value of whole-life policies amounted to about two-thirds of the premiums paid, and Mr. G. H. Ryan has noted the same peculiarity with regard to the 11^{th} 3 per-cent values in his article on Surrender Values which appeared in the *Bankers' Magazine* for May 1891.

In the following table a comparison is given between the reserve values of a whole-life policy for £100 and the amount of with-profit premiums paid, the valuation reserves being respectively on the following bases:

H^M $3\frac{1}{2}$ per-cent.

H^M and $H^{M(5)}$ $2\frac{1}{2}$ per-cent.

„ 3 „

„ 4 „

and in each case the premiums (with-profit scale) are taken from the prospectus of a representative office valuing upon the particular basis referred to.

TABLE C.

Age at Entry	Duration	VALUATION BASIS											
		H^M $3\frac{1}{2}$ %			H^M and $H^{M(5)}$ $2\frac{1}{2}$ %			H^M and $H^{M(5)}$ 3 %			H^M and $H^{M(5)}$ 4 %		
		V	nP'		V	nP'		V	nP'		V	nP'	
20	5	3.9	9.6	40	7.4	9.3	80	6.8	9.3	73	5.8	9.2	63
	10	8.5	19.3	44	12.0	18.6	65	10.9	18.6	59	9.1	18.4	49
	20	19.4	38.6	50	24.1	37.3	65	22.1	37.2	60	18.8	36.8	51
	30	33.2	57.9	57	38.5	55.9	69	36.1	55.7	65	31.9	55.1	58
30	5	5.6	11.8	48	8.1	12.2	67	7.5	12.2	61	6.4	11.8	54
	10	11.9	23.6	50	15.1	24.4	62	14.0	24.4	57	12.0	23.6	51
	20	26.9	47.2	57	31.3	48.7	64	29.4	48.8	60	26.2	47.2	55
	30	44.2	70.7	62	48.6	73.1	66	46.6	73.3	64	42.8	70.7	60
40	5	8.2	15.9	51	10.4	16.2	64	9.8	16.1	61	8.7	15.9	54
	10	17.1	31.9	53	20.1	32.5	62	19.0	32.3	59	17.0	31.8	54
	20	36.7	63.7	58	40.3	65.0	62	38.7	64.5	60	35.7	63.6	56
	30	56.3	95.6	59	59.7	97.5	61	58.1	96.8	60	55.0	95.4	58
50	5	11.6	23.1	50	13.5	22.7	60	12.9	22.3	58	11.8	22.2	53
	10	23.6	46.1	51	26.2	45.4	58	25.2	44.6	56	23.3	44.4	53
	20	47.4	92.2	51	50.1	90.7	55	48.9	89.3	55	46.4	88.8	52
	30	67.1	138.4	48	69.4	136.1	51	68.3	133.9	51	66.1	133.1	50
60	5	15.7	36.0	44	17.3	36.2	48	16.8	33.6	50	15.8	36.1	44
	10	31.1	72.0	43	33.0	72.4	46	32.2	67.3	48	30.6	72.2	42
	20	56.9	144.0	40	58.9	144.7	41	58.0	134.5	43	56.1	144.3	39
	30	74.9	216.0	35	76.7	217.1	35	76.0	201.8	38	74.5	216.5	34

Upon an inspection of the percentages in this table I think it will be admitted that for purposes of bonus distribution (remembering that the interest profit is only a comparatively small percentage upon the reserve value) the reserve values may be taken to bear a fixed ratio to the amount of premiums paid, ranging from 50 per-cent to 70 per-cent, according to the valuation basis.*

If in the case of the H^M $3\frac{1}{2}$ per-cent valuation this ratio is taken to be 60 per-cent, and assuming, as already mentioned,

* The reserve values given in Table C do not include the value of existing reversionary bonuses or reductions of premium, it being assumed that these have been apportioned upon a non-profit basis.

that the surplus interest amounts to 3 per-cent for the triennial period, we arrive at the result $\cdot 03 \times \cdot 6 = \cdot 018$ as the proportion which the interest profit bears to the total amount of premiums paid.

Dealing, then, with policies assumed to be effected at the beginning of a triennium, we arrive at the following results as regards the apportionment of the profit from interest:

For policies 3 years in force

$${}_0V_x \times \cdot 03 = \text{nil.}$$

For policies 6 years in force

$${}_3V_x \times \cdot 03 = 3P'_x \times \cdot 018 \text{ approximately.}$$

For policies 9 years in force

$${}_6V_x \times \cdot 03 = 6P'_x \times \cdot 018 = 3P'_x \times \cdot 036 \text{ approximately.}$$

For policies 12 years in force

$${}_9V_x \times \cdot 03 = 9P'_x \times \cdot 018 = 3P'_x \times \cdot 054 \text{ approximately.}$$

&c.,

&c.

Thus the portion of the surplus derived from excess interest may be calculated as a percentage upon the premiums paid *in the inter-valuation period*, the ratio being itself an arithmetical progression according to the duration of the policy. Then, supposing the remaining surplus to be sufficient to provide an uniform percentage upon the inter-valuation premiums of $15\frac{1}{2}$ per-cent, the total bonus will be calculated as follows:

For policies 3 years in force

Cash bonus 15.5 per-cent.

For policies 6 years in force

Cash bonus $\cdot 155 + \cdot 018 = 17.3$ per-cent.

For policies 9 years in force

Cash bonus $\cdot 155 + \cdot 036 = 19.1$ per-cent.

For policies 12 years in force

Cash bonus $\cdot 155 + \cdot 054 = 20.9$ per-cent.

&c.,

&c.

The following table shows the percentages of cash bonuses arrived at:

- (a) Giving $15\frac{1}{2}$ per-cent upon the 3 years' premiums to policies 3 years in force, the ratio increasing by 1.8 per-cent for every additional 3 years' duration.
- (b) Allowing 3 per-cent interest upon the reserve value (${}_{n-3}V_x$ at $3\frac{1}{2}$ per-cent), and $15\frac{1}{2}$ per-cent upon the 3 years' premiums.

TABLE D.

Percentage of Cash Bonus to Premiums paid in the Triennium—Whole-Life Policies.

Duration of Policy	Upon Basis (a), All Ages at Entry	UPON BASIS (b), AGE AT ENTRY				
		20	30	40	50	60
3	15.5	15.5	15.5	15.5	15.5	15.5
6	17.3	16.7	16.8	16.9	16.9	16.8
12	20.9	19.5	19.9	20.2	20.2	19.4
24	28.1	26.5	27.4	27.5	26.4	23.7
36	35.3	35.5	36.0	34.8
48	42.5	45.8	44.3	39.9
60	49.7	55.0

It will, I think, be admitted that the approximate results are sufficiently near the truth for all practical purposes. For whole-life policies the approximate reserve-values obtained by taking 60 per-cent of the premiums are rather in excess of the true values.

It will be seen from Table E that the results are slightly more accurate in the case of endowment assurances than for whole-life policies, because of the greater proportionate reserve-values of the former. The table now given shows that the application of the method to such policies is very successful, and certainly points to the fact that endowment assurances are entitled to be as liberally treated in the matter of bonuses (under this particular scheme of distribution at any rate) as are policies for the whole of life.

TABLE E.

Percentage of Cash Bonus to Premiums paid in the Triennium—Endowment Assurances.

Duration of Policy	Upon Basis (a), All Ages at Entry	UPON BASIS (b), AGE AT ENTRY					
		20	30	40	20	30	40
		Endowment Age 50			Endowment Age 60		
3	15.5	15.5	15.5	15.5	15.5	15.5	15.5
6	17.3	17.2	17.5	17.9	17.0	17.1	17.5
9	19.1	19.3	19.8	20.5	18.7	19.1	19.6
12	20.9	21.5	22.4	...	20.5	21.2	22.1
18	24.5	26.8	28.6	...	25.0	26.2	27.9
24	28.1	33.6	30.3	32.3	...
30	31.7	37.0

A table is now given to show, for whole-life policies, the reversionary bonuses per £100 policy, for three years, obtained by three different methods, the cash bonus being obtained as described, and the conversions from cash to reversion being always made by the H^M 3 per-cent Table.

- (a) Giving $15\frac{1}{2}$ per-cent to policies three years in force, the ratio increasing by 1·8 per-cent for every additional three years' duration.
- (b) Allowing 3 per-cent interest upon the reserve value ($_{n-3}V_x$ at $3\frac{1}{2}$ per-cent), and $15\frac{1}{2}$ per-cent upon the premiums.
- (c) Giving an uniform percentage of $22\frac{1}{2}$ per-cent upon the premiums paid in the triennium (an assumed average of the results of *a* and *b*), irrespective of the duration of the policies.

TABLE F.

Reversionary Bonuses for Whole-Life Policies of £100.

Duration of Policy	AGE AT ENTRY 20			AGE AT ENTRY 30			AGE AT ENTRY 40			AGE AT ENTRY 50			AGE AT ENTRY 60		
	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>c</i>
3	2·5	2·5	3·3	2·7	2·7	3·6	3·0	3·0	4·0	3·6	3·5	4·8	4·8	4·8	6·5
6	2·7	2·6	3·1	2·9	2·8	3·4	3·2	3·1	3·8	3·8	3·7	4·5	5·1	5·0	6·2
12	2·9	2·7	2·8	3·1	3·0	3·1	3·4	3·3	3·5	4·1	4·0	4·1	5·7	5·3	5·7
24	3·1	2·9	2·3	3·4	3·3	2·5	3·8	3·7	2·8	4·7	4·4	3·5	6·9	5·8	5·1
36	3·2	3·2	1·9	3·5	3·6	2·1	4·1	4·0	2·5	5·4	4·8	3·2	7·9	6·0	4·9
48	3·2	3·4	1·6	3·6	3·8	1·9
60	3·2	3·6	1·4

The tables of specimen bonuses, cash and reversionary, do not represent any actual distribution, but are chosen so as to give bonuses which, on the average, are about equal to a reversionary bonus of £1 per-cent per annum upon the sum assured.

For deferred profit policies, as the reduction made in the premiums has discounted a cash bonus of 10 per-cent upon the premiums, the bonus allotted may be conveniently taken so that its cash value is 10 per-cent (calculated upon the premiums) less than for ordinary with-profit policies.

The method of bonus distribution which I have thus described may be applied to offices of different kinds and under varying conditions. The great advantage it possesses is that both the

interest and the loading profits can, if the premiums are suitably chosen, be calculated upon the premiums paid during the valuation period, so that a large amount of labour is saved as compared with that involved in the application of the ordinary contribution method. It also possesses great elasticity, which renders it adaptable to the changing circumstances of a company. If the valuation rate of interest is the same as that assumed in the calculation of the premiums, the assessment of the interest profit is a comparatively simple matter. If, however, the valuation rate is reduced, it is not clear that the whole of the interest earned beyond the valuation rate should be distributed as a percentage upon the reserve values. If this were done it seems to me that a serious injustice might be inflicted as regards the policies of shorter durations by reducing the valuation rate. In effect, a portion of the loading of the younger entrants would be taken to provide the increased reserves of the older policies—these older policies would then have bonuses allotted to them, not only upon larger amounts of reserve than before, but also at an increased rate, because of the increase in the interest margin. An office desiring to strengthen its reserves can either reduce its valuation rate of interest, or increase its margin of loading, and yet arrive at the same final result; and it does not seem fair that the accident of the former method being the more fashionable should be the means of entirely upsetting the incidence of the bonus apportionment. The *form* of the valuation, providing the resulting reserve is the same, in no way affects individual policyholders—but the system of bonus allotment touches the pocket of each member.

If an office values at $2\frac{1}{2}$ per-cent, it will hardly be contended that this rate is adopted because the office believes that it is the rate of interest likely to be earned during the currency of the existing policies, and, in fact, the office is probably accepting new business at premiums based upon the assumption of a higher rate of interest. One effect, as is well known, of adopting a $2\frac{1}{2}$ per-cent net-premium valuation is to reduce very materially the valuation margin of loading. In fact, it does this to so great an extent that some offices valuing at that rate have thought it well to increase, arbitrarily, the margin of loading, the result being that each new policy effected requires a considerable reserve at the moment of issue, thus, for a time, reducing the available surplus.

The question is one of great difficulty, but, for the reasons indicated, there seems to me to be a weakness in the contribution

plan of distribution, if the interest profit is taken to be the difference between the valuation rate and the rate actually earned, irrespective of the rate of interest assumed in the calculation of premiums. I am far from proposing that the valuation rate should be entirely ignored, but, under the method I have described, an adjustment may be made in fixing upon the increment in the bonus rate to modify the effect of this upon policies of short duration. On the other hand, if there is a large amount of profit from mortality chiefly derived from policies of long standing, some of this may be added to the interest profit, and therefore find its way principally to policies of longer duration.

In fact, there are an infinite number of ways in which the two parts—fixed percentage and increment—can be chosen, so as to absorb the amount of the ascertained surplus, and it will not be found difficult in practice to choose these values so that the surplus may be fairly apportioned among the policies which have created it, and in a manner which meets the popular demand for increasing bonuses.

If an office values at a high rate of interest the bonus increment must of necessity be small; in fact, if the valuation is at a “true” rate of interest there will be no increment, and the uniform cash bonus method will be the result. If in the valuation some of the loading is anticipated, then the fixed percentage of bonus will be reduced. An example of the latter would be the case of an office which is valued by the well-known formula given by Mr. Sprague:

$$A_{x+n} - (1 + a_{x+n})P_{x+1} \dots \dots \dots (a)$$

Although not coming strictly within the scope of this paper, it may not be out of place to remark that if it is necessary or desirable to depart from the net-premium method of valuation, the formula just given, however suitable for young ages at entry, is not very satisfactory at the older ages, and does not apply to other than whole-life policies. Moreover, it does not afford any means of showing how far the results of a valuation by the formula differ from those derived from a net-premium valuation. I would suggest that an alternative formula, possessing some advantages over that referred to, is as follows:

$$A_{x+n} - (1 + a_{x+n})P_x(1 + k) \dots \dots \dots (b)$$

where k is a small fraction.

This formula is more easily applied than the other if the net premiums have already been calculated. It possesses the great advantage of showing by exactly how much the reserve is reduced as compared with that of a net-premium valuation, and it can be applied uniformly to policies of all classes. Taking the H^M Table and 4 per-cent interest, the ratio of P_{x+1} to P_x is as follows:

Age 20	1.027
„ 30	1.032
„ 40	1.038
„ 50	1.045
„ 60	1.053

The adoption of Mr. Sprague's method would therefore probably have the effect of increasing the net premium valued by nearly 4 per-cent on the average.

Another plan would be to assume that a small percentage upon the sum assured, t , was anticipated from the loading to provide for the initial expenses. The formula for valuation would then be

$$A_{x+n} - (1 + a_{x+n}) \left(P_x + \frac{t}{1 + a_x} \right) \dots \dots \dots (c)$$

A table of $P_x + \frac{t}{1 + a_x} = \frac{A_x + t}{1 + a_x}$ could be prepared, and the values inserted in place of the net premiums. This formula could be adapted to classes of assurance other than whole life, but it is not so easily applied as the one I have mentioned.

It may be useful to compare the value at the opening of the policy of the portion of the loading anticipated under each of the three methods.

By Mr. Sprague's plan we have, from formula (a), the value of the anticipated loading at the commencement,

$$\begin{aligned} & (P_{x+1} - P_x)(1 + a_x) \\ &= \frac{1 + a_x}{1 + a_{x+1}} \cdot {}_1V_x. \end{aligned}$$

At the end of the first year this value is reduced to ${}_1V_x$.

Under plan (b) the amount anticipated is $kP_x(1 + a_x) = kA_x$.

Under plan (c) the amount anticipated is, evidently, t . As regards whole-life and endowment-assurance policies, it would be easy to pass from a net-premium valuation to one upon plan (c). Thus the deduction from the net-premium value in each case would be

$$(1 + a_{x+n}) \cdot \frac{t}{1 + a_x}$$

$$= t \frac{1 + a_{x+n}}{1 + a_x}$$

$$= t(1 - {}_nV_x),$$

or, for all the policies,

$$t\Sigma(1 - {}_nV_x).$$

The correction would not, however, apply to limited-premium policies, &c.

If either of the methods be adopted, care should be taken that negative values are eliminated.

I do not wish to enter upon a discussion of the circumstances under which a departure from the net-premium method of valuation is justifiable, nor as to what should be the values of k and t in the formulas given. Much must depend upon individual judgment in dealing with the question. As a general rule the net-premium system works well in practice, and affords an admirable standard by which to measure the financial progress of a company.

It is a matter of surprise that while so many offices admit the principle of "selection" in their valuations by using the H^M and $H^{M.5}$ Tables, none has at present adopted the values of $P_{[x]}$ for the valuation premiums. It seems difficult to find any valid argument in favour of the H^M and $H^{M.5}$ combination excepting that it gives strong reserves, and the full application of the select method does not appear to greatly increase the work of the valuation.

DISCUSSION.

The PRESIDENT (Mr. B. Newbatt) having called upon the referees, Mr. A. H. Bailey and Mr. T. G. C. Browne, to open the discussion.

Mr. A. H. BAILEY said that the desire for a compound reversionary bonus belonged not to the public but to the middleman. The middleman's argument was that life assurance was an excellent investment, and he endeavoured to persuade his client that the amount he paid would always be less than the amount ultimately received, which was diametrically opposed to the principles of life assurance. The compound reversionary bonus system did real injustice, because, seeing that the premiums were regulated

by competition, he did not think it would be practicable to compute premiums in the way Mr. Rothery suggested. The author might have given an illustration of offices which gave something like a compound reversionary bonus, employing the Northampton Table, and in which the rate charged at age 60 was £6. 7s. 4d. per-cent. If a man at 60 wished to effect an assurance he would go to the office which charged £6. 7s. 4d., and not to that which charged £7. 4s. 9d., and he would not make any enquiry as to compound reversionary bonus. The first volume of the *Journal* contained a paper by the late Mr. Jellicoe on the "Determination and division of surplus, and the mode of returning it to contributories" (*J.I.A.*, i, 22*). Mr. Jellicoe said: "The first consideration is to determine what part of the premium charged must be calculated upon as provision for the sum guaranteed under the policy. This, again, involves a further enquiry: what rate of mortality may be expected in future to prevail and what rate of interest will actually be realized?" He agreed with the author in thinking that nobody expected the rate of interest to be realized in the next generation to be only $2\frac{1}{2}$ per-cent. The author had referred to what he called "minor sources of profit", one being "mortality lighter than the estimate." That had been a major source of profit for some considerable time past. The mortality of the country generally during the last 15 years had changed in a marked degree. For many years the rate of mortality was 22 per 1,000, but now it might be taken as 19. The result was that the H^M Table of Mortality, being founded on observations ending in 1863, did not now represent the mortality experienced by assurance companies. In almost every report they took up they were told the claims had been so much less than was calculated upon. He did not despair of getting rid of the absurdity of two rates of premium in every prospectus, with and without profits. Why should they charge avowedly higher rates of premium than they thought would be required? He could not think that the system of increasing bonuses was one they ought to encourage. The essential feature of life assurance was that those who lived to be old must pay for those who died young, but the premium was not an investment of capital, but a payment out of income, and when that income ceased the capital became available.

Mr. T. G. C. BROWNE said that the question of dividing surplus was comparatively simple when actuaries had only one class of policyholders to deal with. In such a case they might adopt an inexact system which could not be defended theoretically, but which in practice would work satisfactorily. Now, however, they had two great classes of policyholders—the whole-term and the endowment assurance—and their conflicting interests had to be reconciled. That could not be done without making an analysis of the profits, and adopting in one or other form the contribution method of division. It was a common practice for offices valuing at a rate of interest about 1 per-cent below the actual rate to give a reversionary bonus to the endowment policyholder at a substantially less rate than that given to the whole-term policyholder, and that, no doubt, was, under such circumstances, inequitable. The author had handicapped himself by starting with a hypothetical bonus of 1 per-

cent. The contribution method must be based upon actual facts, and the sources from which the profit was derived must be analyzed. In making this analysis the author took his valuation at $3\frac{1}{2}$ per-cent, the actual rate he mentioned being $4\frac{1}{4}$ per-cent. The surplus rate was therefore $\frac{3}{4}$ per-cent. He added another $\frac{1}{4}$ to it, which he said was derived from what he called minor sources of profit, but to which Mr. Bailey had referred as major sources of profit, and which he (Mr. Browne) had had to deal with in the light of major sources of loss. The author then arrived at $15\frac{1}{2}$ per-cent on the three years' premiums as a fair estimate of what he called loading profit, but that contained the balance of the miscellaneous sources of profit. No doubt the results came out very closely to those which he desired, namely, an equivalent to a reversionary bonus of 1 per-cent. The author said that "the contribution method in its simplest form divided the profit into two portions only, namely, interest and loading profits, and it seems to me that it is undesirable to attempt to carry the apportionment further, but in order to get the two ratios of division the profit must be analyzed much more completely than the author had done. This necessitated an annual valuation, and an annual estimate of profit and loss from mortality; but if the books of the company were properly planned it did not involve the amount of labour that might be anticipated. It must be remembered that the contribution method, as laid down originally by Mr. Sheppard Homans, only took into contemplation what might be called the intrinsic profits, but there were a good many extraneous profits that had to be dealt with, and those were by no means unimportant. By extraneous he meant, for instance, profits from surrender and lapses, which did not depend upon vitality or the rate of premium paid by the participating policyholder, but were owing to the misfortune or want of foresight of other policyholders. The profit from loading from year to year varied very little. If the loading was debited with the whole expenses of management, it was fairly entitled to the benefit of extraneous profits from surrenders and lapses; and, further, he would give it a rateable contribution of the profit and loss from mortality and investments, and also of the interest on accrued profits—the interest that grew up year by year in consequence of their not dividing their profits annually—a substantial amount. To illustrate what he had said as to the possibility of mortality being a "major" loss, he might mention that on one occasion he had to face a loss equal to 26 per-cent of the total sum divided in respect of mortality alone. One of the best tests of a system was that under varying conditions at different divisions the general character of the bonus should remain the same for policies at different ages at entry and different durations. The system he had referred to undoubtedly did yield such a result. There was one argument in favour of the contribution plan which he had referred to in a letter to an insurance paper some years ago, as follows:—"To my mind it appears that the best reason for adopting an analytical method is that it removes one fruitful element of fluctuation in the rates of bonus. Assuming that the actual profit earned remains the same, if a system be adopted which favours a particular class—it may be the old policies or lives,

or the young policies or lives—it is obvious that the rates of bonus must vary as the claimants in the favoured class to those outside of it vary. If the system favour the old policies, it will go on smoothly so long as there is any influx of young lives who do not object to benefit their seniors in the hope that their day will come; but, if there be any check to the supply of these complacent individuals, you have a period of declining bonuses and consequent discontent, although your rates of interest, expenses, &c., remain the same. The system of an equal reversionary addition to the sum assured, exclusive of existing bonuses, greatly favours the young policyholders, and no doubt attracts new business for a time; but no office can afford to ignore its old clients, and although they may not realize that every increase in the new business ‘waters their stock’, their inarticulate discontent may do much harm. Under all systems that are not based on an analysis of the profits and on their distribution as they have been earned, the bonuses are to some extent dependent on the proportionate increase or decrease of a favoured class, and, therefore, on the amount of new business transacted. In the long career of a successful life office, the latter is sure to fluctuate, and I submit that it is most desirable to eliminate this uncertain element from the bonus system of a company.” It was a very important consideration indeed that the bonus of a company should not be affected in any appreciable degree by fluctuation in the new business, and he claimed for the system which he advocated that it avoided that fluctuation.

Mr. H. W. MANLY presumed the origin of the paper was a desire to meet what had now become a very favourite demand with all insuring persons: that was, an increasing reversionary bonus. Given the desire to obtain that result, the author’s method was ingenious, and did substantial justice to all the policyholders. No doubt an increasing bonus on policies increasing in age must prove an increasing strain upon the profit. That might account for the recent adoption of what he considered a ridiculous rate of interest in their valuations, namely, $2\frac{1}{2}$ per-cent. This was only to be justified by the attempt to set aside out of profits which had been earned by the existing policyholders a sufficiently large sum to enable the office to keep up a constantly increasing reversionary bonus in the future. He joined issue with the author in his statement that “if the valuation rate is reduced, it is not clear that the whole of the interest earned beyond the valuation rate should be distributed as a percentage upon the reserve values. If this were done it seems to me that a serious injustice might be inflicted upon the policies of shorter durations by reducing the valuation rate.” Where they made a valuation at $2\frac{1}{2}$ per-cent they were taking from the present policyholders a large sum for the benefit of future policyholders. Where the existing policyholders were young in age they might live a sufficient time to get a share of that sum which had been set aside; but when they were old they would not last long enough to get their share. He thought that if that method was adopted, a much larger bonus ought to be given to the old policyholders than to the young. It had become popular to give bonuses on policies immediately they were issued. With the present methods of obtaining

new business, he very much doubted whether any new policy produced profit during the first three years, if the sources of profit were strictly analyzed. The author, remarking upon the method of valuation proposed by Mr. Sprague under certain conditions, suggested, in place of using P_{x+1} in the valuation, the adoption of some method which would either increase the net premiums by a fixed percentage, or reduce the reserve by a certain fixed percentage on the value of the sum assured. He (Mr. Manly) thought it was dangerous to change from something which was definite to something which was indefinite. P_{x+1} was definite, but when they came to increasing the net premium, 4 per-cent might easily become 5, 6 or 7 per-cent, and similarly in reducing the reserve by a percentage on the value of the sum assured the percentages could be varied to produce a bonus.

MR. G. H. RYAN said that the central point in the paper was the new means of facilitating the distribution of surplus interest according to what had been called the contribution method of apportionment of profits. There were many strong arguments in favour of this method, and the chief objection to it was that it was cumbrous and troublesome to apply. Mr. Browne had devoted himself to that point with great clearness, but he had not succeeded in demonstrating that the particular method he himself adopted was very simple in its application. If the result, therefore, of the paper was to make this method of apportionment more easy in application, its next effect would be that it would make it more popular with insurance companies. Dealing with a few scattered points in the paper, he would first refer to the author's remarks relative to current premium rates. It was an old idea of Mr. Bailey's that competition fixed the rate of premium. It seemed, however, to be a moot question whether it was purely competition, whether it was a nice balance between benefits and contributions, or whether it was the actuary's ingenuity that regulated them. Probably they would find a combination of all those causes at work; and when they came to older ages, he was inclined to think that the actuary's formula had more to do with the rates than competition or the strict relation of benefit to contributions. In days to come they might possibly make benefit and contribution more closely interdependent, and then they would no longer find that (at the older ages especially) companies giving the smallest bonuses sometimes charged the highest rates. In another place the author mentioned that a particular table of premiums was inapplicable to the system of compound reversionary bonuses. No doubt the author also had in his mind the method and basis of valuation adopted by the company to which he was referring. As to the loading of the proposed scales of premiums, on comparing Mr. Rothery's formula for non-profit premiums with that for the with-profit premiums, he found that he introduced a lower rate of interest in the calculation of the latter. Mr. Rothery later on took objection to Mr. Sprague's plan of valuing P_{x+1} , pointing out that when using this method they did not know exactly what they were doing. He was afraid that in adopting the $3\frac{1}{2}$ per-cent rate of interest for with-profit premiums Mr. Rothery had really exposed himself to the same criticism. Having looked into this question, he

found exactly what increase of loading the substitution of $3\frac{1}{2}$ per-cent for 4 per-cent made. At age 20 it meant a difference of 7 per-cent on the 4 per-cent rate: at 30, 6 per-cent: at 40, 5 per-cent; at 50, $3\frac{1}{2}$; and at 60, $2\frac{1}{2}$ per-cent. With those figures it was easy to see the effect of the loading proposed upon with-profit premiums. Unless the premium rates were originally based upon some uniform and well-devised system, such as that suggested by Mr. Rothery, it appeared to him that the rate of interest at which the premiums were computed had very little whatever to do with the matter; and at any rate he (Mr. Ryan) would contend that the rate of mortality must be equally considered. Nothing was clearer than that rates of premium which worked extremely well and equitably in practice were often originally framed upon tables other than those used in the valuation, and at a rate of interest that did not represent the facts, and it was just the balance of these assumptions that made the premiums work satisfactorily. Mr. Manly had referred to valuations at $2\frac{1}{2}$ per-cent interest, a large question which there was no time adequately to discuss at the present meeting. If they valued at $2\frac{1}{2}$ per-cent, the more successful they were in attracting desirable business, the less could they afford their success. For by valuing at that rate they were setting aside for the purpose of strengthening their reserves a fund of undivided surplus which lay conveniently to their hand, and if they succeeded in obtaining a very large new business they would be less able to make so strong a valuation, because the reserves could not be immediately provided by such business itself. From the closing paragraph of the paper it would be seen that the author expressed surprise that "select" premiums should not enter into the valuation of life policies, and he seemed to associate the "select" premiums with a valuation by the H^M and the $H^{M.5}$ Tables. He presumed that it was not really suggested that they should use those tables in combination, for if that was done great confusion would result. Mr. Rothery afterwards referred to the "full application of the select method", so that no doubt he had in mind a strict valuation by select tables throughout, or some approximate estimate to provide for selection. He would suggest that if they used select premiums, they might employ the $H^{M.4}$ Table for recently-selected business, and the $H^{M.5}$ Table for business of more than five years' duration.

Mr. W. J. H. WHITTALL said that Mr. Rothery had expressed the fear that 4 per-cent was a somewhat high rate of interest to take, and that fear would be shared by many others. In Mr. Sprague's rates of mortality the benefit from the effect of selection was already discounted, and some actuaries thought it was not right to count upon very much more profit from that source, although many offices, no doubt, by careful management might make some. They had then to fall back upon the loading—which consisted of 2 per-cent upon the sum assured and 8 per-cent upon renewal premiums. It was somewhat uncomfortable to feel that that was all one had to fall back upon, and it was possible that the commission of the future might encroach considerably upon that provision. The worst of it was, that they found they were all very much in the same boat; for narrow as the margin appeared to be when it was set out on paper

the resulting premiums were very fair average premiums, which suggested that most offices were doing business upon rather close lines. Passing on to the author's ingenious method for providing an increasing element in his bonus system, looking at Table C he noticed that at the higher ages the percentage of reserve value to premiums paid was very much under the 60 per-cent average which the author used. The effect of this on the increasing element was that the bonus with which he credited the older ages at entry was a little exaggerated. He thought that 3 per-cent upon the premiums paid was a liberal allowance for the $\frac{3}{4}$ per-cent margin between the rate of interest adopted in the calculation of premiums and that actually realized, and this, together with the adoption of an average of 60 per-cent of the premiums as representing the reserve value, partially accounted for what was found in Table F, namely, that the bonuses for ages above 60 at entry increased more rapidly than the bonuses for younger ages at entry. He (Mr. Whittall) had been engaged in an investigation as to increasing bonuses. A table he had before him was based upon the assumption that every previous reversionary bonus was credited with a rate of premium corresponding to the age at entry; and it produced results very similar to those of Table F. It did not prove a very suitable table, for the reason that the increasing element was such as to give a nearly stationary reversionary bonus to young ages at entry, while it gave a considerable increase to older ages at entry. The office in question was already giving high cash bonuses to new entrants, and it was, of course, desirable that they should not be greatly reduced. The difficulty was got over by making the new cash bonus a function, not of the equivalent premiums on the existing reversionary bonuses, but of the reversionary bonuses themselves. The increasing element could then be so adjusted as to give in all cases a very slowly increasing progression. He feared that the facility for variation in Mr. Rothery's system was, in some respects, a disadvantage, as the parts might be arranged according to the desire of the moment, and the power might be dangerous in the hands of an unscrupulous person.

Mr. E. JUSTICAN said that the assumption of a constant percentage of the premiums paid as a representative of a policy-value, however convenient in practice, was hardly borne out by the facts, and he was doubtful whether it was practicable to represent this function in the manner suggested. On the general question of bonus distributions, for many reasons it would be an advantage to get rid of the bonus system altogether, as forming no part of the main principle of life assurance.

Mr. A. F. BURRIDGE said that Mr. Rothery had shown that, although the premiums might be higher than was necessary for the younger entrants and lower for the older ones, provided the system of bonus gave an increasing reversionary addition to the younger entrants, the latter had no reason to complain. He might draw attention to an early paper by Mr. Sprague, written in 1857, and read before the Institute, in which he made rather a damaging criticism of the compound reversionary bonus system. If Mr. Sprague had been present it would have been interesting to have heard his observations upon that system. In an office with which he was for many years

familiar, the contribution method of bonus distribution was adopted with marked success, and the criticisms brought forward in the present discussion had not, in his opinion, shaken the position which that system of distribution occupied as being one of the most successful that could be applied. Mr. Rothery was, no doubt, approximately right in taking 3 per-cent as the three years' interest profit. As to profits from surrenders, he thought they would not be so large as to make any important difference. This brought him to a point not dwelt upon in the paper, namely, that the reserves for bonus should be included in the first portion of the reserve. Mr. Rothery said the 60 per-cent of premiums paid was a very good approximation to the reserve value of the sum assured. If they added the reserve for bonuses of any magnitude they found that the 60 per-cent was upset. He had made out a table of what the total reserve would be in a particular case, and found that instead of 60 per-cent, it came out at an average of over 100 per-cent on the premiums paid, and varied, moreover, for each age at entry. As to endowment assurances, he considered it to be a sign of the times that Mr. Rothery should have made a special arrangement for them. Mr. King had in a previous discussion shown that under the compound reversion system endowment assurances were entitled to as large a share in the profits as were whole-life policies (*J.I.A.*, xxviii, 275).

Mr. A. B. ADLARD said that it was not often that actuaries had to prepare a complete set of tables for all classes of assurance, but if occasion arose for doing so the general principles upon which Mr. Rothery had proceeded appeared to him correct. He could not, however, agree that so high a rate of interest as 4 per-cent should be assumed, nor that there should be any difference in the rates adopted in the calculation of with-profit and without-profit premiums. Allusion had been made to the unfairness produced as between old and new members by adopting a new table of premiums, unless at the same time a new section was commenced. A few years ago he had to consider this point in connection with an office which had formerly charged the Northampton premiums and had afterwards adopted a more modern scale, under which the premiums for ages at which assurances are generally effected were lower than the Northampton. It appeared to him that as a new series of policies had not been commenced at the time the alteration was made, and it would have been impracticable to apply the new scale of premiums to the old class of policies, it was only fair that before making the general distribution that portion of the surplus which had been created during the five years by the additional contributions of the older members should first be allotted to their policies. On the subject of bonuses to endowment assurances he could not altogether agree with the author. Having been engaged in the preparation of a table of premiums for with-profit endowment assurances, he came to the conclusion, after making many experiments, that he could not frame a table of moderate premiums which would admit, without injustice to other policyholders, of the direct application of the contribution method of allotment (the surplus beyond that portion arising from interest being allotted in proportion to the premiums paid), especially in the case of short-term endowments. In some cases

he found it would practically have been returning the premiums paid with compound interest at over $3\frac{1}{2}$ per-cent. The following method was adopted. The premiums were calculated by adding to the net endowment assurance premiums the actual loading contained in the whole-life premiums, an adjustment being made to provide for the larger amount of commission which would have to be paid. This produced a table of very moderate premiums, which admitted of the second portion of the distribution being made in proportion to the *whole-life* premiums for the same respective ages at entry. This method would not give so large reversionary bonuses to endowment assurances as to whole-life policies, and this, he thought, was as it should be. The author's "modified contribution system" was ingenious, and where the sums assured were comparatively small the results would, no doubt, be sufficiently accurate; but the system had the disadvantage pointed out by Mr. Burrige, of not taking into account the interest upon the bonuses attached to the policies at the date of the valuation.

Mr. F. SCHOOLING said he found that out of 29 offices adopting the uniform reversionary bonus system, 8 adopted the compound and 21 the simple reversionary bonus; and Mr. Sprague, in a series of letters to an assurance paper some time ago, pointed out that where the rate of interest used in the valuation was slightly over $\frac{1}{2}$ per-cent less than the realized rate, the simple uniform method was roughly equitable. The profit from mortality on endowment assurances was very much less than it was on whole-life policies, and, in fact, it was hardly appreciable; and endowment assurances should only have the share of mortality profit which they had earned, otherwise the whole-life policies would suffer. With regard to Mr. Rothery's method, he thought it would be difficult to keep up the rate of bonus. Some companies adopted the method of giving a percentage on premiums paid as cash bonus and calculating the reversionary bonus upon it; but on looking at the records of those companies it would be found that the percentage did not increase at each successive quinquennium sufficiently to give increasing bonuses.

Mr. F. B. WYATT said he was an advocate of the contribution plan of dividing surplus, but was fully alive to the difficulties attending its application. These difficulties arose from the three chief sources of profit—loading, light mortality, and interest above valuation rate—and it had occurred to him, in the course of his investigations, that a very simple and equitable plan could be adopted if these three sources of profit were reduced to one, namely, the loading. That would be the case if the valuations were made by the "re-assurance" method advocated by Mr. J. A. Higham, the value of a policy being $(P_{x+n} - P_x)(1 + a_{x+n})$, where P is the office premium and a_{x+n} is derived from a table representing the company's own mortality, with a rate of interest approximating to the actual rate realized in the valuation period. He did not think this method of valuation had received the attention it deserved. Whether it was applicable to any particular company would depend on the circumstances of that company, but a trial could very readily be made if the mortality had been examined. He had himself derived the greatest

benefit from making a collateral valuation by Mr. Higham's method, and by its means had been able to solve a very troublesome problem. The "contribution" formula for a quinquennial period resulting from that method of valuation was

$$\frac{\phi_x N_{x-1} - \phi_{x+5} N_{x+4}}{D_{x+5}}$$

where ϕ was the margin in the office premium over the net premium according to the special table used, less an allowance for expenses, and $x+5$ the age at date of distribution.

MR. ARCHIBALD DAY was sorry that Mr. Bailey did not seem to fully appreciate what appeared to him to be the demands of the public. Mr. Browne, on the other hand, thoroughly understood that the public did want an increasing reversionary bonus. Mr. Justican would like to sweep away the bonus system altogether, but really that was a hope practically impossible of realization. The bonus part of the question was the speculative element which attracted the public to the insurance office. But his real object in rising was to commend to the actuaries of the present day (he was no longer on the active list) an extract from a novel of Oliver Wendell Holmes ("A Mortal Antipathy"), in which he said in regard to a certain thing, "The public wants it, and what the public wants it is bound to have, and we are bound to furnish it." That very much explained the present situation.

THE PRESIDENT said it was because he entirely agreed with the sentiment expressed by Mr. Day that he had brought himself to think differently from Mr. Bailey with regard to this question. Until the other day Mr. Bailey and he, in their official capacities, had stood on the same level with regard to this matter: but he had found out, strange as it might seem to the actuarial mind, that a series of 1 to 10 was considered to be of greatly more value than a series of 10 to 1. Obviously that was not so, but yet such was human nature that the public would have its ascending series, although in reality it was of less value to them than a descending series, and, in obedience to the public want, he had lately been concerned, as his colleague, Mr. Whittall, had already indicated, in endeavouring to give the public what it demanded. With regard to the $2\frac{1}{2}$ per-cent valuation, he would say to Mr. Manly that he agreed that by a descent in the rate of interest from 3 to $2\frac{1}{2}$, or any other rate, unless they gave an increasing benefit to the older policyholders, out of whose accumulations the surplus interest was chiefly earned, they did a great injustice. Mr. Rothery's paper was the outcome of two distinct things: it was an interesting illustration of that refinement of treatment of which on a former occasion he had spoken as characteristic of the modern actuary; but it was also the outcome of that need for looking at the question of bonuses as a separate question. Although Mr. Bailey might disagree with the bonuses, and although Mr. Justican might denounce them, as he himself had denounced them 20 years ago, they had, nevertheless, like an octopus, got such a hold of the insurance system that it was impossible at the present day to cut oneself adrift from them. Whether in consequence of the

increasing importance of the middleman with whom so many of their friends just now were quarrelling, or on account of the keener appreciation of the public themselves, the question of the bonuses they were to give in any particular case was becoming more and more important. That was an illustration of a thing in itself to be welcomed—of the growing intelligence of the public themselves, of which he was quite sure actuaries would have in the long run no cause to complain.

Mr. ROTHERY, in reply, said that Mr. Manly had objected to his suggested modification of Mr. Sprague's well-known valuation formula, which involved P_{x+1} , as indefinite. That was to some extent true, but he might point out that Mr. Sprague's formula, as stated in its general form, involved the quantity P_{x+t} instead of P_{x+1} , and was thus equally indefinite. Of course each method must be applied with great care and judgment. Referring to Mr. Ryan's remarks, he would use the "select" method of valuation in its fullest sense, but would permit himself to adopt any reasonable modification to simplify its practical application. The contribution method had in the discussion found strong adherents, and he did not think anything had been seriously urged against it. The argument of his paper was this: One bonus system or another had to be adopted, and it was a question of compromise between theory and practice. Theory had a great deal to say in favour of the contribution method, but in applying it practically difficulties at once arose. Mr. Browne tried to apply the contribution plan with great exactness. He analyzed the profit or loss from mortality, interest, loading, and from many other sources, but having obtained his results, he did not, as might have been expected, apply them strictly, but apportioned the items into two main divisions only. Theoretically the mortality profit or loss ought to be applied to the various ages where a light or a heavy mortality had occurred. It seemed to him a needless labour to make so minute an examination into sources of profit, and then after all only to divide the surplus into two parts by some very arbitrary process. Objections could, of course, easily be given against any bonus system, but he submitted that, under the plan explained in the paper, and by suitably choosing the percentages, the surplus could be apportioned with fair equity and without undue complication. Whether the reserve was to be assumed to be 60 per-cent or any other percentage of the premiums paid, would depend entirely upon the circumstances of the particular office.

THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1891, published in 1892.]

I N C O M E	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	165,809,131	8,259,058	174,068,192
Adjustments: for one Ordinary Return transferred, and for one Industrial Return discontinued (Deficiency Balance).	- 33,728	+ 6,646	- 27,082
	165,775,406	8,265,704	174,041,110
Premiums	11,833,359	5,031,826	19,865,185
Consideration for Annuities	1,184,705	75	1,184,780
Interest and Dividends (less Tax)	6,744,876	274,562	7,019,438
Increase in value of Investments	177,365	20	177,385
Fines, Fees, &c.	7,293	463	7,756
Capital Paid-up	35,684	10,896	76,580
Customs Timber Measuring, &c.	3,125	...	3,125
Donations (Itinerant Methodists)	3,781	...	3,781
Transfers from other Accounts	47,392	...	47,392
Miscellaneous	19,270	6,500	25,770
	188,832,256	13,620,046	202,452,302

O U T G O	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	11,783,890	2,184,851	13,968,741
Cash Bonuses and Reduction of Premiums	1,057,400	...	1,057,400
Surrenders	792,073	12,534	804,607
Annuities	871,070	1,134	872,204
Commission	731,412	1,377,173	2,108,585
Expenses of Management	1,390,797	844,716	2,235,513
Bad Debts	2,167	72	2,239
Decrease in value of Investments	116,501	483	146,984
Interest on Capital and Dividends and Bonuses to Shareholders	498,344	53,901	552,245
Transfers to other Accounts	3,174	69	3,243
Miscellaneous	8,259	...	8,259
Balance* at the end of the Year	171,547,169	9,145,113	180,692,282
	188,832,256	13,620,046	202,452,302

* This Balance includes the whole of the Life and Annuity Funds (£174,793,243), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1891).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . . .	11,352,139	519,744	11,871,883
Life and Annuity Funds . . .	165,920,161	8,873,082	174,793,243
Fire Funds of Companies trans-acting Life Business . . .	9,815,476	...	9,815,476
Marine Funds of Companies trans-acting Life Business . . .	549,906	...	549,906
Reserve Funds . . .	4,520,904	...	4,520,904
Other Funds . . .	464,315	212,064	676,379
Profit and Loss Balances . . .	2,908,720	...	2,908,720
Depreciation and Investment Bal-ances . . .	877,969	10,134	888,103
Globe Annuitants (Liverpool and London) . . .	1,102,800	...	1,102,800
Outstanding Claims . . .	3,492,085	29,393	3,521,478
Outstanding Accounts . . .	458,964	15,371	474,335
Temporary Loans . . .	139,682	2,238	141,920
	201,603,121	9,662,026	211,265,147

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages (including Rent-charges)	82,801,589	299,150	83,100,739
Loans on Policies . . .	9,217,205	23,795	9,241,000
„ Rates . . .	19,401,604	3,147,471	22,549,075
British Government Securities	5,915,374	524,468	6,439,842
Indian and Colonial Government Securities . . .	12,695,955	113,618	12,809,603
Foreign Government Securities . .	3,533,849	...	3,533,849
Debentures . . .	21,856,801	1,453,930	23,310,731
Shares and Stocks . . .	12,931,402	177,467	13,108,869
Companies' own Shares . . .	537,610	...	537,610
Land and House Property and Ground Rents . . .	12,851,277	2,960,721	15,811,998
Life Interests and Reversions . .	3,480,319	793	3,481,112
Loans on Personal Security . .	1,316,908	13,192	1,330,100
Agents' Balances and Outstanding Premiums . . .	4,374,699	424,768	4,799,467
Outstanding Interest . . .	1,926,577	94,309	2,020,886
Cash, Deposits, Stamps, &c. . .	8,607,418	108,581	8,715,999
Customs Timber Measuring Bal-ances, &c. . .	2,247	...	2,247
Book-Room Grant (Itinerant Methodists) . . .	75,000	...	75,000
Deficiencies, Preliminary Expenses, &c. . .	77,287	319,733	397,020
	201,603,121	9,662,026	211,265,147

INCREASE (+) or DECREASE (—) in the Chief Items of this Year's SUMMARY compared with the corresponding Items for the previous Year.

	Ordinary Companies	Industrial Companies
	£	£
INCOME.		
Premiums	+ 619,973	+ 178,091
Consideration for Annuities	— 90,960	+ 75
Interest and Dividends (less Tax)	+ 217,333	+ 23,726
Net Result of Realization and Re-valuation of Investments	— 143,966	— 1,242
OUTGO.		
Claims	— 1,156,794	+ 256,445
Annuities	— 73,643	— 142
Surrenders	— 27,385	— 1,273
Commission	— 53,088	— 72,644
Expenses of Management	— 28,884	— 6,606
LIABILITIES.		
Paid-up Capital (including sundry Shareholders' Balances)	— 24,990	— 49,990
Life and Annuity Funds	+ 5,434,789	— 847,759
ASSETS.		
Mortgages (including Loans on Rates)	+ 2,005,132	+ 735,483
Life Interests and Reversions	— 53,130	+ 73
Loans on Policies	+ 302,891	— 1,654
British Government Securities	— 211,542	— 198,196
Indian and Colonial Government Securities	— 61,137	— 7,967
Foreign Government Securities	— 5,802	—
Debentures	+ 2,922,533	+ 38,229
Shares and Stocks	— 33,975	— 3,780
Companies' own Shares	— 8,764	—
Land and House Property and Ground Rents	— 759,416	— 245,553
Loans on Personal Security	+ 70,631	+ 4,317

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 97, of which 85 have been classed as Ordinary, 8 as Industrial, and 4 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately.

During the year four names have been removed from the official List of Companies, namely, the Industrial of Great Britain (Limited); Provincial; Scottish Economic (Limited); Scottish Provincial; which have either ceased to do or have transferred their business. And five new names have been added, namely, the Provident Assurance Company (Limited) (included in Summary); and the British Natural-Premium Provident Association (Limited); Imperial Insurance Company; Pioneer Life Assurance Company (Limited); Sun Life Assurance Company of India (Limited); in which cases the Board of Trade have issued their Warrant under the provisions of Section 1 of "The Life Assurance Companies Act, 1872."

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies.*
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.		£		£		£	£	£
Whole Term of Life	669,593	341,598,769	81,029	55,761,779	750,622	397,360,548	18,589,052	378,771,496
Limited number of Premiums . . .	28,498	16,338,685	4,850	2,301,500	33,357	18,640,185	623,885	18,016,300
Endowments . . .	698,091	357,937,454	85,888	58,063,279	783,979	416,000,733	19,212,937	396,787,796
Endowment Assur- ances . . .	2,689	456,825	6,535	1,068,012	9,224	1,524,837	6,000	1,518,837
Joint Lives . . .	154,465	34,517,330	28,498	8,701,293	182,963	43,218,623	733,131	42,483,492
Last Survivor . . .	7,372	1,982,890	1,941	915,802	9,316	2,898,692	304,389	2,594,303
Contingent . . .	1,197	926,815	1,198	1,250,547	2,395	2,177,362	291,140	1,886,222
Issue . . .	43	43,204	2,930	4,860,247	2,973	4,903,451	1,001,364	3,812,087
Miscellaneous . . .	6	10,577	813	3,235,588	819	3,246,165	959,276	2,286,889
	179	109,421	3,933	4,892,791	4,112	5,002,212	1,187,703	3,814,509
	864,042	395,984,516	131,739	82,987,559	995,781	478,972,075	23,787,940	455,184,135
ANNUITIES.								
Immediate	18,752	867,400	12,344	855,056
Deferred	4,508	155,609	8,022	147,587
	23,260	1,023,009	20,366	1,002,643

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assur- ances	Net Amount
	No.	Amount	No.	Amount	No.	Amount	Amount	
ASSURANCES.						£		£
Whole Term of Life	9,665,465	87,825,165	...	87,825,165
Limited number of Premiums	6	237	...	237
Endowments	9,665,471	87,825,402	...	87,825,402
Endowment Assur- ances	38,406	502,432	...	502,432
Joint Lives	72,525	1,142,684	...	1,142,684
	103,526	1,513,243	...	1,513,243
	9,879,928	90,983,761	...	90,983,761
ANNUITIES.								
Immediate	1	15	...	15

The above figures are based on Returns deposited for the most part during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time.

CORRESPONDENCE.

OPINION OF THE LATE DR. WILLIAM WHEWELL AS TO
THE GRAPHIC METHOD.*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—Having regard to the nature of the objections that Mr. Woolhouse has made against the Graphic Method of adjusting Mortality Tables, and, in particular, his expression of opinion that it is unscientific, I think it will be useful to lay before your readers the opinion regarding the method given by the late Dr. William Whewell, who must be admitted to have been an eminently scientific man. This opinion is contained in a passage in his *Norram Organo Renovatum*, which has lately come under my notice; although, as I have had the book for many years, I may possibly have read the passage previously and forgotten all about it. In the chapter entitled *Special Methods of Induction applicable to Quantity*, he has a section devoted to what he calls the "Method of Curves"; and it is from this section that the following passages have been extracted.

It will be noticed that, although Dr. Whewell had not in view the application of the Method of Curves to the adjustment of mortality observations, many of his remarks are quite as appropriate as if they had been written with special reference to this subject. In this connection I would draw attention especially to his statements. *First*, that though the observed facts may appear irregular, the correct facts which they represent are really regular; and, *Secondly*, that the irregular and abrupt deviations, which appear in the course of the original observations as plotted down in a curve, are in most cases small in extent when compared with those bendings which denote the effects of regular law. This is equivalent to saying, in the case of Mortality observations, (1) that although the observed probabilities of death appear irregular as we pass from one age to the next, yet the true probabilities must be really regular; and (2) that the irregularities in the observed series of probabilities are, in most cases, small in extent as compared with the general sweep of the curve considered as a whole. These are the fundamental principles on which we rely in graduating a mortality table by the Graphic Method; and the extract not only admirably describes the process of adjustment, but is even more useful as explaining the principles which establish its claim to be a thoroughly scientific process and not a mere empirical contrivance.

I am, Sir,

Your obedient Servant,

Edinbro,

12 March 1892.

T. B. SPRAGUE.

Extract from WHEWELL'S "*Norum Organon Renovatum*",
Book III, ch. vii, p. 204. (3rd Ed. 1858.)

The Method of Curves proceeds upon this basis: that when one quantity undergoes a series of changes depending on the progress of another quantity, * * * * this dependence may be expressed by means of a *curve*. In the language of mathematicians, the variable quantity, whose changes we would consider, is made the *ordinate* of the curve, and the quantity on which the changes depend is made the *abscissa*. In this manner, the curve will exhibit in its form a series of undulations, rising and falling so as to correspond with the alternate Increase and Diminution of the quantity represented, at intervals of Space which correspond to the intervals of Time, or other quantity, by which the changes are regulated. Thus, if we set up, at equal intervals, a series of ordinates, representing the Height of all the successive High Waters brought by the tides at a given place, for a year, the curve which connects the summits of all these ordinates will exhibit a series of undulations, ascending and descending once in about each Fort-night; since, in that interval, we have, in succession, the high spring tides and the low neap tides. The curve thus drawn offers to the eye a picture of the order and magnitude of the changes to which the quantity under contemplation, (the height of high water), is subject.

Now the peculiar facility and efficacy of the Method of Curves depends upon this circumstance:—that order and regularity are more readily and clearly recognized, when thus exhibited to the eye in a picture, than they are when presented to the mind in any other manner. To detect the relations of Number, considered directly as Number, is not easy; and we might contemplate for a long time a Table of recorded Numbers, without perceiving the order of their increase and diminution, even if the law were moderately simple. * * * *

But the Method of Curves not only enables us to obtain laws of nature from *good* Observations, but also in a great degree, from observations which are very *imperfect*. For the imperfection of observations may in part be corrected by this consideration;—that though they may appear irregular, the correct facts which they imperfectly represent, are really regular. And the Method of Curves enables us to remedy this apparent irregularity, at least in part. For when observations thus imperfect are laid down as Ordinates, and their extremities connected by a line, we obtain, not a smooth and flowing curve, such as we should have if the observations contained only the rigorous results of regular laws; but a broken and irregular line, full of sudden and capricious twistings, and bearing on its face marks of irregularities dependent, not upon law, but upon chance. Yet these irregular and abrupt deviations in the curve are, in most cases, but small in extent, when compared with those bendings which denote the effects of regular law. And this circumstance is one of the great grounds of advantage in the Method of Curves. For when the observations thus laid down present to the eye such a broken and irregular line, we can still see, often with great ease and certainty, what twistings of the line are probably due to the irregular errors of observation; and can at once reject these, by drawing a more regular curve, cutting off all such small and irregular sinuosities, leaving some to the right and some to the left; and then proceeding as if this regular curve, and not the irregular one, expressed the observations. In this manner, we suppose the errors of observation to balance each other: some of our corrected measures being too great and others too small, but with no great preponderance either way. We draw our main regular curve, not *through* the points given by our observations, but *among* them: drawing it, as has been said by one of the philosophers (Sir J. Herschel,*

* Mr. Sorley, in his "Observations on the Graduation of Mortality Tables" (*J.I.A.*, xxii, 309) has mentioned Sir J. Herschel's connection with the Graphic Method, and has made some lengthy quotations from the paper here referred to.—T. B. S.

Ast. Soc. Trans. vol. v, p. 1) who first systematically used this method, "with a bold but careful hand". The regular curve which we thus obtain, thus freed from the casual errors of observation, is that in which we endeavour to discover the laws of change and succession.

By this method, thus getting rid at once, in a great measure, of errors of observation, we obtain data which are *more true than the individual facts themselves*. The philosopher's business is to compare his hypotheses with facts. But if we make the comparison with separate special facts, we are liable to be perplexed or misled, to an unknown amount, by the errors of observation; which may cause the hypothetical and the observed result to agree, or to disagree, when otherwise they would not do so. If, however, we thus take the *whole mass of the facts*, and remove the errors of actual observation, (*Ibid.*, vol. v, p. 4), by making the curve which expresses the supposed observation regular and smooth, we have the separate facts corrected by their general tendency. We are put in possession, as we have said, of something more true than any fact by itself is.

One of the most admirable examples of the use of this Method of Curves is found in Sir John Herschel's *Investigation of the Orbits of Double Stars*. The author there shows how far inferior the direct observations of the angle of position are, to the observations corrected by a curve in the manner above stated. "This curve once drawn", he says, "must represent, it is evident, the law of variation of the angle of position, with the time, not only for instants intermediate between the dates of observations, but even at the moments of observation themselves, much better than the individual *raw* observations can possibly (on an average) do. It is only requisite to try a case or two, to be satisfied that by substituting the curve for the points, we have made a nearer approach to nature, and in a great measure eliminated errors of observation." "In following the graphical process", he adds, "we have a conviction, almost approaching to moral certainty, that we cannot be greatly misled."

A LONG-LIVED FAMILY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The vital history, as annexed, of a family of 11 children is so remarkable that it would seem worthy of insertion in the *Journal*: the father died in 1845, aged 85, and the mother in 1811, aged about 45. I would add that the father, one of the founders of a great life office, was the head of a well-known house now rivalling, in the vigour of its official existence, the longevity of certain of its members.

	Sex	Born		Died		Age at Death
1	F.	7 Dec.	1789	24 Jan.	1892	102·1
2	F.	23 Feb.	1791	10 Nov.	1886	95·7
3	M.	28 March	1792	10 Jan.	1881	88·8
4	F.	9 April	1793	10 Aug.	1875	82·3
5	F.	16 Feb.	1795	27 March	1891	96·1
6	M.	4 Sept.	1796		1796	(say) ·1
7	M.	20 Nov.	1798	28 Sept.	1841	42·9
8	M.	15 Dec.	1799	8 June	1869	69·5
9	M.	23 Jan.	1801	29 Dec.	1888	87·9
10	M.	21 April	1802	3 Feb.	1892	89·8
11	F.	27 June	1806	8 May	1807	·9

The average age at death was therefore 68·7; or, leaving the two who died in infancy out of consideration, 83·9.

I am, Sir,

Your obedient Servant,

C. D. HIGHAM.

81 *King William Street, E.C.*,
27 April 1892.

THE INSTITUTE OF ACTUARIES.

NEW SYLLABUS AND RULES FOR THE EXAMINATIONS.

[These Rules will come into force as regards Parts I and II of the Examination in 1893, and as regards Part III in 1894.]

RULES prescribed by the Council of the Institute to regulate the Examinations qualifying for admission to the Classes of Students, Associates, and Fellows respectively.

GENERAL REGULATIONS.

1. The Examinations shall be conducted in writing, or partly in writing and partly *virâ voce*, as the Examination Committee may think fit, and under such conditions as the Examination Committee may prescribe, subject to the approval of the Council.

2. The Examiners shall place successful Candidates in three Classes, and the names in each Class shall be arranged in alphabetical order.

3. No Candidate will be allowed to present himself for examination until he has paid all Subscriptions, Entrance Fees, and Examination Fees that may be due.

Examinations in the United Kingdom.

4. Except in the case of Examination for admission to the Class of Students, at least one month's public notice will be given of the days and hours when, and of the place or places where, the Examinations in the United Kingdom will take place.

5. Candidates for any Examination in the United Kingdom must give fourteen days' notice in writing, addressed to the Honorary Secretaries, of their intention to present themselves for Examination, specifying the particular Examination for which they intend to present themselves.

Examinations in the Colonies.

6. Except in the case of Examinations for admission to the Class of Students, public notice will be given, not later than in October in

each year, by advertisement in at least one newspaper of the Colony in which an Examination is to be held, of the days when such Examination will take place.

7. Candidates for any Examination in a Colony must give notice in writing, addressed to the Honorary Secretaries, of their intention to present themselves for examination, specifying the particular Examination for which they intend to present themselves. Such notice must be posted in time to reach the Honorary Secretaries in London not later than the 31st December preceding the date of the Examination.

STUDENTS.

1. Examinations for admission to the Class of Students will be held as often as the Examination Committee may prescribe.

2. Candidates who have paid the Application Fee of £1. 1s., and whose applications for admission have been approved by the Council, shall, unless exempted, be examined in the following subjects:

- (1) Writing from Dictation.
- (2) English Composition.
- (3) Arithmetic.
- (4) Elementary Algebra (not beyond Equations of the first degree).

3. In the event of a Candidate passing the Examination he will be admitted a Student after he has signed the Form of Obligation prescribed by the Council, and paid his Subscription of £1. 1s. for the current year.

4. In the event of a Candidate failing to pass the Examination, he shall be permitted to offer himself for Examination once more within a period not exceeding six months from the date of the first Examination. If he fail to pass on the second occasion, or do not present himself a second time for examination within such prescribed time, the fee paid upon application for admission shall be forfeited.

ASSOCIATES.

1. The Examination to qualify Students for admission to the Class of Associates shall consist of two parts, and, subject to the exceptions hereinafter mentioned, no Student shall present himself for Examination in the second part unless he has previously succeeded in passing the first part of the said Examination.

2. Examinations will be held in April of each year, or oftener if the Council shall think fit.

3. Candidates for any Examination in either the first or second part must, at the time of giving notice of their intention to present themselves for examination, pay a fee of £1. 1s.

4. In the first part of the Examination, Students shall be examined in the subjects set forth in Part I of the annexed Syllabus; and in

the second part of the Examination, in the subjects set forth in Part II of the said Syllabus.

5. Graduates in Mathematical Honours of any University in the United Kingdom will be exempt from Examination in Part I of the annexed Syllabus.

6. Fellows of the Faculty of Actuaries by Examination will be considered by the Council to be eligible under Bye-Law No. 44 as Associates, without passing the Examination for admission to the Class of Associates.

FELLOWS.

1. Examinations will be held in April of each year, or oftener if the Council shall think fit.

2. Subject to the exceptions hereinafter mentioned, and to the exceptions mentioned under the heading "Associates," the Examination to qualify an Associate or a Student for admission to the Class of Fellows shall consist of three parts, the third of which shall consist of two sections. The first and second parts of the Examination shall be in the same subjects, and subject to the same rules, as those prescribed for admission to the Class of Associates; and in the third part, for neither section of which shall they present themselves until an interval of at least twelve months after passing the second part, Candidates shall be examined in the subjects set forth in Part III, Section A and Section B, of the annexed Syllabus.

3. Candidates may take both sections of Part III in the same year, or they may take either section in one year and postpone the other section to a subsequent year.

4. Candidates for Examination in the third part must, at the time of giving notice of their intention to present themselves for examination, pay a fee of £1. 1s. in respect of each section for which they enter.

5. Associates who prior to the commencement of the Bye-Laws (26th day of February, 1886,) passed the first or second year's examination, shall be required to pass only the second and third parts, or only the third part, respectively, of the Examination to qualify for admission to the Class of Fellows.

6. Associates admitted after the 26th February, 1886, shall be required to pass only the third part of the Examination to qualify for admission to the Class of Fellows.

By order of the Council, June 14th, 1892.

THOS. H. COOKE, }
GEORGE KING, } *Hon. Secs.*

SYLLABUS OF EXAMINATIONS REFERRED TO IN THE
ANNEXED RULES.

PART I.

- (1) Arithmetic and Algebra, including the Theory and Practice of Logarithms, and the Elements of the Theory of Probabilities.
- (2) Euclid—Books I, II, and III.
- (3) The Elements of the Theory of Compound Interest, including Annuities-Certain.
- (4) The Principles of Book-keeping.
- (5) Practical Examples in all the foregoing subjects.

PART II.

- (1) The Theory of Compound Interest, including Annuities-Certain.
- (2) The application of the Theory of Probabilities to Life Contingencies.
- (3) The Theory of Life Contingencies, including Annuities and Assurances on Lives and Survivorships.
- (4) The methods of construction (excluding graduation) and the use of Mortality Tables.
- (5) The methods of construction and the use of monetary and other Tables relating to Life Contingencies.
- (6) The Elements of the Law relating to Life Assurance Policies and their assignment.
- (7) The Investments of Life Assurance Companies other than Life Interests and Reversions.
- (8) Practical Examples in all the foregoing subjects.

N.B.—Questions will not be set involving combinations of more than three lives, or the application of the Differential and Integral Calculus, or of the Calculus of Finite Differences.

PART III.—*Section A.*

- (1) The Elements of the Law of Real and Personal Property, and of the Law relating to Life Assurance Contracts, and to Joint-Stock Companies and Friendly Societies.
- (2) The Principles of Banking and Finance, including a knowledge of the Constitution and Operations of the Bank of England, and of the National and Local Debts of the United Kingdom.
- (3) Elements of the Calculus of Finite Differences, including Interpolation and Summation, with its Application to Life Contingencies.
- (4) The methods of constructing and graduating Mortality Tables, Sickness Tables, and other Tables relating to Life Contingencies, with special reference to existing well-known Tables.
- (5) Practical Examples in all the foregoing subjects.

PART III.—Section B.

- (1) The methods of Valuation of the Liabilities and Assets of Life Assurance Companies and analogous Institutions, and of dealing with the results shown by such Valuations.
- (2) Investments generally, including Life Interests and Reversions.
- (3) Miscellaneous subjects connected with the foregoing, such as the practical valuation of Life Interests and Reversions, the preparation of Statements, Accounts, Reports, &c.
- (4) Subjects in Life Contingencies not included in Part II.
- (5) The Application of the Differential and Integral Calculus to Life Contingencies.
- (6) Practical Examples in all the foregoing subjects.

EXAMINATIONS OF THE INSTITUTE, APRIL 1891.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).

Examiners—F. E. COLENSO, Esq., M.A.; G. F. HARDY, Esq.;
G. MARKS, Esq.; H. C. THISELTON, Esq.

First Paper.

1. A retail dealer, who reckons his average cash sale to be 1s. 8d., adds to the wholesale price 25 per-cent for profit, and further additions to provide for discount of 10 per-cent for cash, and to meet expenses at the rate of 2d. per average cash sale. He buys one set of articles wholesale at 81s. per hundred. What should be his nominal retail selling price?

2. Give a brief description of the method of book-keeping by double entry, and show how the books of a company, kept by that method, may be made to furnish a complete account of the state of its affairs.

3. Solve the equations:

$$(1) \quad x^3 + ax^2 + ax + 1 = 0.$$

$$(2) \quad x^4 + ax^3 + bx^2 + ax + 1 = 0.$$

4. If each of two quantities varies as a third when the other is constant, show that when both vary the third varies as their product.

Assuming that the quantity of work done varies as the number of days when the number of working hours per day is constant, and that the quantity of work done per hour varies as the excess of 12 over the number of working hours per day, find what the number of working hours per day must be in order that the work of 27 days may be equivalent to the work of 20 days of 9 hours each.

5. Obtain the conditions under which the expression

$$ax^2 + 2bx + c$$

is of one sign for all real values of x .

Find the greatest value of

$$x + 2 \\ 2x^2 + 3x + 6$$

6. Define the logarithm of a given number to a given base, and find the logarithms of $\frac{16}{81}$ to the base $\frac{2}{3}$, $\frac{52}{243}$ to the base $\frac{4}{3}$, and $\cdot 0000157609$ to the base $\cdot 00397$.

Between what negative integers does the logarithm of $\frac{13}{579}$ to the base 3 lie?

7. Prove the Binomial Theorem for a positive integral exponent; and find the general term in the expansion of:—

$$(1) \quad \left(\sqrt{\frac{3}{2}} \cdot x + \sqrt{\frac{1}{2}} \right)^5;$$

$$(2) \quad \frac{1}{\sqrt[5]{(a^5 - x^5)}}.$$

8. In how many trials may a person undertake, for an even wager, to throw an ace with a single die?

A undertakes to throw at least one 6 in a single throw of six dice. B to throw at least two 6's in a single throw of twelve dice. Which has the better chance of success?

9. Explain the nature of the Calculus of Finite Differences, and furnish reasons why the value usually given to the difference of the independent variable is unity.

Prove that if u_x be a rational and integral function of x of the n th degree, its n th differences are constant.

10. Deduce a formula for the sum of the series

$$u_x + u_{x+1} + u_{x+2} + \dots + u_{x+n-1}.$$

Hence find the sum of the fourth powers of the first n natural numbers.

11. Obtain the conditions under which the equation

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

represents:—

(1) a circle;

(2) two straight lines.

12. If a side of a triangle be produced, the exterior angle is equal to the two interior and opposite angles, and the three interior angles of any triangle are together equal to two right angles.

Draw a straight line DE parallel to the base BC of a triangle to cut the side AB in D and the side AC in E , so that DE may be equal to BD and CE together.

Second Paper.

13. A sum of £7,687. 10s. is invested in $2\frac{3}{4}$ per-cent. Consols at $95\frac{3}{4}$, the broker's charge for the investment being $\frac{1}{8}$ per-cent on the Stock. The Consols are subsequently realized at 98 and the proceeds re-invested, as far as possible, in North-Eastern Railway $\frac{1}{2}$ per-cent Debenture Stock at 130, this Stock being transferable only in multiples of £1. The broker's charge on the transfer being $\frac{1}{8}$ per-cent on the re-invested money, and the balance being deposited in the Post Office Savings Bank (at $2\frac{1}{2}$ per-cent), calculate the difference in income due to the transaction.

14. Two vehicles start at the same moment from two towns, A and B respectively, and travel towards each other. They meet after $10\frac{1}{2}$ hours, one taking $\frac{1}{32}$ hour more to a mile than the other. If the distance from A to B is 104 miles, what are the rates at which the vehicles travel?

15. Sum to n terms the three series, the n th terms of which are $(a+nb)^2$, $(n-2)2^{-n}$, and $(2n-1) \cdot x^n$ respectively.

16. Prove that a quadratic equation cannot have more than two roots, and investigate the relations which subsist between them and its co-efficients.

Given that the roots of the two equations:

$$ax^2+bx+c=0,$$

$$a'x^2+b'x+c'=0,$$

are in Geometrical Progression, prove that $\frac{a'c'}{ac} = \left(\frac{b'}{b}\right)^2$.

17. Reduce to its simplest form:

$$\frac{1}{1024} [-1 + \sqrt{5} \pm \sqrt{-10 - 2\sqrt{5}}]^5,$$

and show that

$$\left[-\frac{1}{2} \sqrt[3]{a} + \sqrt[3]{-\frac{3}{4} \sqrt[3]{a^2}} \right]^3 = a.$$

18. Show how to find the number of permutations of n things taken all together, which are not all different.

Find the sum of the different numbers which can be formed with three 6's and four 2's, all these seven digits being employed in the formation of each number.

19. Separate $\frac{306x^2-450x+162}{(8x-7)(5x-4)(2x-1)}$ into three fractions, of which the denominators are respectively $8x-7$, $5x-4$, and $2x-1$.

20. Write down the usual series for $\log_e(1+x)$, and find the value of x to four places of decimals in the following equations:

$$(a) \quad 50 \log_e \sqrt{1+x} = 49x:$$

$$(b) \quad \sqrt{\log_{10} \left(\frac{1}{1-x} \right) + \log_{10} \left(\frac{1}{1+x} \right)} = \frac{2}{3}x.$$

21. Two cards are drawn from a complete pack. What is the chance that the sum of their values will be

(a) exactly 20:

(b) at least 20:

assuming that each ace and court card is reckoned as 11?

22. There are two independent events (A) and (B), whose probabilities are respectively a and b .

A third event (C) must happen if (A) and (B) both fail, must fail if (A) and (B) both happen, and has an even chance of happening if one only of (A) and (B) happen.

Find the probability

(1) that (C) will happen:

(2) that (C) having happened (A) and (B) have both failed.

23. Having given the following values of annuities, find by interpolation the value of a_{13} at $3\frac{1}{2}$ per-cent:

	3 per-cent	$3\frac{1}{2}$ per-cent	4 per-cent
a_{40}	17.176	16.103	15.135
a_{42}	16.566	15.568	14.664
a_{44}	15.924	15.001	14.162

24. Find the equation to two straight lines such that the co-ordinate axes are the diagonals of the parallelogram contained by those straight lines and by the straight lines represented by the equation

$$(y - mx)^2 = c^2.$$

25. If a straight line be divided into any two parts the square on the whole line is equal to the squares on the two parts together with twice the rectangle contained by the parts.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

Examiners—F. BELL, Esq.; R. CROSS, Esq.; W. SUTTON, Esq., M.A.;
E. WOODS, Esq.

First Paper.

1. Explain the term *discount*, giving both for compound interest and simple interest, the formula for discount on 1 payable n years hence, the annual rate of interest being i .

If i be a nominal rate of interest convertible momentarily, x a number of years, what will be denoted by y , when $y = e^{ix}$?

2. Given a table of amounts and present values of annuities payable yearly, interest being convertible once a year, find the

constant factor by which the values in such tables can be multiplied so as to give the corresponding results where the annuities are payable m times a year, interest still convertible once a year.

What does the factor become when $m = \infty$?

3. Explain and illustrate what is meant by a table of mortality.

Assuming the existence of such a table, show how to obtain for any age x

- (1). The probable future lifetime (*vie probable*);
- (2). Average duration of life.

Distinguish between curtate and complete expectation of life.

4. In a stationary population where l_x = number living at age x , l_{x+1} = number living at age $x+1$, and so on, and A_x = present value of 1 payable at end of the year in which an individual age x dies, show that $\sum l_x A_x$ for all ages from x inclusive to the end of life $= l_x(a_x + A_x)$.

Can you give any verbal interpretation to this result?

5. Obtain by general reasoning the first approximate formula for the value of an annuity payable m times a year, namely,

$$a_x^{(m)} = a_x + \frac{m-1}{2m}.$$

Is this formula in excess or deficiency of the true value?

6. How would you answer the objection raised by a non-expert to the formula in the previous question that it could not be right because the correction is the same for all rates of interest?

7. Explain the meaning of the term "Policy-value."

Show that ${}_nV_x = A_{x+n} - P_x(1 + a_{x+n})$ (i)

and also ${}_nV_x = \frac{P_x(N_{x-1} - N_{x+n-1}) - (M_x - M_{x+n})}{D_{x+n}}$ (ii)

Distinguish clearly between the Prospective Method and the Retrospective Method of treatment of Policy-values.

8. Find a formula for the amount of a policy to be issued free from payment of premiums in lieu of receiving the value of the original policy in the following cases:

- (1). Ordinary whole-life policy;
- (2). A whole-life policy with limited number of premiums;
- (3). An endowment-assurance policy.

9. Find the formula for the value of an annuity on the life of x during the lifetime of y , and for t years after y 's death.

10. The owner of an estate has to pay off a charge in the following way in n years:

B being the amount of the charge he has to pay, $\frac{B}{n}$ each year as repayment of capital and interest at the rate j per annum on the amount still unpaid at the beginning of each year. For what lump sum might his annual payments be commuted?

11. Given the implicit relation, $x^2y^2 = a^3(x+y)$, find $\frac{dy}{dx}$.

12. Show that in the logarithmic curve, $y = (1+i)^x$, the distance from the origin along the axis of x from the ordinate y to the point where the tangent at any point (x, y) cuts the axis of x is constant. What is its value in actuarial notation?

Second Paper.

1. If an annuity-certain of S per annum for n years be payable in consideration of an advance of M , write down the proportions of the original advance repaid out of each of the annual payments of S .

In financial discussions as to the various methods of repaying the capital advanced in the case of loans effected by public bodies, what is the real question at issue? Illustrate this in the two cases of loans repayable, with interest, by equal annual payments for n years, and by annual payments where each annual payment consists of $\frac{1}{n}$ th instalment of the original loan and interest on the amount of loan unpaid at beginning of each year.

2. Define the terms "force of interest" and "force of mortality."

Assuming $l_x = kq^x s^x$, find μ_x .

In what respect does the value of μ_x differ from that obtained from Gompertz's formula?

3. Describe Orchard's Tables (Single Premium), and explain the method of construction.

How can these tables be used for endowment assurances?

4. State Simpson's rule for finding the value of an annuity on three joint lives by means of tables of annuities on single and on two joint lives.

In which direction do the values obtained generally err?

5. Obtain a formula for the net annual premium for a deferred annuity, the premiums paid to be returned in the event of death before entering upon the annuity.

What formula would you use if the premiums actually paid, *i.e.*, office premiums, are to be returned in the event of death before entering upon the annuity, and

Office premium = net premium $(1+k) + c$?

6. Describe what is known as the Hypothetical Method (or Re-insurance Method) of valuation, and find, in the case of an ordinary whole-life policy with annual premiums, under what conditions as to loading the magnitude of the policy-values brought out is the same as by the net-premium method of valuation.

7. Two methods may be used in a valuation of a life office's policies to give effect to immediate payment of claims. Describe them, and compare the results.

8. Show by actual demonstration that the theory that the average yearly sickness at each age bears a constant ratio to the deaths at that age cannot hold good.

Assuming the average rate of sickness at each age to be given, show how you would proceed to construct a table of sick-pay values for a friendly society.

9. Describe clearly the two methods that have been proposed and used in the construction of commutation columns for joint lives.

State the advantages of each method.

10. Given that

$$A_{xy}^1 = \frac{1}{2} \left(A_{xy} + \frac{a_{x-1:y}}{p_{x-1}} - \frac{a_{x:y-1}}{p_{y-1}} \right),$$

express the right-hand side in each of the commutation methods of the previous question.

11. Given the following data, find the value of ${}_5^{\circ}70$:

Age	Alive at beginning of Year	Entered during the Year of Age	Left during the Year of Age	Died during the Year of Age
70	1,562	27	39	82
71	1,468	15	27	100
72	1,356	20	40	102
73	1,234	11	20	95
74	1,130	15	25	94
75	1,026	17	27	99

NOTE.—The answers are to be obtained without the use of any tables. Special weight will be given to correct answers to this question, and the complete work is to be sent in with the answer.

12. Taking the equation to the logarithmic curve $y = (1+i)^x$, find the value of $\int_0^n y dx$, and explain the result obtained.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III).

Examiners—H. W. MANLY, Esq.; H. J. ROTHERY, Esq.;
G. H. RYAN, Esq.

First Paper.

1. How would you deduce the experience of a life office in respect of discontinuances? For what purpose would such an investigation be useful?

2. State what you know regarding the *Thirty American Offices' Experience*, particularly with reference to the extent of the experience, the classification of the data, the continuation of the experience to final series, and the main features of the table compared with the Institute of Actuaries' Experience.

3. Describe the methods employed by Mr. A. J. Finlaison in the graduation of the Government Annuitants' Experience, 1883.

4. What information would you require, and in what form, in order to deduce the experience of a friendly society giving sick pay at full benefit for the first six months' illness, half benefit for the second six months, and quarter benefit afterwards: and making payments on the confinement of members' wives, and on the death of members and their wives?

5. A life assurance company has resolved to lower its valuation rate of interest from 3 per-cent to $2\frac{1}{2}$ per-cent. Discuss the effect of this course upon the bonuses (a) of the present policyholders, and (b) of future entrants.

6. What method or methods of distribution of surplus would appear to you at the present time most suitable in the case of a valuation at $2\frac{1}{2}$ per-cent?

7. If an office desires to adopt a reduced scale of premiums, to be formed by deducting a sum in anticipation of future profits from the ordinary with-profit rates, state fully how you would calculate such reduced rates, and how the policies should be treated in the valuation.

8. What data would you use for the following objects:

(a) The construction of a set of premium tables for an assurance office transacting ordinary business.

(b) The construction of a table of weekly premiums for an office transacting industrial business.

(c) The construction of a table of annual contributions for

the use of a society for providing pensions to its members' widows.

9. What legal questions arise in connection with assurances and endowments granted to minors?

10. What suggestions would you make with the view of simplifying the assignment of life policies?

11. What difference is there between—

(a) An agreement under hand and an agreement under seal?

(b) A life policy under hand and a life policy under seal?

12. Explain Makeham's first and second developments of Gompertz's law of mortality, and demonstrate how, in each case, l_x may be obtained from μ_x .

13. What conclusions as regards the relative policy-values by different mortality tables, can be drawn from a comparison of the values of A and B in the formula

$$\mu_x = A + Bc^x ?$$

Second Paper.

14. Discuss the advisability of retaining the Sixth Schedule, or any part of it, in a revision of the Life Assurance Companies Acts.

15. Explain the circumstances that led up to the financial panic of 1890, and the measures that were taken by the Bank of England to restore credit.

Alternative Questions. { 16a. What is the difference between "real" and "accommodation" bills of exchange? Give instances of financial crises that have been attributed to an improper use of the latter.

16b. State the law relating to post-dated cheques payable to "order" and to "bearer" respectively.

17. What are the conditions that affect the bank rate in its variations?

18. Explain how you would ascertain what amount of profit or loss resulted, from mortality alone, to a life office in respect of its single-life annuity contracts in any given year.

* Candidates are only required to answer one of these questions, and no extra marks will be given if both are answered.

19. It is desired to offer to a proposer whose life has been accepted with a certain addition to the age, an alternative in the form of a debt on the policy, the normal premium being alone required. How should the amount of this debt be calculated, first, if it remains constant during the selected term, and, secondly, if it diminishes yearly by equal decrements?

Should policies subject to such contingent debts be specially dealt with in the valuation? Give your reasons.

20. Discuss the practice of purchasing reversionary life interests, option of redemption being allowed within a certain period. What legal difficulties have to be borne in mind?

21. Would you advise a company to advance on the security of reversions if the funds included *inter alia*, Bank Shares, the shares of high-class industrial companies fully paid up, and mortgages of leasehold property? Give your reasons in each case.

22. In what ways can the guarantee of a mortgage insurance company be utilized in connection with the investments of a life office?

23. What steps would you take to eliminate negative values in the valuation of an industrial assurance company in which the gross premiums are brought into account, less a certain percentage?

24. Under what, if any, circumstances do you consider it would be proper to include negative values, wholly or partially, in your estimate of the financial position of

(a) An industrial life office

(b) A friendly society?

25. A, aged 30, is entitled to an absolute reversion expectant on the decease of the survivor of two lives—B, aged 50, and C, aged 70. B has, however, *by voluntary settlement*, relinquished to A his right to the life interest which he would otherwise take after the death of C (the present life tenant). A now wishes to borrow upon his reversion a sum greater than the value of the absolute reversion if expectant on the death of the survivor of the two lives, but less than the value if C is treated as the only life tenant. What precautions should a lender take under the circumstances?

PROCEEDINGS OF THE INSTITUTE.—SESSION 1891-92.

First Ordinary Meeting, 29 November 1891.

The President (Mr. B. NEWBATT) in the Chair.

THE PRESIDENT, in welcoming the members at the opening of the new session, trusted that he should not disappoint their expectations by the omission of the Presidential address. It had been his intention to set a new precedent for which his successors would no doubt be grateful, and he did so on this occasion the more readily to make way for the consideration of a subject of the highest importance. The range of subjects open to a President of the Institute was circumscribed. The personal equation now constituted the chief interest of addresses delivered from the Presidential chair, and that he thought was usually exhausted in one such address.

He mentioned with great regret, that since the last meeting two notable names had been removed from their lists. In the death of Mr. Marr, they had to lament the loss of one whose capacity and integrity had given him a high place amongst practical actuaries, and whose genial nature had endeared him to a large circle of friends; while, by the still more recent death of Mr. Makeham, actuarial science had lost one of its brightest ornaments. The concluding instalment of his paper on the theory of inverse probabilities appeared in the October number of the *Journal*.

It had been arranged that Mr. Makeham should be proposed that evening for election as an Honorary Member, but the intention of thus doing honour to him and to the Institute remained, unfortunately, an intention only.

A paper entitled "On Legislation affecting Life Assurance Companies, more especially with reference to the Life Assurance Companies Acts, 1870 to 1872, and their Amendment", was read by the author, Mr. George King.

The following gentlemen took part in the discussion:—Messrs. Bailey, Higham, R. P. Hardy, Crisford, Strachan, Sheppard Homans, Young, Warner, A. G. Mackenzie, and the President.

Second Ordinary Meeting, 21 December 1891.

The President (Mr. B. NEWBATT) in the Chair.

Mr. Nightingale read his paper entitled "Formulas and Tables of Values for Life Interests and Reversions."

The following gentlemen took part in the discussion:—Messrs. Manly, Rothery, Ryan, Bailey, Bumsted, G. King, G. F. Hardy, and the President.

Third Ordinary Meeting, 23 January 1892.

The President (Mr. B. NEWBATT) in the Chair.

A paper entitled "On the Formulæ for determining the Value of Benefits, according to the Principle of Collective Assurance", was read by the author, Mr. R. P. Hardy.

The following gentlemen took part in the discussion:—Messrs. Adler, G. King, Bailey, Young, A. Hendriks, and the President.

Fourth Ordinary Meeting, 29 February 1892.

The President (Mr. B. NEWBATT) in the Chair.

The under-mentioned gentleman was elected a Fellow of the Institute:

A. B. Adlard, Associate.

A paper entitled "An Attempt to Measure the Extra Risk arising from a Consumptive Family History when the Life proposed for Assurance is physically Sound and Healthy", was read by the author, Mr. H. W. Manly, who also read a note upon the same subject, written by Dr. Glover Lyon.

The following gentlemen took part in the discussion: Messrs. Chisholm and G. King, Dr. Lyon, Dr. Pollock, Dr. Mackenzie, Mr. Harding, and the President.

Fifth Ordinary Meeting, 28 March 1892.

The President (Mr. B. NEWBATT) in the Chair.

A paper by Mr. H. J. Rothery, "On a System of Bonus Distribution considered in relation to the Office Premiums and Modes of Valuation", was read by the author.

The following gentlemen took part in the discussion:—Messrs. Bailey, Browne, Manly, Ryan, Whittall, Justicau, Burridge, Adlard, Schooling, Wyatt, Day, and the President.

Sixth Ordinary Meeting, 25 April 1892.

The President (Mr. B. NEWBATT) in the Chair.

A paper entitled:

"i. On a Method for Determining the Gain or Loss from Mortality in an Annuity Company;

"ii. On a Means of Calculating the expected Death-Strain in a Life Office".

was read by the author, Mr. G. H. Ryan.

The following gentlemen took part in the discussion:—Messrs. C. D. Higham, Colenso, Young, R. P. Hardy, G. F. Hardy, A. Hendriks, Ackland, Frankland, Bell, Burridge, G. King, Searle, and the President.

The Forty-Fifth Annual General Meeting, 11 June 1892.

The President (Mr. B. NEWBATT) in the Chair.

The proceedings at the Annual General Meeting will be found on page 185.

REPORT, 1891-92.

"The Council have pleasure in reporting to the members upon the progress of the Institute during the session of 1891-92, the forty-fourth year of its existence.

"The increase in the number of members was 25, as compared with 19 in the preceding year. Since the date of the Charter the number of members has been as follows:

1884-85	.	.	434	1888-89	.	.	563
1885-86	.	.	441	1889-90	.	.	601
1886-87	.	.	481	1890-91	.	.	620
1887-88	.	.	521	1891-92	.	.	645

"The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1892.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1891 .	2	168	202	237	11	620
ii. Withdrawals by						
(1) Death	4	2	45
(2) Resignation	2	31	...	
(3) Default in pay- ment of Sub- scriptions	6	...	
iii. Additions to Member- ship	2	164	198	200	11	575
(1) By Order of Council	68	...	70
(2) By Re-instatement	.	1	1	
iv. Transfers	2	165	199	268	11	645
(1) By Examination: from Associates to Fellows	5	5
(2) By Examination: from Students to Associates . . .	2	170	194	268	11	645
	8	8
(3) By Ballot: from Associates to Fellows . . .	2	170	202	260	11	645
	1
v. Number of Members in each Class on 31 March 1892 .	2	171	201	260	11	645

"It is with great regret that the Council have to report the loss by death of four Fellows during the year—namely, Mr. Goddard, Mr. Lucas, Mr. Makeham, and Mr. Marr. The two latter gentlemen were both former members of Council, while Mr. Makeham by his writings had acquired a world-wide reputation.

"The accounts for the year are satisfactory, the total amount of funds now being £4,353. 5s. 6d., showing an increase of £239. 4s. 1d.

"The income and expenditure account and balance sheet are given herewith.

1891-92.	£	s.	d.	£	s.	d.	
Amount of Funds, 31 March 1891—							
Messenger Legacy Fund		286	2	4	425	10	0
Brown Prize Fund		280	1	9			
Library Fund		293	0	0	104	9	10
General Fund		3,254	11	7			
				1,111	1	5	
Annual Subscriptions—							
Town Fellows	£233	2	0				
Country "	69	6	0				
Fellows admitted since 1884 :							
(1) By Examination	115	10	0				
(2) By Ballot	29	8	0	117	6	0	
Town Associates	207	18	0				
Country "	79	16	0				
Associates admitted since 1881 :							
(1) By Examination	30	9	0				
(2) By Ballot	25	4	0	313	7	0	
Students				282	9	0	
				1,075	2	0	
Fines on Reinstatement of Members					16	16	0
Entrance Fees—							
Fellow				5	5	0	
Students				71	8	0	
					76	13	0
Examination Fees					152	6	0
Sales of Publications—							
Text Book, Part I.				21	6	9	
" "				73	19	1	
Log. Cards				2	8	5	
Mortality Experience				8	16	0	
Life Tables				17	5	0	
					126	15	6
Dividends and Interest					113	9	1
Donation to Library Fund					5	5	0
					£5,658	8	0

1891-92.	£	s.	d.	£	s.	d.
Journal—						
Cost of Nos. 159, 160, 161, 162, 163						
Less Sales						
Library—						
Binding and Purchases						
General Expenditure—						
Rent				255	0	0
Salaries				341	10	0
Lectures				52	10	0
Examination Charges				61	6	3
Meetings				58	9	4
House Expenses				18	8	10
Corporation Duty				6	0	0
Fire Insurance				11	8	0
Postage and Stationery				142	3	3
Donation to Congress of Hygiene and Demography				10	10	0
Sundries				9	3	7
				906	9	3
				1,305	2	6
Funds 31 March 1892—						
Messenger Legacy Fund				291	15	1
Brown Prize Fund				288	0	9
Library Fund				298	5	0
General Fund				3,475	4	8
				4,054	3	8

Examined and found correct, 29 April 1892.

(Signed) ARTHUR B. WOODS, }
ELW. B. TREW, } *And others.*
W. REED MANHAM, }

Balance Sheet, 31 March 1892.

LIABILITIES.	£	s.	d.	£	s.	d.
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	263	17	8			
Accumulated Dividends	87	17	5			
				291	15	1
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200	0	0			
Accumulated Dividends	288	0	9			
	288	5	0			
Library Fund				3,475	4	8
General Fund						
				£4,353	5	6
LIABILITIES.						
Consols (£21,000), cost						
Metropolitan Railway 4 per-cent Debenture Stock (£2,700), cost						
Great Eastern Railway 4 per-cent Debenture Stock (£2,800), cost						
On Deposit at the London and Westminster Bank						
On Current Account at London and Westminster Bank						
Less Outstanding Cheque				187	5	6
Arrears of Subscriptions				7	0	0
				180	5	6
				39	18	0
				£4,353	5	6

Examined and found correct, 29 April 1892.

(Signed) ARTHUR B. WOODS, } Auditors.
EDW. B. TREW, }
W. REID MACKENHAM.

Examined and found correct, 29 April 1892.

(Signed) ARTHUR B. WOODS, } Auditors.
EDW. B. TREW, }
W. REID MACKENHAM.

"The annual subscriptions, together with fines for re-instatement, admission and other fees, amounted to £1,298. 17*s.*, being identical with those of the previous year. The total income was £1,544. 6*s.* 7*d.*

"The total expenditure for the year was £1,305. 2*s.* 6*d.*, which shows some increase, owing chiefly to expenses in connection with the *Journal*. During the past year five numbers have been published, whereas in the ordinary way only four appear in a year.

"The stock in hand of the Institute publications at date is as follows:

No. of Copies	Description of Work
453	<i>Text-Book</i> , Part I.
660	" " II.
153	Mortality Experience Tables.
19	Mortality Experience.
172	Logarithm Cards.
410	Messenger Prize Essay (Friendly Societies).
518	Index to 10 Vols.
118	" 20 "
8,808	Parts of <i>Journal</i> .

"The Council, in June last offered, out of the fund presented to the Institute by the late Mr. Samuel Brown, two prizes, of the value of fifty guineas and twenty-five guineas respectively, for the best two Essays, to be written by members, on 'THE ENFRANCHISEMENT OF LEASEHOLDS AND THE TAXATION OF GROUND RENTS, CHIEF RENTS, AND KINDRED CHARGES ON LAND IN ENGLAND AND WALES.' Two Essays were received in response, and the Adjudicators considering them of equal merit, and both worthy of distinction, the Council determined to award a prize of forty guineas to each of the competitors, who were Mr. Thomas Kyd and Mr. Arthur Wyndham Tarn.

"The following papers were submitted at the sessional meetings of the Institute, namely:

" 30 *November* 1891—'On Legislation affecting Life Assurance Companies, more especially with reference to the Life Assurance Companies Acts, 1870 to 1872, and their Amendment'—by Mr. George King.

" 21 *December* 1891—'Formulas and Tables of Values for Life Interests and Reversions'—by Mr. H. E. Nightingale.

" 25 *January* 1892—'On the Formulae for determining the Value of Benefits, according to the Principle of Collective Assurance'—by Mr. R. P. Hardy.

" 29 *February* 1892—'An attempt to measure the Extra Risk arising from a Consumptive Family History when the Life proposed for Assurance is physically Sound and Healthy'—by Mr. H. W. Manly.

" 28 *March* 1892—'On a System of Bonus Distribution considered in relation to the Office Premiums and Modes of Valuation'—by Mr. H. J. Rothery.

" 25 *April* 1892—I. On a Method for determining the Gain or Loss from Mortality in an Annuity Company. II. On a Means of Calculating the Expected Death-strain in a Life Office'—by Mr. G. H. Ryan.

"An Intermediate Examination in Part I was held on 31 October 1891, when nine candidates presented themselves, of whom five passed, as follows:

In Class I None.

In Class II { A. C. R. Cockman.
J. Crompton.
G. M. Searle.

In Class III { G. Baker.
J. Burn.

"It is intended that an October Examination in Part I shall be held in 1892, but that in 1893 and subsequent years the October Examination shall be discontinued.

"For the Examinations held in the United Kingdom on 29 and 30 April last, fifty-six candidates presented themselves, namely:

23	for Part	I,	of whom	6	were successful.
23	"	II,	"	10	"
10	"	III,	"	5	"

"The following are the successful candidates, the names in each Class being arranged alphabetically:

PART I.

Class I:

{ F. G. Channon.
G. H. Ward.

Class II:

{ K. W. Elder.
O. Kentish.

Class III:

{ J. W. Salisbury.
W. Tuck.

PART II.

Class I:

R. Todhunter.

Class II:

{ J. A. Archer.
F. H. Kitchin.

Class III:

{ H. J. Baker.
J. F. E. Hall.
W. Hutton.
P. B. Moodie.
J. F. Moran.
L. Stahlschmidt.
W. G. Titmuss.

PART III.

Class I:

None.

Class II:

{ H. Foot.
 { G. J. Lidstone.
 { E. R. Straker.

Class III:

{ J. R. Hart.
 { H. Milton.

"For the first time, Examinations were held in the Colonies. For Part I thirty-one candidates sent in their names, for Part II two candidates, and for Part III one candidate. The Council consider it to be very satisfactory that Colonial students should have availed themselves so largely of the facilities thus offered to them. The names of the successful candidates in the Colonies will be published in due course.*

"With a view to make admission to the Class of Associate less difficult in future, and yet to maintain the value of the Associateship as an honourable degree, it has been determined, on the one hand, to withdraw from the Associateship Examinations some of the more advanced mathematical subjects, and, on the other hand, to introduce into them the elementary portions of some of the financial and legal subjects.

"A revised Syllabus will shortly be issued,† which will come into force as regards Parts I and II in April 1893, and as regards Part III in April 1894. It is not intended to lower the standard for admission to the Class of Fellows, but the Examination in Part III will be divided into two sections, and candidates will be allowed to take both the sections in the same year, or to postpone either section to a subsequent year. In future, graduates in mathematical honours of any university in the United Kingdom will be exempted from the Examination in Part I, and Fellows of the Faculty of Actuaries by examination will be considered by the Council to be eligible, under Bye-Law No. 44, as Associates, without passing the Examinations for admission to the Class of Associate.

"These alterations will, the Council hope, have the effect both of consolidating and extending the membership and of enlarging the authority and usefulness of the Institute.

"B. NEWBATT,

"June 1892."

"President."

* The results of the Examinations in the Colonies are given on page 188.

† See page 164.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Saturday, 11 June, the President, Mr. B. Newbatt, in the chair.

The Report of the Council (given on p. 179) having been read.

The PRESIDENT, in moving that the Report of the Council and Statement of Accounts be received and adopted, congratulated the members on the fact that, owing principally to the accession of some 30 Australian fellow-subjects, the numbers at the close of the year showed a net increase of 25, bringing up the total membership to 645. The expenditure was more by £264. 9s. than in the previous year, but there was a balance of receipts over expenditure of £239. 4s. 1d., raising the total funds to £1,353. 5s. 6d. Of this sum £579. 15s. 10d. was held in connection with the Messenger and Brown funds, thus leaving as the unfettered property of the Institute £3,773. 19s. 8d. The chief item of extra outlay was in connection with the *Journal*, of which there was issued an extra part containing Mr. Young's exhaustive paper on the "German State Insurance Laws", so that the larger expenditure was not one with which those who rate highly the educational functions of the Institute would quarrel. As regards the recent competition for the Brown prize, not only the successful competitors, but the Institute itself, were to be congratulated. The subject of the essay was "The Enfranchisement of Leaseholds, and the Taxation of Ground Rents, Chief Rents, and kindred charges on Land in England and Wales"—a subject treating of, and in its treatment not unlikely to influence, one of the burning questions of the day; and two prizes of 50 guineas and 25 guineas respectively were offered. Two essays only were received, but, while differing a good deal in method of treatment, the merits of both were so conspicuous, and in the judgment of the adjudicators were so nicely balanced, that equal prizes were awarded. That award was improved upon by the Council, who would gladly have given both competitors first prizes of 50 guineas, but the funds at their disposal not admitting of this, 40 guineas were awarded to each gentleman—Mr. Thomas Kyd and Mr. A. W. Tarn. The report next refers to the papers of the year, which reached a high standard of excellence. For the first time the April examinations this year had been carried on in the Colonies as well as at home, and the result of the Colonial examinations was not as yet wholly known. They would endorse the language of the Council, in expressing satisfaction that so many Colonial students—34—had availed themselves of the new facilities offered them. He hoped also that they would equally sympathize with his satisfaction at that approach to the principle of federation or affiliation for which he pleaded in his presidential address. He offered hearty congratulations to the five gentlemen who had passed in Part III, and welcomed them to the full honours of the Fellowship. The concluding paragraphs in the report treated of the future examinations. The new departure recorded the outcome of much laborious consideration on

the part of the Examination Committee, and particularly of its chairman, Mr. Young, and of the hon. secretary specially attached to it, Mr. King; and the object of the committee, supported by the hearty concurrence of the Council, was to give effect to his desire to make admission to the class of associates less difficult, while yet maintaining the value of the Associateship as an honourable degree. How that has been done—how there had been withdrawn from the Associateship examinations some of the more advanced mathematical subjects, and how there had been introduced into them the elementary portions of some of the financial and legal subjects—the new syllabus about to be issued would explain. He expressed his pleasure at the further action of the Council in agreeing to make the Fellowship by examination of the Faculty of Actuaries in Scotland that sufficient evidence of professional eligibility which Bye-Law No. 44 required from those who sought to be elected Associates of the Institute without passing the prescribed examinations. That was a first step towards the attainment of the identity in the examinations of the Institute and of the Faculty which many hoped to see established.

Mr. T. E. YOUNG, in seconding the proposition, indicated the elements of success which it had been the happy fortune of the Institute to secure; the loyal devotion of the members to their *alma mater*, the vigorous and prudent official administration, and particularly the good fortune in the succession of presidents—men in whom the power of initiative, the absence of fear of reforms, judiciousness in carrying those reforms into conciliatory execution, and not the least invaluable, in whom urbanity and suavity had been admirably combined. He cordially congratulated the President upon the fact that in surveying at leisure his occupancy of the chair he could unfeignedly feel that, by the work of the society during his *regime*, *Quorum pars magna fuit*, he had succeeded in taking a most effective part in helping and extending the permanent fortunes of the Institute to the benefit of them all.

The resolution was unanimously adopted.

ELECTION OF OFFICERS.

A ballot having been taken, the Scrutineers, Messrs. BELL and E. WOODS, reported that the following list had been adopted:

President.

AUGUSTUS HENDRIKS.

Vice-Presidents.

THOMAS G. C. BROWNE.
GEORGE STEPHEN CRISFORD.

RALPH PRICE HARDY.
CHARLES D. HIGHAM.

Council.

ARTHUR HUTCHESON BAILEY.	CHARLES DANIEL HIGHAM.
GEO. WM. BERRIDGE.	GEORGE HUMPHREYS, M.A.
THOMAS G. C. BROWNE.	GEORGE KING.
JAMES CHISHOLM.	HENRY WILLIAM MANLY.
HENRY COCKBURN.	BENJAMIN NEWBATT.
FRANCIS ERNEST COLENZO, M.A.	HOWARD JAMES ROTHERY.
ERNEST COLQUHOUN.	*GERALD H. RYAN.
THOS. HOMANS COOKE.	*JAMES SORLEY.
GEORGE STEPHEN CRISFORD.	THOMAS BOND SPRAGUE, M.A.
*STANLEY DAY.	*A. W. SUNDERLAND, M.A.
DAVID DEUCHAR.	WILLIAM SUTTON, M.A.
THOMAS CHARLES DEWEY.	GEORGE TODD, M.A.
*ALEX. J. FINLAISON, C.B.	WILLIAM J. H. WHITTALL.
*RALPH P. HARDY.	FRANK BERTRAND WYATT.
AUGUSTUS HENDRIKS.	THOMAS EMLEY YOUNG, B.A.

* New Members of Council.

Treasurer.

HENRY WILLIAM MANLY.

Honorary Secretaries.

THOMAS HOMANS COOKE. | GEORGE KING.

Mr. AUGUSTUS HENDRIKS, in acknowledging his election to the office of President of the Institute, said they had conferred upon him the greatest honour to which any actuary could aspire. He expressed great regret that the office should be vacated by Mr. Newbatt, who had so faithfully carried on the traditions which had descended to him, of holding intact the privileges of the Institute, upholding its dignity, and in every way conducing to its prosperity. It would be his duty to carry on the traditions which had been handed down to him. The former president had referred to the good understanding which has existed between the Institute and the Faculty of Actuaries in Scotland. That good understanding it would be one of his first duties to preserve. There were other bodies in the United Kingdom, the Colonies, and abroad, all working for the common good, and he would continue that cordiality of feeling with regard to them which had been maintained by Mr. Newbatt. It was an unwritten bye-law, that a new President had, in the dark days of November, to throw such light as he was able to do upon the current subjects of the hour, and therefore he would not detain the members, except to express his cordial thanks for the great honour conferred upon him.

Messrs. W. R. MAKEHAM, A. S. HARRIS, and J. W. MILLER were elected Auditors for the ensuing year.

Mr. J. B. TENNANT proposed, and Mr. JOHN COLES seconded, a vote of thanks to the President, Vice-Presidents, Council, and Officers of the Institute for their services during the past year.

The resolution was cordially adopted, and Mr. T. G. C. BROWNE responded.

A vote of thanks was accorded to the Auditors, on the motion of Mr. G. H. RYAN, seconded by Mr. A. W. SUNDERLAND.

The proceedings then terminated.

COLONIAL EXAMINATIONS.

Examinations were held on 29 and 30 April 1892, at Montreal, Melbourne, Sydney, and Wellington, with the following results:

PART I.

Twenty-nine Candidates sent in their names, of whom twenty-two presented themselves, and ten passed as follows:

Class I:

Adams, C. E. (Wellington).
Blackadar, A. K. (Montreal).
Hardcastle, E. E. (Wellington).

Class II:

Home, N. C. M. (Montreal).
Muter, P. (Wellington).
Stuckey, J. J. (Melbourne).

Class III:

Fraser, T. J. (Melbourne).
Meyers, H. W. (Melbourne).
Paull, R. A. (Melbourne).
Townsend, H. V. (Melbourne).

PART II.

Three Candidates sent in their names, and presented themselves, of whom two passed as follows:

Class I:

None.

Class II:

None.

Class III:

Holliday, J. (Montreal).
Moors, E. M. (Sydney).

PART III.

One Candidate sent in his name, but did not present himself for examination.

THOS. H. COOKE, }
GEORGE KING, } *Hon. Secretaries.*

London, July 1892.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

- I. *On a Method for determining the Gain or Loss from Mortality in an Annuity Company.*
II. *On a Means of Calculating the Expected Death-Strain in a Life Office.*

By GERALD H. RYAN, *Fellow of the Institute of Actuaries, and Actuary of the Royal Exchange Assurance.*

[Read before the Institute, 25 April 1892.]

I.—ON A METHOD FOR DETERMINING THE GAIN OR LOSS FROM MORTALITY IN AN ANNUITY COMPANY.

EVERY student of actuarial literature can call to mind the writings of leading experts that deal with the methods to be employed in order to ascertain what pecuniary profit or loss has resulted from the mortality experienced by a life assurance company. In this path of knowledge, Mr. C. D. Higham may be said to have been, if not the first to tread the soil, at least the earliest to write, for the benefit of the community, a record of his travels. Certain it is that since his paper on the "Value of the Death-Strain" (*J.I.A.*, xx, 153), the importance of considering the true amount rather than the nominal amount at risk in such investigations as I have referred to, has been freely recognized and acted upon; and the death-strain as a function in actuarial matters has been much written and talked about.

Events have led me to consider the subject in regard to its bearing upon the operations of annuity companies, and though the interest of my enquiry may be comparatively restricted, I need make no apology for submitting to the Institute the conclusions at which I have been able to arrive.

In dealing with the true effect of mortality in life assurance transactions, the formula which supplies the base of operations is

$$({}_nV_x + P_x)(1+i) = {}_{n+1}V_x + q_{x+n}(1 - {}_{n+1}V_x) \quad . \quad . \quad (1)$$

or, to translate it verbally, the reserve at the commencement of a year, together with the premium just due, accumulated at interest for a year, is equal to the reserve at the end of a year *plus* the probability of dying in the year multiplied into the actual sum at risk, or, as it has been termed, the “measure of the death-strain.”

An analogous expression can easily be obtained for life annuities.

We have

$$\begin{aligned} a_x &= v p_x (1 + a_{x+1}) \\ &= (1 - q_x) v (1 + a_{x+1}) \end{aligned}$$

$$\text{and} \quad (1+i)a_x = (1 + a_{x+1}) - q_x(1 + a_{x+1}) \quad . \quad . \quad . \quad (2)$$

or, again, explaining the equation verbally, the reserve at the commencement of the year, accumulated at interest, is equal to the reserve at the end of the year *minus* the probability of dying in the year multiplied into the actual sum that will be released if death takes place.

In comparing the formulas numbered (1) and (2), certain points arrest attention. First, from the nature of the case, in annuity investigations it is the expected *gain* from mortality, and no longer the expected loss, or strain upon the funds, that we are concerned with. Parenthetically, it should be remarked that in using the convenient and simpler term “expected gain”, we must, however, be careful to bear in mind that it is merely an abbreviation for the more exact phrase “expected cancelment or release of liability by death”, and does not in any sense denote true profit from mortality. Next, that the reserve at the end of the year is identical with the sum subject to mortality, that is, with the “measure of the strain” (or gain). This, also, is evident from a consideration of the elements involved, though the brief mathematical analysis brings it home to us clearly and forcibly. The analogy between the two formulas is, however, complete, and

just as we have $q_{x+n}(1 - {}_{n+1}V_x)$ as an expression for the value of the death-strain in life policies in the year succeeding age $x+n$, so we have $q_{x+n}(1 + a_{x+n+1})$ to represent the value of the death-gain in single-life annuities.

We now find ourselves possessed of an instrument for measuring the effect of mortality on annuity funds, which can be applied with ease and accuracy. A schedule will show the simplicity of the whole operation.

Age attained	Amount of Annuities in course of Payment	VALUATION		MORTALITY EXPERIENCE		
		Unit of Value	Value of Annuities at end of Year of Age	Expected Rate of Mortality	Expected Cancellments of Annuities by Death	Expected "Gain" or Liability released by Death
x	N	$(1 + a_{x+1})$	$N \times (1 + a_{x+1})$	q_x	$N \times q_x$	$q_x N(1 + a_{x+1})$
(1)	(2)	(3)	(4)	(5)	(6)	(7)

The summation (in groups, or in a single total) of the results obtained in col. 6 has now to be compared with the amount of the annuities actually cancelled by death; and the summation of col. 7 with the corresponding summation of the "actual gain by mortality", that is, of the amount of annuities cancelled by death multiplied into $(1 + a_{x+1})$ for each age (x). We thus obtain the comparisons familiar to us, under slightly different conditions, in life assurance investigations, namely, "expected cancellments by death" as against actual cancellments by death, and "expected gain from mortality" as against actual gain therefrom.

A few words may here be interpolated, with the object of showing how the formula just deduced can be used by direct application to the valuation schedules. In connection with the valuation of life policies, it has been usefully suggested that we should use $(1 + a_x)$ in computing the value of the premiums, and make any corrections that may be required by the circumstances of the case at the end, and I would venture to suggest that single-life annuities might be valued by $(1 + a_{x+1})$, and any needful adjustment subsequently made. In this way allowance could be made for the incidence or distribution of the annuities over the year of age, for half-yearly or quarterly payments, for

apportionable final payments up to the date of death, and so on, under the conditions applicable to the case in hand.

Now if we adopt this mode of estimating the reserves (subject to correction, as explained), we can immediately pass to the "expected gain" by merely multiplying the values at each age by q_x ; and thus the whole process of valuing the annuities and ascertaining the expected cancelments and expected gain from mortality could be gone through year by year with the smallest expenditure of time and labour. But in spite of the formula, it will clearly in practice be accurate enough, for the purpose of an estimate, to use the actual valuation schedules, though they may be based upon some other mode of selecting the units of value. If a company, for instance, values by $(\frac{1}{2} + a_x)$, the error in the "expected gain", obtained by multiplying q_x into the valuation results, will be quite unimportant. One point, however, remains to be briefly mentioned. The formula assumes all annuities to be entered upon or brought forward at exact age x , and the lives to be exposed to the contingency of mortality throughout the year of age x to $x+1$. Hence, the process explained leaves out of account new entrants who join the company during the course of the year of experience; and some correction will be necessary on this score. Unless a company is transacting a very large new business, it will probably be sufficient to disregard the experience of the new entrants of the year altogether, care being taken to deduct from the "annuities cancelled by death" any that may have arisen on contracts not in force at the date of the investigation.

As an exercise in the use of the method, not without some degree of interest in the light thrown upon the working of certain tables in relation to annuity experience, the following table has been prepared:

Single-Life Annuities.—Valuation and Calculation of Expected Gain by Mortality.

Age attained	Annuities in Force	VALUATION 3 PER-CENT INTEREST			EXPECTED RELIEF OF LIABILITY IN ENSUING YEAR, ON VALUATION BY THE CARLISLE TABLE		
		Carlisle Table	Gov. (Female) 1883, Average	Gov. (Female) 1883, Select	Carlisle Mortality	Gov. (Female) Average Mortality	Gov. (Female) Select Mortality
x	N	$N(1 + a_{x+1})$	$N(1 + a_{x+1})$	$N(1 + a_{[x+1]})$	Carlisle $q_x \times (3)$	Gov. Av. $q_x \times (3)$	Gov. Sel. $q_{[x]} \times (3)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
50	2,239	33,128	36,003	35,712	449	375	261
51	2,438	35,497	38,325	38,106	507	430	284
52	2,795	39,613	42,931	42,736	602	504	324
53	3,246	44,795	48,658	48,528	723	618	376
54	3,710	49,751	54,277	54,203	841	744	428
55	4,134	53,783	58,992	58,910	964	853	474
56	4,544	57,300	63,162	63,116	1,089	956	517
57	4,984	60,505	67,434	67,433	1,273	1,091	563
58	5,495	65,061	72,259	72,314	1,575	1,224	617
59	6,116	70,273	78,040	78,224	1,987	1,373	682
60	6,872	76,819	85,007	85,350	2,573	1,575	764
61	7,704	83,820	92,217	92,833	3,000	1,816	935
62	8,500	89,815	99,760	99,195	3,361	2,051	1,122
63	9,185	94,146	102,505	103,791	3,601	2,326	1,316
64	9,726	96,482	104,555	106,111	3,837	2,600	1,511
65	10,078	96,547	104,307	106,021	3,967	2,812	1,693
66	10,204	94,183	101,428	103,367	4,003	3,025	1,850
67	10,215	90,607	97,553	99,606	4,022	3,161	1,992
68	10,150	86,276	92,873	95,105	4,008	3,238	2,123
69	10,006	81,205	87,553	90,054	3,988	3,318	2,237
70	9,821	76,015	82,103	84,951	3,926	3,388	2,344
71	9,684	71,371	77,182	80,571	4,200	3,519	2,374
72	9,516	66,993	72,322	76,032	4,564	3,679	2,404
73	9,276	62,613	67,158	71,240	4,891	3,853	2,423
74	9,008	58,642	62,155	66,389	5,288	4,002	2,448
75	8,667	54,429	56,942	61,189	6,199	4,108	2,450
76	8,204	49,716	51,275	55,541	5,119	4,096	2,413
77	7,653	44,694	45,535	49,591	4,802	4,020	2,339
78	7,100	39,689	40,044	44,120	4,319	3,867	2,239
79	6,488	34,841	34,711	38,539	4,125	2,638	2,118
80	5,897	30,192	29,898	32,080	3,675	3,483	1,978
81	5,341	26,171	25,637	26,972	3,502	3,311	2,443
82	4,831	22,561	21,884	22,464	3,174	3,101	2,361
83	4,298	19,126	18,395	18,353	2,886	2,877	2,247
84	3,800	16,074	15,352	15,314	2,552	2,632	2,655
85	3,280	13,218	12,530	12,497	2,317	2,377	2,392
86	2,767	10,708	9,961	9,961	2,072	2,077	2,089
87	2,268	8,573	7,733	7,711	1,854	1,826	1,836
88	1,829	6,712	5,908	5,889	1,475	1,552	1,556
89	1,439	5,037	4,403	4,389	1,085	1,263	1,267
90	1,129	3,939	3,263	3,263	1,024	1,046	1,050
91	878	3,143	2,414	2,414	898	915	918
92	677	2,498	1,760	1,760	699	768	768
93	513	1,919	1,257	1,257	498	627	629
94	378	1,421	862	858	355	486	488
95	263	973	555	555	227	368	370
96	181	644	382	346	140	258	258
97	124	420	262	211	93	168	187
98	94	294	198	133	63	118	144
Totals	257,745	2,132,952	2,269,920	2,335,405	122,392	100,543	69,257

The groundwork of the above table is supplied by the "model office", formed from the returns of certain companies transacting business of this class, for purposes explained by the title of a paper I had the honour to read before the Actuarial Society of Edinburgh [*On the Several Mortality Tables Employed in the Valuation of Annuity Contracts*. Actuarial Society Transactions (New Series) No. 13]. Taking the total amount of annuities in force at each age from this source, the succeeding columns give the value of the contracts *at the end of the year*, according to the Carlisle and the New Government (Female) Annuitants' Experience. Columns are then added for the purpose of showing the "actual and expected gain" from mortality, assuming the actual experience of mortality to follow a different table from that employed in the valuation. The results in this respect may be shortly epitomized:

The annuities in course of payment amount to £257,745, and their present value (at the end of the year), with interest at 3 per-cent, comes out at

£2,132,952 by the use of the Carlisle Table.

£2,269,920 " " " New Government (Female) Annuitants' Table
(Average).

and £2,335,405 " " " New Government (Female) Annuitants' Table
(Select: assuming all contracts to have been
just issued).

Now, assuming that such a company which has valued its contracts by the Carlisle Table experiences in the ensuing year in respect of all its annuities a rate of mortality coinciding with that shown by the Government (Female) Average Table, the expected gain to the fund by death, or the amount of liability expected to be cancelled from this cause, would be £122,392, whereas the actual gain on the hypothesis named is shown to be £100,543 only. In other words, the "model" company would have suffered a loss on valuation of £21,849 by reason of the death rate having been so much lighter than that assumed in the valuation tables. Again, assume the actual experience of the company to follow the Government (Female) Select Table. In this case, the amount of liability cancelled by death would be £69,257 only, and the loss on valuation from mortality in the year under observation would be £53,135. The magnitude of the above differences will not escape attention, and if the present investigation has any importance at all, it should impress upon us afresh the necessity of watching with extreme closeness the working of this class of business, which, as is well known, has

operated unfavourably in the case of several offices in the past.

As a final illustration, an examination of the experience of a certain company transacting annuities to a not inconsiderable extent, over a period of five years, showed that notwithstanding the use of the Government (Female) Average Table in the valuation of all its annuities, male and female, the actual gain on valuation—not, of course, strict profit—to the fund by death (obtained by valuing all the annuities cancelled from that cause as at the end of the year in which death occurred) was £10,251 only, as against an expected gain of £12,449 according to the valuation table. This may have arisen from an exceptionally light mortality, unlikely to recur; but as the annuities payable over the five years amounted to £43,786, the data were not very meagre. At any rate, it will not be denied that another warning is here sounded, however little real importance should be attached to it, that business of this class demands close attention.

In dealing with contingent reversionary securities, it is sometimes customary to take out annuities on the joint lives of the reversioner and life-tenant to secure payment of the premiums necessary to protect the advances, and occasionally of the interest also. In such transactions, there would presumably be little or no selection against the company; and it is reasonable to suppose that annuities of this description would be subject to a higher rate of mortality, and (presuming they were purchased on the same terms) would yield a higher rate of profit than those purchased from prudential motives. We might even imagine the case of a company showing a profit on its annuity branch, which profit, on close investigation, is found to be attributable entirely to the working of the two-life annuities, the general provident business of the branch having yielded no profit at all. The moral of this surely is that we should not rest satisfied until we have investigated the true effect of the mortality experience on the annuity fund: and thus we shall guard against being led into erroneous conclusions based on a consideration of the aggregate results only.

It should not, however, be inferred that I wish to give expression to pessimistic views on the profitableness of granting annuities. The experience of each company will, I imagine, vary far more here than in life assurance business, and some offices may very likely reap a profit while their neighbours are lamenting a loss. But the point I would venture to enforce is, that the

foregoing illustrations of the method I have submitted for the consideration of the profession enable us to see in a clear light the extent of the loss which a deviation from the rates of mortality assumed in the valuation may cause. And it is, perhaps, not too much to claim for the method itself that it places in the hands of actuaries a very short and easy means of following up and watching the results of mortality experience in annuity business—a course which, on many grounds, seems most desirable, if not, indeed, absolutely necessary.

II.—ON A MEANS OF CALCULATING THE EXPECTED DEATH-STRAIN IN A LIFE OFFICE.

The foregoing investigation has suggested to me an examination of the larger subject of the modes of computing the expected strain by claims upon the funds of a life office, and I take this opportunity of laying before the members of the Institute certain suggestions on this point. My remarks will form a brief corollary to Mr. C. D. Higham's important paper before referred to; and Mr. Higham has been allowed to remain so long in undisputed possession of this field of knowledge, that he will not regard me, I hope, as an interloper or trespasser on his preserves if I follow him, *longo intervallo*, in the path he has marked out.

The examination of the true effect of the claims experience of a company upon its funds is not, I am afraid, so common a thing as might be expected considering its undeniable importance. For this fact—if fact it be—we must look for the cause in the great difficulty of applying the principle explained by Mr. Higham upon the lines laid down by him; and if a shorter means of obtaining the desired result can be suggested, it is fair to assume that these investigations will become more frequent. That the troublesomeness of the operation has not been exaggerated is sufficiently proved by quotations from Mr. Higham's paper. In accordance with his recommendations, it is required to have: "for each age
" the sums assured and reserve value—first, of the gross number
" of the policies in force; secondly, of the death claims; thirdly,
" of such a proportion of the assurances terminated otherwise than
" by death as is left by the foregoing adjustment [*i.e.*, one-half
" the sum assured, &c., for lapsed and surrendered assurances];
" and, lastly, of a similar proportion of the new business." There is no doubt that the conditions here imposed can only be complied with at a great expenditure of time and labour, more especially

in the case of an office transacting a large new business with its attendant waste of business from surrenders and lapses.

Now, considering Mr. Higham's expression for the value of the expected death-strain,

$$q_{x+n}(\Sigma S - \Sigma_{n+1} V_x),$$

we see that the result obtained relates to the year from $x+n$ to $x+n+1$, and that the policy-reserves at the end of the year, $\Sigma_{n+1} V_x$, are those that attach to the policies in force at the commencement of the year, the sums assured under which may, for greater clearness, be designated by $\Sigma(S)_{x+n}$. These two quantities do not facilitate the use of the valuation schedules, or class-book, in their present form, and Mr. Higham therefore replaces $\Sigma(S)_{x+n}$ by $\Sigma(S)_{x+n+1}$, modifying the last-mentioned values to allow for the claims of the year, the discontinuances and the new business, corresponding adjustments being made in $\Sigma_{n+1} V_x$. He thus obtains, as a working formula,

$$q_{x+n}[\Sigma(S')_{x+n+1} - \Sigma_{n+1} V'_x],$$

the accents denoting that the terms are specially adjusted. Upon this basis his numerical illustration of the process recommended has been founded (see *J.I.A.*, xx, 159).

The formula, as originally given may, however, be slightly expanded. We have, obviously,

$$\text{Expected strain} = q_{x+n}[1 - A_{x+n+1} + P(1 + a_{x+n+1})],$$

or, generally,

$$\text{Total strain} = q_{x+n}[\Sigma(S)_{x+n} - \Sigma A_{x+n+1} + \Sigma P(1 + a_{x+n+1})].$$

Confining our attention for the moment to the first expression, we see the operation of the formula in a very clear light. The constituent parts are plainly exposed to view. If we assume no premium to be payable, the expression reduces to $q_{x+n}(1 - A_{x+n+1})$, as separately deduced by Mr. Higham, and gives us the value of the death-strain on paid-up policies and reversionary additions. If, again, we assume no sum assured, or death-benefit, to be payable, the formula takes the form $q_{x+n} \cdot P \cdot (1 + a_{x+n+1})$, and is suitable to the case of reductions of premium, or, replacing P by the appropriate quantity, to annuities, agreeing with the formula for the death-gain in annuities obtained in the preceding paper (see p. 190). The new form of the expression is, perhaps, preferable in use to that with which Mr. Higham has familiarized us. A blank schedule will show the process in full:

Calculation of the Expected Death-Strain.

Age at Date of Investigation	Total Sums Assured and Reversionary Bonus at $x+n$	Value of (2) at $x+n+1$	Total Net Premiums payable at $x+n$, valued at $x+n+1$	Expected Amount of Claims	Value of Expected Gross Liability cancelled by Death	Value of Expected Loss by Cancellation of Premiums through Death
$x+n$	$\Sigma(S)_{x+n}$	$\Sigma(S)_{x+n} \times A_{x+n+1}$	$\Sigma P \times (1+a_{x+n+1})$	$\Sigma(S)_{x+n} \times q_{x+n}$	$\Sigma(S)_{x+n} \times A_{x+n+1} \times q_{x+n}$	$\Sigma P(1+a_{x+n+1}) \times q_{x+n}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
⋮						
⋮						
⋮						
Totals for all Ages }						

From the totals of the various columns we should have—

Expected amount of claims = Σ col. (5).

Expected death-strain in } ensuing year = Σ col. (5) - Σ col. (6) + Σ col. (7).

This now appears by no means a formidable undertaking. The necessity for valuing the sums assured and bonuses, and net premiums appertaining to age $x+n$ at one year advance of age, prevents us from using the ordinary valuation schedules; but the re-arrangement is simple enough, and would not take very long to work out. No account is, however, taken of the new entrants or discontinuances of the year, and some adjustment would be necessary on this score.

Some of our difficulties will, however, disappear if we discard the usual units of value in favour of *continuous values*. It is obvious we may assume the function V to grow by moments, instead of annual increments. The expression for the value of the death-strain then becomes

$$\mu_{x+n}(1 - \bar{A}_{x+n} + P\bar{a}_{x+n}),$$

and generally, $\mu_{x+n}[\Sigma(S)_{x+n} - \Sigma\bar{A}_{x+n} + \Sigma P\bar{a}_{x+n}]$;

or, if continuous values are not actually employed in the valuation,

$$\mu_{x+n}\left[\Sigma(S)_{x+n} - \Sigma A_{x+n}\left(1 + \frac{i}{2}\right) + \Sigma P\left(\frac{1}{2} + a_{x+n}\right)\right].$$

Here, then, we have a formula which brings the quantities and their values into strict correlation as to age, thus permitting direct use of the results already obtained in the usual valuation schedules. The "expected claims", $[\mu_{x+n}\Sigma(S)_{x+n}]$, and the "expected strain" do not, however, relate to the full year starting from the date of our investigation, but give the expectation for that year on the assumptions that the force of mortality is constant and the amounts at risk at $x+n$ remain undisturbed, throughout the year, the place of any claims maturing or discontinuances being immediately filled up by other risks of the same amount, age, duration and class (see *J.I.A.*, xiii, 329). But the results also give, with quite sufficient accuracy, in the case of an office not subject to rapid expansion or contraction, the mean expected claims and strain of the year immediately preceding and that immediately succeeding the date of investigation. It will also be noticed that in adopting the new formula we at once get rid of the many troublesome and complicated adjustments in respect of new entrants and discontinuances, for no such disturbances need be considered to take place at the instant of time, $x+n$, to which the formula strictly refers.

Investigations of this nature, performed at reasonable intervals of time, are of real value and importance; but there is certainly no necessity to undertake the task too frequently. If an examination of the true pressure of the claim experience on the funds of a company is made at each periodical valuation (or once every three or five years) it is, in my opinion, quite as often as the circumstances require. And an application of the new method at two successive valuations would enable the expected claims and strain of the period to be assessed with as much closeness as the materials at our command permit, and would take into due account any important changes in the nature or distribution of the risks that might have occurred.

The whole scheme is thus reduced to the greatest simplicity. Again taking advantage of the clearness which a draft schedule conveys, I append a skeleton table:

Valuation of Contracts, and Calculation of Expected Claims and Expected Strain.

Age at Date of Investigation	* VALUATION				EXPECTED CLAIMS AND EXPECTED STRAIN		
	Total Sum Assured and Reversionary Bonuses	Total Net Premiums Payable	Value of Sum Assured and Reversionary Bonuses	Value of Net Premiums	Expected Claims	Value of Expected Gross Liability cancelled by Death	Value of Expected Loss of Premium Income by Death
$x+n$	$\Sigma(S)_{x+n}$	ΣP	$\Sigma(S)_{x+n} \times A_{x+n}$	$\Sigma P \times (\frac{1}{2} + a_{x+n})$	$\frac{\mu_{x+n}}{\Sigma(S)_{x+n}}$	$\mu_{x+n} \times \Sigma(S)_{x+n} \times A_{x+n}$	$\frac{\mu_{x+n}}{\Sigma P(\frac{1}{2} + a_{x+n})}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
⋮							
Totals for all Ages)							

Then we have

Expected claims = Σ col. (6).

Expected strain = Σ col. (6) - Σ col. (7) $\times \left(1 + \frac{i}{2}\right) + \Sigma$ col. (8).

In applying these methods to the case of my own office, I slightly depart from the formula arrived at. As the office values its contracts by the H^M and $H^{M(5)}$ Tables, I substitute q for μ , and use $q^{(0-4)}$ (see *J.I.A.*, xxii, 256, xxiv, 283, and xxvii, 252) in connection with the H^M figures, and $q^{(5)}$ in connection with the $H^{M(5)}$ figures. In this way effect is given to the influence of selection still current on business of less than five years' duration, and the valuation schedules can be used precisely as they stand. We also probably obtain far more trustworthy results than would be derived by using the formula strictly, and taking μ from a general or average table. The shortness of the entire operation is surprising, and if Crelle's Tables are used—I do not hesitate to use them myself for this purpose—we can obtain results quite accurate enough for the object in view in the

* The other columns of the usual valuation schedules have been omitted, as having no special bearing on the subject of this note. In the case of a company making reductions of premiums to any material extent, columns would have to be added, both to the valuation and mortality experience sections of the above form.

space of a few hours. Students will hardly need to be reminded that $\mu_x < = > q_x$, according as $d_{x-1} < = > d_x$ approximately, Mr. Sprague having demonstrated this result some years ago (*J.I.A.*, xxiv, 43).

A rough estimate for the expectations under special policies completes the investigation. It may be noted, however, that if the method is applied in full to endowment assurances, and the valuation schedules made use of, the adjustment in the total value of the sums assured and reversionary bonuses (ΣA) to convert it into the *continuous* form will not be represented by $\left(1 + \frac{i}{2}\right)$, as the pure-endowment part of the function does not need correction. This has already been pointed out by several writers.

In conclusion, opportunity may be taken to point out that the method developed in connection with the expected strain in life policies may also be applied to the case of annuities, if circumstances should require it; and that the expected gain (or strain) in the latter may be represented by the expression, $\mu_x \bar{a}_x$, subject to the conditions and limitations already mentioned.

DISCUSSION.

The PRESIDENT (Mr. B. Newbatt) having invited the referees, Mr. C. D. Higham and Mr. F. E. Colenso, to open the discussion.

Mr. C. D. HIGHAM, after thanking Mr. Ryan for his courteous references to himself, said that Mr. Ryan complained of the trouble that the carrying out of his (Mr. Higham's) suggestions would give, but he must plead that he urged in his paper that to simplify the method the total figures should be taken from the valuation class books and quinquennial groups of ages used; while with regard to his figures not falling in with the valuation date, he would remind them that he was referring to a yearly valuation. He was sorry to find Mr. Ryan suggesting that a yearly valuation was not necessary. To keep up the class-books continuously was not more laborious than to re-write them every five years, and by a slight modification of Mr. Woolhouse's method of approximate valuation (*J.I.A.*, xxvii, 433), a valuation and claim estimate could then be very easily made. Dividing Mr. Woolhouse's groups so as to make A_x proceed by differences of .05 instead of .10, and making a little allowance for the heaping up and falling off of business at the younger and older ages respectively, they could obtain the results with so little labour that he did not see any justification for not making a valuation every year, either as a rough estimate or a check on more detailed calculations. He then gave some specimens of the results of valuations by the detailed and approximate methods, showing very close agreement. As regarded claim estimate, q_x could be made to fit the

same groups of ages, and needed similar adjustment for the beginning and end of life. The number of policies and the sums assured (with the risk premiums) estimated to become claims could then be obtained, as also their reserve value by the same method as for the valuation just described. When the reserve value of the policies expected to fall in had been ascertained, half a year's interest thereon and half the corresponding premiums should be added to bring the figures up to the middle of the year. He was accustomed to have to deal with surrenders and lapses to the extent only of $\frac{1}{8}$ per-cent of the total sum assured, and when that was halved for the half-year at risk, and multiplied by q , the amount was not worth consideration, but an allowance would naturally be made in other circumstances. He made a rough addition for the claim liability of the new business as also for some of the special policies. They would have then, as well as the valuation, the particulars of the expected claims as to number, amount, quality (that was to say, the proportion the strain should be of the sum assured), and the actual strain itself. In conclusion, he thought it was sometimes forgotten that the more they increased their reserves the less opportunity they had of making profit (or loss) by mortality, because the higher the reserve value the less difference or strain there was to become subject to the variation between the expected and actual rates of mortality.

Mr. F. E. COLENSO said that referring to the analogy between formulas (1) and (2), he thought it might be usefully displayed by conceiving that the annuitant, at the time of purchasing his annuity of 1 per annum, effected an assurance of $1 \div (\pi_x + d)$, or $(1 + a_x)$. There would then be no "strain" upon the office funds, *quâ* mortality, in respect of the combined contracts. Under the annuity the fund available at the end of the n th year, the payment of 1 being just due, would be

$$1 + a_{x+n} - q_{x+n-1}(1 + a_{x+n});$$

and under the policy

$$(1 + a_x)[{}_nV_x + q_{x+n-1}(1 - {}_nV_x)],$$

or

$$a_x - a_{x+n} + q_{x+n-1}(1 + a_{x+n}).$$

The terms on the right of these two expressions cancelled each other, and the reserve under the combined contracts was always $(1 + a_x)$ just before the payment of the annuity. He thought that it would be interesting to compare the terms of various offices upon the hypothesis of an assurance being effected in this way, the premium and annuity being calculated by the same table of mortality. Thus, if they took the Government Select Tables for females at age 60, the 4 per-cent rate, loaded by Mr. Sprague's formula, would give an annual premium of £5. 3s. 10d. per-cent, against an ordinary office premium of £6. 3s. 10d. It would be found, he thought, that in some cases annuitants might in this way get a very fair rate of interest at the lower ages, even when ordinary non-profit premiums were charged. It was obvious that if the two contracts were combined it would, in theory, be immaterial to the office what mortality table was employed. Turning to the second section of the paper, he asked whether any real practical benefit would be gained by applying this

method of measuring the strain of mortality at intervals of five years. If they considered the hypothesis underlying the formula it came to this—that on the assumption that it was required to maintain, at the end of the year, the standard of reserve adopted at its beginning, in respect of the three elements, mortality, premium valued, and interest, they merely deducted a fund estimated according to the basis adopted, from the actual accumulations of the reserve, after putting aside all accretions to capital arising from surplus interest, unspent loading, &c., the difference being the gain or loss to be attributed to mortality alone. Thus, if l_{x+n-1} unit policies were assumed to be in force at the beginning of the n th year of assurance, the funds available to provide claims and reserves at the end of the year would be

$$(1) \quad . \quad . \quad . \quad l_{x+n-1}(n-1)V_x + \pi_x)(1+i) + f,$$

where f was due to surplus interest, &c. The first term of this expression might be written in the form

$$(2) \quad . \quad . \quad . \quad . \quad l_{x+n,n}V_x + d_{x+n-1}.$$

Now, if the actual deaths among the l_{x+n-1} assured were at the rate q' , so that $q'.l_{x+n-1} = d'$, the fund actually required, if the standard of reserve were to be maintained, would be

$$(3) \quad . \quad . \quad . \quad [l_{x+n} - (d' - d_{x+n-1})]_n V_n + d'.$$

The difference between (2) and (3), or $(q' - q_{x+n-1})(1 - {}_nV_x)$ per policy, was the sum by which f would be diminished or increased according as q' exceeded or fell short of q_{x+n-1} . Now, of the two practical objects with which such an investigation would be undertaken—(1) where the bonus system of an office required it, in which case it would have to be annual, or (2) where it was desired to measure the sufficiency of the valuation standard adopted—he thought that the latter would require that a number of years should be included in the observations. Mr. Browne had recently given them figures showing how widely the effects of mortality in an office fluctuated from year to year. In the case of annuity business it certainly seemed that offices were in the habit of cutting their cloth more closely than in regard to life business. In some instances it was very doubtful whether the Government Select Tables were the basis of the rates, and there was little, if any, direct provision made for expenses, the margin in the rate of interest being relied upon to supply this.

Mr. T. E. YOUNG said that the expediency of an extended annuity business was one of the most serious practical problems they had to consider. Many companies had abandoned the business altogether on account of its unprofitableness, and it seemed unlikely, as one or two reports of companies which he had lately seen appeared to imply, that a profit might be expected in the future. He found that whereas ten years ago the annuity payments made in one year were £439,000, last year in the Board of Trade Returns they stood at £797,000, excluding American offices, notwithstanding that a great number of companies had discontinued that class of business. Mr. Ryan had stated that he was not a pessimist upon the value of annuity business on the whole, but he (Mr. Young) confessed

himself distinctly a pessimist in this matter. He himself quoted a suitable rate when enquiry was made, though practically he had discontinued the class of business. He did so simply for the selfish reason that he should not send a customer to another more accommodating office, lest he should transfer his other orders also. He was grateful to Mr. Ryan for introducing a simplified formula, so that there was no excuse for neglecting a strict investigation into this class of risks. With regard to the death strain in relation to ordinary assurances, he said that he would adopt Mr. Ryan's plan in future, instead of the rougher plan he had hitherto used. He felt, however, that for companies singly, investigations of this character were more of the nature of actuarial luxuries, and possessed little value or influence upon practice. In conclusion, although it was rather belated at the present time to make the remark, he observed that, for mathematicians accustomed to precision and purity of language, the vocabulary of assurance had been signally unhappy in its terms. It would be almost impossible to decipher from some of the terms employed, such as "strain", "loading", and so on, what meaning they were intended to convey; and in relation to the particular term appropriate to the subject of this paper, it would be impossible to recognise the meaning which the word connoted with historic precision in the science from which it was derived. It was possible, however, that they might yet have some opportunity of reforming their business tongue.

Mr. R. P. HARDY said that there was no more salutary mental process than that of retrospection—looking back to see how far their anticipations had been confirmed by actual outcome, and then endeavouring for the future to eliminate either the error or the personal equation, whichever it might be. Confining his remarks to that portion of the paper dealing with the annuity fund, he thought that no more practical method could be found than that adduced by Mr. Ryan. But if he might address himself to a class who were still students, whatever their age might be, who were not content with seeing things as they appeared to be, he would apply a somewhat closer analysis; and it would then be seen that the strain was really the resultant of at least two forces. The function $1 + a_x$ was really made up of two quantities: one, the interest upon the fund from time to time in hand, and the other what, for want of better language, he must call the sums "released." Taking the fund each year as $l_x(1 + a_x)$, $l_{x+1}(1 + a_{x+1})$, &c., the present value of the interest thereon (in advance) was

$$d[l_x(1 + a_x) + v l_{x+1}(1 + a_{x+1}) + \dots] \quad (1)$$

and the present value of the sum "released" each year was

$$v[l_x(1 + a_x) - l_{x+1}(1 + a_{x+1})] + v^2[l_{x+1}(1 + a_{x+1}) - l_{x+2}(1 + a_{x+2})], \\ \text{\&c.} \quad (2)$$

Then $(1) + (2) = l_x(1 + a_x)$, and dividing by l_x

$$d \frac{S_{x-1}}{D_x} + \frac{R_x}{D_x} = 1 + a_x \quad (3)$$

Formula (3) meant that the value of the annuity in advance was equal to an increasing life annuity of d , plus an increasing assurance of 1, *i.e.*, that the capital of the payments stood over till death, but that the interest upon the amount so forborne was paid from year to year. That was perfectly true in all its generalities. The strain might be looked upon really as the balance of the year; if more persons claimed the annuity of d , there was a less sum realised from the assurance benefits. He had made some trials and arrived at a conclusion which seemed paradoxical. The practical conclusion was that the lower the valuation rate of interest the more important was the mortality. It would be a very instructive exercise to reproduce Mr. Ryan's results by operating upon the component parts of the function. He had some doubt as to whether the use of the continuous function was sufficiently accurate towards the older ages where the numbers at risk could not be held constant.

Mr. G. F. HARDY thought that by introducing continuous formulas Mr. Ryan had simplified the application of the method originally proposed by Mr. Higham, though some difficulty with respect to special assurances, surcharged cases, &c., still remained. In estimating the profit or loss upon the mortality experience, it was needful to calculate not only the expected, but the actual death-strain, as it was only upon a comparison of the two that any conclusions could be based. The latter question was not dealt with in the paper, it being presumably taken for granted that the actual strain would be obtained by recording the reserve values held against policies becoming claims. This suggested the question whether the profit or loss on the mortality experience could not be ascertained by a shorter and more direct process. Excluding for the moment the question of surrenders and lapses—which in some offices were very small, and in all offices might be allowed for without much difficulty—they had the following relation. To the reserve at the commencement of the year, increased by a year's interest, add the difference between the net premiums and the claims of the year, increased by half a year's interest, and deduct the reserve at the end of the year; the result gave the exact profit or loss from mortality during the year. The method was easily extended to a quinquennium, by accumulating the total reserves at the commencement of the quinquennium for the five years, and to add the accumulated difference between the net premiums and the claims, deducting from the resulting total the reserve at the close of the period. Put into symbols this became

$$V_0(1+i)^5 + \left\{ \begin{array}{l} (P_1 - d_1)(1+i)^4 \\ + (P_2 - d_2)(1+i)^3 \\ + (P_3 - d_3)(1+i)^2 \\ + (P_4 - d_4)(1+i) \\ + (P_5 - d_5) \end{array} \right\} \times \left(1 + \frac{i}{2}\right) - V_5,$$

where V_0, V_5 were the initial and final reserves, and P, d , &c., the total net premiums and claims of each year. They then had a numerical result which was the profit from mortality during the period, including profits from lapses and surrenders. If the profit from withdrawals was to be eliminated, they must record the reserves upon lapsed and surrendered policies, as well as the reserves on claims. This method

avoided the lengthy calculation necessary for obtaining the expected death strain, and it applied to all classes of assurances and annuities and of bonuses. It would be less laborious than the "death-strain" method, even in the simplified form given by Mr. Ryan. The question as to the conditions under which Mr. Ryan's application of the continuous method would be sufficiently exact seemed to depend on the character of the business of the office employing it. It assumed that the amount at risk at the various ages varied uniformly during the quinquennium; that was to say, they might take the average amount at risk at each age during the five years as a mean between the amounts at the commencement and at the end. With respect to the question of annuity business, it might be worth remarking that of the two elements involved in their calculations—the rate of interest and the rate of mortality—a change in either of these operated in the same direction as regards annuity-values. They were generally of opinion that if any change was taking place it was that both the rates of mortality and of interest were diminishing. Hence, sellers of annuities were hit from both sides at once. In the case of assurances, these changes operated in different directions, and they need not, therefore, be under apprehension of loss from a diminished rate of interest if the rate of mortality fell simultaneously.

MR. A. HENDRIKS said there seemed to be a belief that actuaries of companies transacting annuity business did not exercise that watchfulness which was the characteristic of actuaries managing life assurance business. In his opinion the contrary was the case. He believed that companies having a large annuity business were constantly watching the condition of things from the point of view mentioned by Mr. G. F. Hardy, namely, that the two great changes which were taking place with respect to the rates of mortality and interest, were both acting adversely as regards profit from the granting of annuities. He was under great obligation to Mr. Ryan, because his formula enabled all companies to easily measure the death-strain. A chance of profit on annuities still remained in the difference between the rate of interest earned and the rate assumed in the valuation and in the formation of the tables. He had never heard of a table of annuity rates being constructed that did not make allowances for commission and further official expenditure. One of the safeguards of annuity business was that the expenditure was very small indeed as compared with that in regard to life assurance. No doubt it was true, that not only was the rate of mortality improving, but a larger number of people were reaching extreme longevity. In criticism of the Government Annuity Tables he would say that for a table of mortality for annuities they did not want to go back to the beginning or the middle of the century, and they should limit the experience to the last 20 years or thereabouts; and if they could get a table, by the combination of the experience of offices transacting annuity business, they would arrive nearer to the truth, and, if found necessary, the rates allowed to the public could then be reduced.

MR. T. G. ACKLAND said he agreed with Mr. Hendriks as to the lines to be followed in the preparation of annuity tables, having regard to the rate of interest, the margin for expenses, and also to

the extraordinary vitality of annuitants as a body. The formula for the death-strain, produced by Mr. Higham in 1877, did not express the conditions of practice, and, indeed, Mr. Higham in his paper departed from that formula to a considerable extent with regard to its application, and had now suggested further ingenious modifications. The suggestion made by Mr. Ryan, as to annuities being valued (for purposes of the death-strain) by the annuity-due at age $x+1$ reminded him of the Continental practice in regard to the valuation of policies. The value of the future premiums was frequently worked out by the annuity-due, and in Italy, France, and other countries what was called the "mathematical reserve" was thus arrived at, the "definite reserve" being subsequently ascertained by suitable adjustments. Reverting to Mr. Ryan's suggestion as to annuities, it was not altogether clear in what way they were to pass from the valuation of an annuity by the annuity-due at age $x+1$, to the valuation as ordinarily effected by a continuous annuity at age x ; and upon the whole he thought it would be simpler to work out the values of the annuities and of the death-strain by independent computations. He was surprised to find that doubts were sometimes expressed as to annuities being apportionable, as he had understood that by the Apportionment Act there could be no doubt of the legal obligation to pay the annuity up to the day of death. The results of annuity valuations, as brought out by Mr. Ryan in his model office, although they indicated a material difference in the amount of the death-strain under different bases, should be considered in relation to the amount of the annuities at risk. Mr. Ryan spoke of £69,000 and £53,000 and other sums as considerable; but having regard to the amount of annuity at risk—a quarter of a million per annum—those amounts could hardly be considered as relatively large. The case adduced by Mr. Ryan of a company dealing with ordinary annuity business, and making little or no profit thereon, but having also on its books annuities on two joint lives in connection with contingent reversions, in respect of which a material profit might be made, had come practically under his notice. He thought that the use of continuous values materially assisted the computation of the death-strain. His experience in the case of a large progressive company making its valuations triennially was that it was quite sufficient to take the average of the reserves at the beginning and end of the triennium, and the death-strain obtained by taking such an average reserve value was extremely close to that which would be obtained by a more exact formula. In the case of a company doing a large new business, where the average duration of policy was not considerable—perhaps five or six years—it might be altogether incorrect to employ throughout the average value of q_x , in any investigation of the expected and actual rate of mortality. It would appear to be preferable in such a case to make allowance for the actual duration of the policies upon the books, and use, as Mr. Ryan has suggested, a select q_x in respect of policies of short duration, and an average q_x for those exceeding (say) five years' duration. A more stringent test of mortality was thus applied. He believed it was the practice in some companies to investigate the death-strain, with a view to the accurate ascertainment of sources of

profit during the triennium or quinquennium, and that, by methods which were not fully known to himself, the profit could be set out in its component parts, as arising from mortality, interest, and miscellaneous sources. He should be very glad if in a future session they could have laid before them details as to the methods upon which those investigations proceeded, and as to the results thus arrived at.

Mr. F. W. FRANKLAND said the problem attacked in the second half of Mr. Ryan's paper had confronted him some years ago under rather special circumstances. He had taken the management of a Colonial life assurance office, which a few years earlier had established a separate section for total abstainers. The section had been established on the principle of having entirely separate funds from those of the general section, the expenses being duly apportioned. The result at the first valuation, owing to the pressure of new business expenses in that particular section, was such as to prevent the declaration of any bonus. With a view of rectifying this it was proposed to amalgamate the two sections in all respects, except in regard to their mortality experience, and to determine the mortality profit in each section on the lines of Mr. Higham's paper. The full application of Mr. Higham's method would have entailed a large amount of work, and it was desired to obtain a shorter and more expeditious process of determining the expected death strain in the two sections. That was done by disregarding for the purpose in hand the ages of the lives dealt with, and considering only the durations of the policies. In the case of so young an office the duration of the policy was a far more important element in the investigation than the age of the life. An average value of q_x was therefore formed for the various durations. The result of the application of this method was not yet known. There was a feature of Colonial life assurance which stood in special relation to the question of the particular q to be applied in getting the measure of the death-strain, namely, the high rate of accidental death which obtained in Australasia and other colonies. The high accident rate tended proportionately to equalise the q for the successive years of duration of the policy, so that the import of the initial selection was diminished.

Mr. F. BELL explained a short method of investigating the loss by death-strain, which he believed to be new, and which differed from that brought forward by Mr. Ryan. Starting with Mr. Higham's well-known formula for death-strain—

$$q_{x+n}(1 - {}_{n+1}V_x),$$

and substituting for ${}_{n+1}V_x$ its value

$$1 - \frac{1 + a_{x+n+1}}{1 + a_x},$$

the expression became

$$q_{x+n} \left(\frac{1 + a_{x+n+1}}{1 + a_x} \right).$$

The sum assured being S , the expected death-strain was

$$\frac{S}{1 + a_x} \times q_{x+n}(1 + a_{x+n+1}).$$

For reversionary bonuses (and paid-up policies) Mr. Higham's expression became

$$q_{x+n}(1 - A_{x+n+1}),$$

and substituting for A_{x+n+1} its value

$$1 - d(1 + a_{x+n+1}),$$

this was reduced to

$$q_{x+n} \times d(1 + a_{x+n+1}),$$

and the reversionary addition being B , the expected death-strain, in respect of bonus additions to the sum assured, was

$$dB \times q_{x+n}(1 + a_{x+n+1}).$$

For reductions of premium the death-strain (a negative quantity), R being the amount of annual reduction, was

$$-R \times q_{x+n}(1 + a_{x+n+1}).$$

To apply the process it was necessary to introduce into the class-books a new column of $S(1 + a_x)^{-1}$, a quantity which was constant throughout the duration of the policy, and was independent of the age at which the investigation was made. From the class-books they could obtain for each age, ΣB , $\Sigma S(1 + a_x)^{-1}$, and ΣR , and from these they could proceed to the schedule as follows:

Age	Total Sums Assured and Reversionary Bonuses	Expected Amount of Claims	Measure of Sums Assured and Bonuses after allowing for Reductions of Premium	Expected Death-strain
$x + n$	$\Sigma S + \Sigma B$	Col. (2) $\times q_{x-n}$	$\Sigma B \times d + \Sigma S(1 + a_x)^{-1} - \Sigma R$	Col. (4) $\times q_{x-n}(1 + a_{x-n-1})$
(1)	(2)	(3)	(4)	(5)
Totals for all Ages				

Expected amount of claims = Σ Col. (3).

Expected death-strain = Σ Col. (5).

The method applied equally to endowment assurances, the whole-life annuities being replaced by suitable term annuities.

Mr. A. F. BURRIDGE said that it was the particular function of the actuary to analyze the various branches of business, and to ascertain which were profitable and which less profitable. With regard to the market rates of annuities, they had an active and successful rival in the Government. From time to time, when the Government rates had been reduced, there was a similar movement in the rates of individual offices, but the Government fixed the

minimum, and it was difficult for offices, with any chance of success, to reduce their rates below those fixed by Government, although there were at the present moment one or two offices which granted less liberal terms than Government at the chief ages at which annuity business was done. The reason was known why the longevity of annuitants was so great. The element of self-selection was undoubtedly the strongest element which could be brought to bear, and the late Mr. Finlaison, in his report on this subject, said, "It may be doubted whether any step in the way of selection will ever surpass the intuitive perception which exists in the minds of self-nominated annuitants at the time of purchase." No great amount of work would be involved if an annual investigation were made upon the lines suggested in the paper. He could not quite agree with the statement that in contingent reversionary securities "there would presumably be little or no selection against the company." He thought it was an ascertained idea, that life tenants, as a class, gave very reasonable proof of their powers of longevity. The other class of lives, the reversioners, were subject to medical examination, because there was life assurance connected with it. He had to criticise Mr. Ryan's statement, that from one investigation alone he was able to get the returns for two years, namely, the one preceding the valuation and the one succeeding. In the latter case the new bonuses required to be added, so that a separate investigation must be made for them. The plan he advocated was to make a test valuation at quinquennial ages by the assistance of the class-book at the end of the first year of a new period. Then by applying the appropriate q_x at each group of ages, he obtained the required results for the past year, and, by applying q one year older to the same figures, the figures for the coming year. The process could be repeated at the end of the third year of the quinquennium, so as to obtain the figures for the third and fourth years. Thus, by means of a test valuation, repeated twice in a quinquennium, in addition to the ordinary valuation, he obtained the figures relating to each year of the period. The necessary corrections must, of course, be applied in respect of new business and of discontinued policies, but it would be found in practice that the whole operation did not involve much trouble.

MR. GEORGE KING said, in regard to annuity business, that he had looked at the accounts of various companies, and found in many cases there had been a loss, and he was afraid that that was not altogether of the past, but it was also a thing of the future. It had been pointed out that the fall in the rates of mortality and of interest, while they had opposite effects on insurance transactions, both tended to make annuity transactions unprofitable; and, therefore, to have an easy and accurate method of testing the course of annuity business was most valuable. He was afraid that if some of the actuaries put on loading for expenses and commission they must use very queer annuity tables for the purpose of getting out their net annuity rates. He had found that one or two British companies charged rates at certain ages equivalent to Government rates at something like 5 per-cent interest. He did not see how they could make profits with such rates. With regard to the second part of the paper he did not share Mr. Ryan's

views that investigations of that kind were extremely valuable. He did not think the mortality experience of a single company was of very much use: either they did not get enough facts, or those facts were extended over too long a period. What was wanted in these days was to get mortality experience in sections. To be useful they ought to have it each 20 years, and the experience of a single company was not sufficient. Mr. Ryan dismissed in a line and a half the subject of special policies, and Mr. Higham was equally curt with regard to them. But an investigation of the whole-life business only was of very little use, while, if they brought in the special cases, a rough estimate was not at all trustworthy. In the case of endowment assurances and limited-payment policies, the death-strain was very different from that of whole-life assurances. Then, again, the disturbance of bonus was very acute in some companies. With regard to assurance business it was of very little use to have investigations of that kind. Business once on their books was beyond their control. They might find it useful to watch closely the mortality under new policies, because there, by modifying their examinations and forms and elucidating further facts, they might be able to control the business; but old business on the books was entirely beyond them, and they must trust to averages.

Mr. T. J. SEARLE wished to explain a new method of finding the death-strain. Taking the left-hand side of Mr. Ryan's equation No. 1, which involved only the policy-value at the beginning of the year, one could deduce by a very simple multiplication the expected death-strain, and by deducting this from the same side, obtain the policy-value at the end of the year. The valuation could thus be continued from one year to the next, and so on. The formula No. 1 had reference to a policy of 1. It could be made more general as follows:

$$(V + P)(1 + i) = {}_{+1}V + q(S - {}_{+1}V).$$

S represented the sum assured and bonuses, and P the premiums actually valued on the page of the class lists relating to any particular date of birth, V being the total policy-values for the same valuation age. The formula held good for endowment assurances, limited payments, and any sort of assurances whatever on single lives. If $(V + P)(1 + i)$ be represented for shortness by V' , then it could be shown that $(S - V')k$ was the expected death-strain, where k was a factor the value of which could be easily found. Thus, assuming

$$(S - V')k = (S - {}_{+1}V)q$$

and

$$V' - (S - V')k = {}_{+1}V,$$

and deducting both sides of the last equation from S , they obtained

$$(S - V') + (S - V')k = S - {}_{+1}V,$$

whence $(S - V')(1 + k) = S - {}_{+1}V = (S - V') \frac{k}{q}$;

$$\therefore 1 + k = \frac{k}{q}$$

whence

$$k = \frac{q}{1 - q} = \frac{q}{p}.$$

There was thus a very simple means of proceeding from the valuation at the beginning of the year to the expected strain, and thence to the valuation at the end of the year, and so on. The office premiums, less the premiums brought into the calculation, would give the exact loading, and that, less the expenses, would give profit from loading. The whole interest earned, less the interest required for the accumulation, would give the profit from interest, and the expected strain would give the profit from mortality. With regard to lapses and surrenders, they had to take away the values when they occurred, and for new policies they had to put on the values. Through the courtesy of Mr. Chisholm, he had been employed in applying the system to a portion of the business in his office.

THE PRESIDENT, in proposing the usual vote of thanks to the author, said that it had been somewhat of a regret to him that he had not been able to put in motion during his presidency the preparation of a new table of mortality, and the observation of Mr. Hendriks, that insurance offices had a vast amount of valuable information with regard to annuities, should be borne in mind when a new investigation was undertaken. The remark made by Mr. King that they wanted their information in time sections with regard to the mortality of insured lives, was one which he hoped would be equally remembered. He (the President) had lately been engaged in making a mortality investigation in his own office on those lines, and the differences and contrasts which were brought out by taking different periods were instructive, and, in some cases, remarkable.

MR. RYAN having briefly acknowledged the vote of thanks, the meeting adjourned.

On an Application of the Graphic Method to obtain a Graduated Mortality Table from a limited Experience, by means of Comparison with a Standard Table. By GEORGE J. LIDSTONE, F.I.A., of the Alliance Assurance Company.

IN a paper by the late Mr. Samuel Brown "On the Rate of Mortality and Marriage among Europeans in India" (*J.I.A.*, xi, 1), reference is made to a method used by Griffith Davies to deduce rates of mortality from the records of the Madras Military Fund. It appears that Davies ascertained the average age of the lives in each of certain groups into which the experience naturally divided itself, and obtained the ratios which the actual deaths in the several groups bore to the corresponding number expected according to the Northampton Table at the average age. These ratios were then set out as ordinates corresponding to abscissae representing years of age, and a continuous curve was drawn in the usual way, by reading off which the graduated ratios for each age were obtained, and from these the graduated rates of mortality were at once formed.

Mr. Brown remarks with reference to this process that it must be admitted that it "was a very rough way of deducing a table from original observations", and this is doubtless the case when the age-grouping is wide and irregular. When, however, the exposed to risk and deaths have been obtained in the usual way for each age, and are then combined in small groups of about five years, there does not seem to be any good reason why the method should not produce satisfactory results; at any rate where the observations are of small extent, when from the nature of the case we can only expect to produce a table which will represent the general progression of the rates of mortality. I am not aware of any case in which the process as above described has been adopted, but a very similar plan was used with satisfactory results by Messrs. G. F. Hardy and H. J. Rothery in deducing their Barbados Mortality Table (*J.L.A.*, xxvii, 170), in which case the ratios were found to be so regular as to be adjustable by inspection. By slightly modifying Griffith Davies' process it may, I think, be made very useful in dealing with small experiences. The following is a short account of an application of the modified process referred to.

Considering first the mortality table to be used in calculating the expected deaths, it is evident that smoothness of graduation is most essential, more so, in fact, than close agreement with the observed rates of mortality, since any irregularities in the standard table would be reproduced and possibly exaggerated in our final results. For this reason a table following a mathematical law—as, for example, Makeham's—will generally be the most suitable to employ. The H^M and Carlisle Tables have both been graduated by Makeham's law, and it will usually be found that one of these may be employed with good results. With regard to the function to be operated upon, I have found it convenient to substitute for

$q_x:q_x$, which was used by Davies, the quantity $\log \frac{p_x}{p'_x} = \log p_x - \log p'_x = r_x$ say, where the unaccented symbols denote values according to the standard table, and the accented symbols the values deduced from the observations. In the first place this will be found to simplify the process of graduation by enabling us to deal with quantities which are not only much smaller than those used by Davies, but which will usually progress more smoothly.

This was the original reason for selecting the function $\log \frac{p_x}{p'_x}$, but it will be found that there are other advantages arising from

the use of this quantity, as in certain cases we are enabled to form an idea of the *nature* of the difference between the rates of mortality exhibited by the observations and by the standard table. It is well known that a constant increase or decrease in the force of mortality at all ages is equivalent to diminishing or increasing the values of μ_x in a constant ratio. In such a case the value of

$\log \frac{\mu_x}{\mu'_x}$ will evidently be constant, and will, therefore, be represented on our diagram by a straight line parallel to the abscissae. This relation is, of course, perfectly general, and applies universally to all mortality tables; but the growing importance which appears to be attached to comparisons based on changes in the values of the constants A and B in Makeham's formula

$$\mu_x = A + Bc^x \quad . \quad . \quad . \quad . \quad . \quad (1)$$

renders it a matter of some interest to consider the special form which the curve representing $\log \frac{\mu_x}{\mu'_x}$ will assume in consequence of such changes. In the following remarks it will, as usual in comparisons of this nature, be assumed that the quantity c remains invariable, whatever the values of A and B. The legitimacy of this assumption is abundantly shown by the results of various applications of Makeham's formula to the graduation of mortality tables (see *J.I.A.*, xxx, 7, from which it appears that c , there denoted by q , never differs greatly from $\log_{10}^{-1} 0.4$). From equation (1) we have

$$-\int \mu_x dx = \log_e l_x = -Ax - \frac{B}{\log_e c} c^x + \log_e \kappa \quad . \quad (2)$$

where $\log_e \kappa$ is the constant of integration, corresponding to the radix of the mortality table, κ ; whence

$$\Delta \log_e l_x = \log_e \mu_x = -A - \frac{B(c-1)}{\log_e c} c^x \quad . \quad . \quad . \quad (3)$$

If we now change A and B to A' and B' respectively, we obtain

$$\log_e \mu'_x = -A' - \frac{B'(c-1)}{\log_e c} c^x \quad . \quad . \quad . \quad (4)$$

and

$$\log_e \frac{\mu_x}{\mu'_x} = A' - A + \frac{(B' - B)(c-1)}{\log_e c} c^x \quad . \quad . \quad . \quad (5)$$

If in equation (5) we make $B = B'$, we find, as before, that a constant addition to μ_x produces a similar addition to μ'_x . In

Let u_n and p_n refer to one table
 μ'_n and p'_n refer to other table

Suppose $u_n - \mu'_n = k$ a constant

$$\text{But } -u_n = \frac{1}{l_n} \frac{d}{dx} l_n = \frac{d}{dx} \log l_n$$

$$\therefore -\int u_n dx = \log l_n$$

$$\therefore \int (\mu'_n - u_n) dx = \log \frac{l'_n}{l_n}$$

$$\text{ie } kx = \log \frac{l'_n}{l_n} \quad \text{if } l_0 = l'_0$$

$$\therefore \frac{l'_n}{l_n} = e^{-kx}$$

$$\text{so } \frac{l'_{n+1}}{l_{n+1}} = e^{-k(n+1)}$$

$$\therefore \frac{p_n}{p'_n} = e^{-k} = \text{a broken fraction since } k \text{ is pos.}$$

\therefore if $u_n > \mu'_n$ by the constant k
 $p_n < p'_n$ in the ratio e^{-k}

to what extent the curve may be allowed to fall without producing the result referred to. We have

$$\log p'_x = \log p_x - r_x \quad \dots \quad (7)$$

$$\text{whence } \text{colog } p'_x = \text{colog } p_x + r_x \quad \dots \quad (8)$$

$$\text{and } \Delta \text{colog } p'_x = \Delta \text{colog } p_x + \Delta r_x \quad \dots \quad (9)$$

In order that q'_x may uniformly increase with the age, we require that the right-hand member of this equation should be positive, and this will be secured if

$$\Delta \text{colog } p_x > (-\Delta r_x) \quad . \quad . \quad . \quad . \quad (10)$$

If, therefore, we set out graphically the values of $\Delta \text{colog } p_x$, we can see by inspection when the curve representing r_x is falling so rapidly as to produce decreasing values of q'_x .

No exposition of a method of adjustment can be considered complete which does not give the results of a practical application, which must always be the best and most severe test. I have therefore used the plan above described to graduate the experience of the Marine and General Life Office in respect of mariners' lives (see *J.L.A.*, xxvi, 413, and Table I), in the hope that the results may be of some interest apart from their value as an illustration. The exposed to risk and actual deaths, together with the expected deaths according to the H^M Table, and the process of deducing the values of r_x , are given in the following table. The ordinary H^M mortality table has in this case been used, instead of the table given in Part II of the *Institute of Actuaries' Text-Book*, and graduated according to Makeham's hypothesis, as we are by this means enabled to make use of the numbers of expected deaths which have already been calculated.

TABLE A.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Age x	Exposed to Risk E_x	Actual Deaths d'_x	Expected Deaths d_x	$E_x - d_x$ $= E_x p_x$	$E_x - d'_x$ $= E_x p'_x$	$\text{Log } (5)$	$\text{Log } (6)$	(7)-(8) $= \text{Log } \frac{p_x}{p'_x}$ $= r_x$
20-24	3,375	57	23	3,352	3,318	·5253	·5208	·0045
25-29	8,074	158	56	8,018	7,916	·9041	·8985	·0056
30-34	7,574	164	61	7,513	7,410	·8758	·8698	·0060
35-39	5,969	158	56	5,913	5,811	·7718	·7643	·0075
40-44	4,394	137	47	4,347	4,257	·6382	·6291	·0091
45-49	2,938	86	40	2,898	2,852	·4621	·4551	·0070
50-54	1,733	72	30	1,703	1,661	·2312	·2204	·0108
55-59	820	35	19	801	785	·9036	·8949	·0087
60-64	365	18	12·3	352·7	347	·5474	·5403	·0071
65-69	} * 196·5	13	11·2	185·3	183·5	·2679	·2636	·0043
70-74								
75-81								

* Assumed average age 70.

The values of r_x , given in column 9, have been graduated by the method above described, and the values of p'_x and q'_x have been deduced from the graduated values of r_x . The original data being arranged in quinquennial groups, we are not able to follow the usual plan of so selecting the groups as to obtain a useful preliminary graduation. Any irregularities due to faulty drawing or reading of the curve may be eliminated by inspection of the differences, as mentioned by Mr. Sprague in his paper on the Graphic Method, and it will be found convenient to apply this process to the final values, p'_x or q'_x , rather than to r_x . The following table gives the results of the graduation:

TABLE B.

*Marine and General Mutual Life Assurance Society's
Experience in respect of Mariners' Lives.*

Values of $p_x^{\text{M. \& G.}}$, $q_x^{\text{M. \& G.}}$, and $r_x = \log_{10} \frac{p_x^{\text{HM}}}{p_x^{\text{M. \& G.}}}$.

Age x	$p_x^{\text{M. \& G.}}$	$q_x^{\text{M. \& G.}}$	r_x	Age x	$p_x^{\text{M. \& G.}}$	$q_x^{\text{M. \& G.}}$	r_x
20	.9845	.0155	.0040	45	.9681	.0319	.0088
21	.9836	.0164	.0043	46	.9673	.0327	.0087
22	.9829	.0171	.0045	47	.9665	.0335	.0088
23	.9824	.0176	.0048	48	.9656	.0344	.0089
24	.9821	.0179	.0050	49	.9649	.0351	.0088
25	.9814	.0186	.0053	50	.9641	.0359	.0089
26	.9809	.0191	.0055	51	.9632	.0368	.0090
27	.9804	.0196	.0056	52	.9623	.0377	.0090
28	.9799	.0201	.0057	53	.9612	.0388	.0090
29	.9794	.0206	.0058	54	.9601	.0399	.0090
30	.9789	.0211	.0059	55	.9590	.0410	.0090
31	.9783	.0217	.0060	56	.9581	.0419	.0087
32	.9777	.0223	.0063	57	.9572	.0428	.0085
33	.9770	.0230	.0065	58	.9561	.0439	.0082
34	.9763	.0237	.0067	59	.9548	.0452	.0080
35	.9756	.0244	.0069	60	.9534	.0466	.0077
36	.9749	.0251	.0071	61	.9517	.0483	.0074
37	.9741	.0259	.0073	62	.9498	.0502	.0071
38	.9734	.0266	.0074	63	.9477	.0523	.0068
39	.9727	.0273	.0076	64	.9454	.0546	.0065
40	.9721	.0279	.0078	65	.9430	.0570	.0062
41	.9713	.0287	.0080	66	.9406	.0594	.0058
42	.9705	.0295	.0083	67	.9382	.0618	.0055
43	.9697	.0303	.0085	68	.9356	.0644	.0051
44	.9689	.0311	.0087	69	.9326	.0674	.0047

It will be seen that the rates of mortality progress with considerable regularity, but it remains to be shown that they will, when applied to the actual number exposed to risk, reproduce the observed number of deaths. In the present case this can only be done in a somewhat rough manner, owing to the exposed to risk not being given separately for each age. We have, using the notation of Table A,

$$d'_x = E_x \times (1 - p'_x) = E_x - E_x \times p_x \div \frac{p_x}{p'_x} = E_x - E_x \times p_x \div r_x \quad (11)$$

and by this formula the expected deaths shown in the following table have been obtained :

TABLE C.
Comparison of Actual and Expected Deaths.

Age	Actual Deaths	Expected Deaths	Expected - Actual		Accumulated Error
			+	-	
20-24	57	57
25-29	158	158
30-34	164	169	5	...	+5
35-39	158	155	...	3	+2
40-44	137	129	...	8	-6
45-49	86	98	12	...	+6
50-54	72	65	...	7	-1
55-59	35	35	-1
60-64	18	18	-1
65-	13	13	-1
All Ages	898	897	17	18	-1

It will be seen from this that there is a very close agreement between the total number of deaths expected and experienced, and that the errors in the separate groups are of small extent and varying sign. It may, therefore, be claimed that satisfactory results have been obtained by the use of the process above explained, and this notwithstanding that we have selected as our standard table one which differs very widely from that formed from the observations. In conclusion, it may be pointed out that, with suitable modifications, the method is equally applicable to the graduation of rates of sickness or secession, which it is sometimes necessary to deduce from data obtained from the records of a friendly society. The valuation of such societies may be materially simplified if we can give effect to any departures

from the rate of mortality shown by the table used in the valuation by means of a change in the rate of interest, or by rating-up the ages of the members, especially if the observed rates of sickness can be conveniently expressed in terms of the tabular rates at the *substituted ages*. The results deduced from equation (5) may, therefore, find a useful application in connection with such valuations.

Life Assurance Business in Australasia.

[Abstracted from the Inaugural Address to the Insurance Institute of New South Wales, delivered by the President, Mr. David Carment, F.I.A., F.F.A., 13 April 1892.]

I SHALL, in the following remarks, mainly confine myself to some considerations connected with the rise and the development of life assurance business in Australasia—a development which, considering the limited population of these colonies, is probably quite as phenomenal as that which has attended its progress in the United States of America with their teeming millions. To such an extent, in fact, has this been the case, that in no part of the world, so far as I am aware, has the amount assured per head of the population reached so high a figure as in Australasia. The figures are approximately as follows: Australasia, £19; Canada, £9; United Kingdom, £12; United States, £10. It is thus apparent that in this important respect the Australian colonies have largely surpassed not only the mother country, but also the most enterprising of the other English-speaking communities of the world. To my mind, this one fact is of itself a convincing proof of the wide-spread material prosperity of these colonies, and the importance of this consideration is enhanced when we consider that the average sum assured per policy is *smaller* here than in England or in America, thus proving that the beneficent practice of life assurance has here been availed of more largely by the great bulk of the people than in the other countries named. In Australia, in fact, many labourers and others of a similar class are assured for amounts of £100 or £200, while in England those of a similar grade in the social scale could not afford to assure at all, except, possibly, in one of the industrial companies for a very small amount. If, now, we enquire the causes which have led to such phenomenal results, I think they will be found partly in the skill and energy and enterprise with which in general our life offices have been conducted, and partly

in the practice so universally adopted here of procuring business through the agency of travelling canvassers with their associated medical referees. This practice, of course, originated through the extreme sparseness of the population in most of our country districts, to which the benefits of life assurance could hardly have been carried in any other way than this. No doubt the method may be, like most other things, susceptible of abuse, but it is certainly also capable of being honestly and safely worked. Of course, in a thickly-peopled country like England such methods would be unnecessary, and, therefore, Australian offices might well be excused if their expenses are, on the whole, somewhat higher than those of the home offices. A somewhat undesirable concomitant of a high rate of new business obtained at considerable pressure is an undue rate of lapsing, and this is undoubtedly one respect in which our Australian offices have a difficult task before them. New business obtained at too high a cost results, of course, in a direct loss to the company if its policies lapse soon after issue, as so many of them are often apt to do. The evil effect of this may, no doubt, be partly remedied by paying the first year's commission to the agent only concurrently with the receipt of the several instalments of the first year's premium in cases where it is payable otherwise than annually; but even in spite of this useful precaution it may happen that loss may arise in cases where the premium is small, and the object to be aimed at should be to so fix the rates of commission paid for the procurement of new business that in the great majority of cases no loss shall occur after allowing for the risk of death within the first year.

In the case of endowment assurance policies, which are now becoming so increasingly popular, the risk of any loss is much lessened and the probability of making some profit increased; for while the premium on such policies is larger, there need to be no more cost incurred in obtaining them if, as I believe to be usual with Australian companies, the commission be calculated as a percentage on the *amount assured*. The practice of paying a renewal commission during the whole continuance of a policy, though almost universal in England, and to some extent followed in America, is not so largely adopted in Australia, it being generally considered that if a policy is once obtained it may be left to the assured himself to keep it up, or that if he neglects to do so, and allows it to lapse or surrenders it, matters can in most cases be so arranged as at least to prevent the occurrence of any

loss to the office. And this leads me to remark the much greater liberality of the Australian offices in the matter of surrender-values than their British compeers. It is not, however, well to be *too* liberal in this respect, and I fear that in these colonies the pressure of competition has induced the companies to give values which are, perhaps, in some cases excessive, as well as to allow surrenders as soon as two years' premiums have been paid. In such cases, as well as in those of early lapses, there may, in fact, be an actual *loss* to the office in respect of the smaller policies after payment of the surrender-value.

In America it is rather the exception for life assurance companies to pay cash surrender-values at all, and when they do they generally make much heavier deductions from the full reserve value by way of "surrender charge" than anything to which we are accustomed in Australia. In fact, many of the American offices act upon the principle that it is highly undesirable, in the interest of the assured himself and his heirs, to permit the original contract to be terminated except in return for a paid-up assurance of such an amount as the equitable surrender-value of the policy would purchase. No doubt there is a considerable amount of truth in this contention, but we in Australia have in this respect followed, properly enough, in my opinion, the almost universal practice of British life offices, which have always been in the habit of allowing a moderate surrender-value after the payment of a certain number of years' premiums.

A considerable diversity of practice exists both here and in England as to the preferable method of assessing the surrender-values of policies on rated-up lives, some companies invariably treating these lives for all purposes (calculation of surrender-values included) as if they were exactly equivalent to select lives of the rated-up ages, while others treat them for surrender-value as being of the *true* age. The latter method will, for ordinary whole-life policies, always bring out a smaller value than the former, but in the case of endowment assurances the result will sometimes be in the contrary direction. While admitting that the practice of treating the lives assured by such policies as always equivalent to select lives of so many years older is hardly scientifically correct, I am inclined to maintain that it may quite safely be carried out by a company which makes its periodical valuations on similar principles.

The discussion of the question of surrender-values naturally leads up to that of the application of the surrender-value of a

policy to keep it in force without any application whatever from the assured in the event of omission to pay the premium within the days of grace, whether that omission be accidental or intentional. This important reform in the practice of life assurance undoubtedly originated in Australia, where it has now been in force for over 20 years, with the most beneficent results to many thousands of policyholders. In this respect the life offices of Australia, I believe, may fairly claim to have shown the way to the more conservative kindred institutions of England, many of which are now following in the same direction. It is, however, a significant fact that in a paper on this subject, read before the Institute of Actuaries on 31 March 1884 by Mr. T. B. Sprague, its then President, no mention whatever is made of the fact that the principle, the adoption of which was apparently at that time only beginning to be advocated in England, had already for a number of years been in successful operation in most, if not all, of the Australian companies. In the discussion which followed the reading of the paper, the Australian practice on this point was referred to with approval by Mr. J. R. Macfadyen, as well as by the late Mr. Cornelius Walford, but most of the speakers on the occasion seemed to be unaware of the precise manner in which the principle is carried out in actual practice, and considerable differences of opinion existed as to the best and most equitable method of applying it. We in Australia can, I believe, claim to have long ago completely solved the problem, and that in a less complicated manner than that suggested for adoption by Mr. Sprague.

I may illustrate by one or two examples the extent to which the non-forfeiture regulation may prove sufficient to keep policies in force, even for many years after cessation of the payment of premiums, thus preserving to a large extent the benefit of the assurance to the policyholder's representatives in the event of his dying within the period during which it is so sustained. Take first the case of an ordinary whole-life participating policy for £1,000, effected at age 30, with a premium of £24. 1s. 8d., payable annually, and assume further that bonuses are added thereto at the rate of 2 per-cent per annum on the amount assured. If such a policy becomes overdue after payment of *ten* annual premiums, it can be shown that it would be sustained by the operation of the non-forfeiture regulation for fully *eleven* years more, allowing for the accumulation of the overdue premiums as a debt against the policy with compound interest at the rate of

8 per-cent per annum. In the case of endowment-assurance policies the result of the operation of the principle in question is still more surprising. Take, for instance, the case of a policy for £1,000, effected at age 30, payable at age 50 or previous death, the annual premium being £45. 3s. 4d., the bonuses being assumed to be at the same rate as in the case of the whole-life policy. If such a policy were to become overdue after *five* years' premiums had been paid it could never lapse at any subsequent time, for at the date of maturity the amount of the accumulated debt against it would be less than the amount of the sum assured with bonus additions. This somewhat extraordinary result is largely due to the fact that after the first few years of its duration the value of the policy is increased after payment of each premium by an amount at least equal to the premium paid. And while on this subject of endowment assurances, I should like to refer to some remarks made by Mr. Newbatt in his opening address as President of the Institute of Actuaries on 24 November 1890. Towards the close of that address the learned President took occasion to criticize somewhat severely the present tendency of the age in England to invest more largely than before in endowment-assurance policies, mostly for comparatively short terms and maturing comparatively early in life—a class of assurances or investments which has for a number of years been popular in America, and the spread of which in England Mr. Newbatt pretty clearly indicates as being largely fostered by the use of “estimates”, which he as clearly intimates are, in his opinion, extravagant. But, though it may, no doubt, seem presumptuous in me to differ to any extent from such a justly-esteemed authority, it appears to me that in illustrating his argument Mr. Newbatt has taken a somewhat extreme case, namely, that of a man effecting an endowment-assurance policy of £2,000 for a comparatively short term, on which the annual premium amounts to £120. Now, in Australia the *average* annual premium on a policy of this class would be, roughly speaking, about £10, and the term of the assurance probably from 20 to 25 years, in place of 15 or 20, as in the bulk of such cases in America. Mr. Newbatt argues that the policyholder who is able to pay £120 a year in premiums could himself invest his money to much greater advantage than the assurance company could do, especially when account is taken of the fact that a considerable percentage has to be deducted from the gross premium payable for expenses of management. Now, whatever possible amount of

truth there may be in this statement as regards England and America, I am prepared to maintain that in Australia the average endowment-assurance policyholder may undoubtedly obtain *greater* benefits by placing his money with a sound and economically-managed life office than if he attempted to invest the same amount himself. For, to say nothing of the advantage of being assured during the term for a constantly increasing amount, instances are by no means uncommon of such policies at maturity yielding an amount equal to all the premiums paid with compound interest at rates varying from $4\frac{1}{4}$ to $4\frac{1}{2}$ per-cent. Now, suppose the policyholder had endeavoured to invest his £10 a year himself, in what way could he, with due regard to safety, have earned a higher rate of interest? Irrespective of speculative investments, and of deposits in banks and building societies of the class which have of late failed in such numbers, the only opening available to him would appear to be deposits with a savings bank, which would earn a rate varying, according to circumstances, between 4 and 5 per-cent, and it must be remembered that such institutions do not usually allow any interest on accounts exceeding £200. Moreover, there is by no means the same inducement to unfailing regularity in the making of such deposits with a bank as there is in regard to paying the periodical premiums on a life policy. With regard to the supposed English policyholder paying £120 a year, such a man would find it even *more* difficult to find absolutely safe investments, for the amount would be much too large to go on depositing in savings banks, while fixed deposits in other banks, or government securities, would probably earn less than 3 per-cent; and most other forms of investment would be either unsuitable for such small annual sums, or would involve some risk, and require constant and careful watching. When, in addition to the above considerations, we recollect the fact so well expressed by the late Professor De Morgan in his *Essay on Probabilities*, that "there is nothing in the commercial world "which approaches, even remotely, to the security of a well-"established and prudently-managed insurance office", I think you will agree with me that most men will probably do better with their money, even from an investor's point of view, by paying it in the form of premiums to a well-managed life office than by endeavouring themselves to attend to its investment.

Before leaving the subject of endowment assurances, I might point out the extraordinary prevalence of this class of policies in

Australia as compared with England, and the wonderful manner in which their numbers have increased within the last few years, this result being largely aided by the operation of the enactments as to the compulsory assurance of railway officials, under policies maturing at age 60 or previous death, in the colonies of New South Wales, Victoria, and South Australia, as well as of all other civil servants in Victoria. Taking the latest available statistics, as found in the valuation reports of the various offices, I find that endowment-assurance policies form about 45 per-cent of the total number of assurance policies in force, though, owing to their being of smaller average amounts than the whole-life policies, the sums assured under this class form only about 33½ per-cent of the total assurances, as against 22·8 per-cent in America and 7·8 per-cent in England. Further analyzing the Australian figures, I have arrived at the following approximate results:

	Average Amount of Policy	Average Rate of Prem. per-cent	Average Premium per Policy	Average Loading per-cent on Gross Premiums
Whole-Life Policies .	£366	£2·981	£10·315	25·2 per-cent
Endowment-Assurance Policies	225	3·935	8·542	19·6 "
All Assurance Policies .	302	3·330	9·975	22·9 "

The rate of premium, £3. 18s. 8d. per-cent, corresponds to an average term of about 23 or 24 years—a much longer term than such policies usually run for in America, where 10, 15, and 20 years are, I believe, the terms most in vogue. Endowment assurance, therefore, as practised in Australia, partakes considerably less of the *investment* element, and more of the pure *assurance* element than the same class of business as practised by our transpacific brethren.

An idea is prevalent in many quarters that an office may, without any danger to its stability, spend the whole of its first year's premiums in obtaining new business, or, at any rate, that the cost of procuring new business, *plus* the amount required to cover the risk of death in the first year, may safely swallow up the first year's premium; and Mr. Sprague has devised formulæ for valuing the liabilities of such an office based on the assumption just mentioned, which formulæ have been made use of to a considerable extent by Australian offices. Now, though such a rate of expenditure in the case of whole-life policies can hardly be regarded as unsafe provided the renewal business be conducted on a scale of strict economy, yet I consider that new business

obtained at such a price is paid for at a rate in excess of what it is really worth to the office; and I desire further to point out that to extend the operation of this maxim to the case of offices, a large proportion of whose business consists of endowment assurances, would be highly improper; and still more so in the case of companies which do a large amount of business in pure endowments, a class of policies wherein practically the whole of the premiums paid is required to be kept invested and accumulated at compound interest in order to produce the amount of the policy at maturity.

I am aware that this question of expenses is a somewhat delicate one to meddle with, especially in Australia, where almost all our offices are still comparatively young, and almost all are of the "progressive" order. I find the average expenses per-cent of 11 Australasian offices, as given in the *Australasian Insurance and Banking Record* for January last, to be 21·28 per-cent, as against 14·35 for the whole of the English offices as deduced from the summary appended to the last volume of Returns to the Board of Trade under "The Life Assurance Companies Act, 1870", and 20·33 per-cent for the companies doing business in the United States. Although the rate of expense in Australia seems high by comparison with England, it must not be forgotten that the fact of the necessary expenses of living and travelling in Australia being so much higher than at home will necessarily tend to keep up expenses here, and that nearly all the Australian companies are at present doing a very large proportionate amount of new business, which necessitates not only commission to agents at higher rates than generally prevail in England, but also the employment to a large extent of travelling medical referees, whose fees and travelling expenses have both to be provided for. It must, nevertheless, be carefully kept in mind that new business may very easily be bought at too high a cost. It would no doubt be difficult, if not impossible, to draw any hard and fast line which should be suitable to all offices at all times and under all circumstances, but it may safely be laid down that the introduction of new members into a mutual assurance society should never involve an expense so great as to diminish the rate of profit which would otherwise accrue to the existing policyholders. An interesting discussion took place, some little time ago, at the Actuarial Society of America, upon a paper read by Mr. B. J. Miller on "The Value of New Blood in Life Insurance", the author maintaining that "New Blood",

viewed simply "as such, is not worth as much to a life insurance company in dollars and cents as it is often thought to be worth."

The bonuses hitherto declared by Australian offices have resulted in a very large degree from their high interest-earnings. No doubt the general and inevitable tendency of interest rates in the course of a long series of years is in a downward direction, especially in newly-settled countries, but it is highly satisfactory to find that so far in Australia this tendency does not seem to have made itself much felt in the history of Australian life offices. In England there has been during the last 20 years a decided, though not very great or rapid, fall in the rate realized on investments. According to Mr. David Deuchar's pamphlet on "The Progress of Life Assurance Business in the United Kingdom during the last Fifty Years", the average rate of interest earned by companies on their life assurance fund, as deduced from the Returns to the Board of Trade under the Act of 1870, was, in the year 1865, £4. 10s. per-cent, and in the year 1885, £4. 4s. per-cent. In Mr. A. G. Mackenzie's paper on the "Practice and Powers of Assurance Companies in regard to the Investment of their Life Assurance Funds" (*J.I.A.*, xxix, 191) there is given a table arrived at in a slightly different manner from Mr. Deuchar's, but showing very similar results. According to this table the annual average rate of interest realized, exclusive of profit on investments, has fallen from $4\frac{1}{2}$ to $4\frac{1}{8}$ per-cent in the 18 years between 1872 and 1890. This tendency to a fall in the rates realizable at home has within the last few years induced several of the leading Scotch offices to seek an outlet for some of their surplus accumulations in the form of mortgages in some of the Australasian colonies, where some millions sterling are now invested in such securities at rates which have materially assisted in helping to sustain the average interest return of the companies in question.

In America the fall in the rates of interest during the past 20 years has been more decided and more rapid than in the United Kingdom. According to some statistics compiled a year or two ago by Mr. W. C. Wright, the average rate realized by 20 leading life insurance companies in the United States of America has fallen by about $1\frac{1}{2}$ per-cent during the period in question, being for the year 1889 as low as £5. 2s. per-cent. In the year 1890 it is given as £4. 19s. 5d. per-cent for the whole of the companies existing in the United States, according to *Life Insurance Ratios*, published in 1891 by the Spectator Company

of New York. In Australia it has so far remained pretty constant, and is, in fact, at the present time fully as high as it was 20 years ago, averaging about £5. 16s. per cent. This result is no doubt largely due to the fact that a more remunerative class of investments has been sought for during the interval, a smaller proportion of funds being now invested in Government debentures and fixed deposits with banks, and a larger proportion in loans on mortgage and loans on policies. But even allowing for these considerations, I think it will be admitted that there has not as yet been such a decided tendency downwards as has been exhibited in the case of the American offices. To predict the course of events in the future is, no doubt, a difficult task, but I see no reason at present to fear any great reduction in the near future. In fact, if the rate of interest at which Governments can borrow affords any indication, it would rather appear as if the rate had at present an upward tendency. In this connection it may be interesting to note the different proportions in which life assurance funds are invested in various classes of securities in England, America, and Australia respectively. As it would be difficult to collate all the facts in a single table, owing to the differing methods in which they are generally summarized in the official returns, I exhibit them separately for each of the countries named. As regards English offices, the proportionate amounts, according to the summary appended to the last published volume of Returns to the Board of Trade, are as follow:

	Per-cent
Mortgages on Property within the United Kingdom	39·28
Mortgages on Property out of the United Kingdom	3·44
Loans on Rates and Rent Charges	11·89
	<hr/> 54·61

	Per-cent
Loans on Policies	4·82
British Government Securities	3·02
Indian and Colonial Government Securities	6·85
Foreign Government Securities	1·96
Railway and other Debentures	9·20
Shares and Stocks	6·82
Land and House Property and Ground Rents	6·47
Life Interests and Reversions	1·79
Companies' own Shares	·25
Loans on Personal Security	·66
Cash, Deposits, Stamps, &c.	3·55

The percentages just quoted are given in Mr. Mackenzie's paper above referred to (*J.I.A.*, xxix, 193).

In America, according to the latest available report of the insurance superintendent of New York, the funds of the companies therein reported on were invested in the proportions shown below:

	Per-cent
Real Estate	10.47
Bonds and Mortgages	40.16
United States Stocks and Securities	8.4
Other Stocks and Bonds	34.45
Collateral Loans	4.62
Premium Notes and Loans	2.58
Cash in Office and in Bank	4.20
Deferred and Unpaid Premiums	1.77
All other Assets91

In Australia, as derived from the summary of "Assets of Australasian Life Assurance Companies reporting in 1891", given in the *Insurance and Banking Record* for January 1892, the percentages are as under:

	Per-cent
Mortgages	55.91
Loans on Policies and Personal Security	17.01
Government Securities and Debentures	7.90
Shares	0.73
Cash on Deposit, in hand, and on Current Account	4.88
Freehold and Leasehold Property	10.49
Agents' Balances	0.10
Outstanding Premiums	1.57
Outstanding Interest	0.87
Sundries	0.54

I will only call attention to the very large proportion of the assets of the Australian companies lent on security of their own policies, as well as the large amount held in the form of real estate, the percentage here and in America being nearly alike and considerably greater than in the case of the English offices. Debentures, other than *Government* debentures, shares, stocks and bonds, are a form of investment hardly as yet known in Australia, and life interests and reversions are also conspicuous by their almost total absence from the balance sheets of our local companies.

Closely connected with the question of the average rate of interest realized is that of the proper rate to be assumed by companies in making their periodical valuations, and I believe I am right in saying that 4 per-cent is the rate almost universally adopted by the Australian offices, thus leaving a margin of at least $1\frac{3}{4}$ per-cent between the rate realized in the past and that calculated upon for the future. In America 4 per-cent is also the standard rate prescribed by law in most of the States, but as I have already shown that the average rate now being realized

does not much exceed 5 per-cent, it follows that at the present time the margin of profit from interest there is much less than it is here. In England the rates at which valuations are made vary in general between 3 and 4 per-cent, but of late years one or two of the stronger companies have adopted $2\frac{1}{2}$ per-cent as a means of further strengthening their position in view of the continued fall in the rate realizable, and, as a matter of fact, all, or nearly all, of the better companies have by degrees been adopting a 3 per-cent basis, thus leaving a margin of something over 1 per-cent to provide for future profits from this source.

The third and last main source of profit in a life assurance company is that arising from a low rate of mortality, and here, too, our Australian offices have so far been singularly fortunate as compared with those located in any other part of the globe. It is an interesting question, but one to which, in the present state of our knowledge of the subject, it would be difficult, if not impossible, to give any satisfactory reply, to what extent this state of things will continue in the future. It must not be forgotten that contracts of life assurance, unlike those in the fire and marine branches of the business, when once entered into may endure for 50, 60, or 70 years, and that, therefore, in endeavouring to penetrate the vista of futurity, we must look ahead for more than a generation. In old-settled countries like England this, in the matter of mortality if not of interest, can be done with a fair approximation to accuracy, for there we have life offices some of which have been in active operation for considerably over a century, and which have tabulated their experience in this respect for the benefit of themselves as well as of others. In England, too, the general rate of mortality among the population has, on the whole, remained wonderfully constant during the half-century over which the Registrar-General's mortality returns extend, though of late years there were distinct indications of an improvement in the death rate—an improvement which has, however, been again checked by the ravages of influenza during the past three years. The history of the past in Australia in the matter of death rates, whether we look at the statistics of the general population or of assured lives, has been phenomenally favourable, but there seems no certainty that the future will in this respect be a reflex of the past. It becomes all the more necessary, therefore, for our assurance companies to proceed carefully in the way of dividing the apparent profit derived by them from light mortality, especially when we consider that the great majority of

the lives have only comparatively recently passed the ordeal of a medical examination. It does not, however, appear to me to be necessary to hoard up the whole of the mortality profit now being made in order to divide it among the next generation of policyholders. The existing members have a fair claim to so much of it as can safely and reasonably be divided. The mortality experience of the Australian Provident Society for its first 30 years was published in 1881 by Mr. Black, its then actuary, and showed that the number of deaths had only been 67·7 per-cent of the number which might have been expected according to the H^M Table. Last year the experience of the same office for its first 40 years, as investigated by Mr. Teece, its present general manager and actuary, was given to the world, and the results are found to be equally favourable with those shown in the former experience.

It is curious to note that, while in the experience of 20 English offices above referred to, only about 7 per-cent of the lives observed were classed as other than "healthy", in that of the Australian Mutual Provident Society no less than 34 per-cent of the whole had been rated up on account of personal or family defects. In America, again, it is quite the exception, as I understand, for lives to be accepted otherwise than at select rates. If a life is decidedly bad it is declined, but if it has some inherent defect it is often, nevertheless, accepted for an endowment assurance, to run for 10 or 15 years, although a whole-life policy would be refused. It has always, however, appeared to me that a life of such a character can hardly be accepted on these terms with perfect safety to the company—certainly not with perfect equity to the other policyholders who are really select lives, unless, perhaps, occasionally under exceptional circumstances. For it can hardly be asserted that a man with some decided personal defect such as would justify us in declining him or rating him up pretty heavily for a whole-life assurance has *as good* a chance of surviving, say, the next 15 years, as another man who has no such defect.

One or two English offices have recently started schemes of "assurance without medical examination", but these are hedged round with so many conditions and safeguards as to render the benefits offered little, if any, more tempting than the ordinary plans of the general run of companies, and I hardly think that very many proponents will after all be found to avail themselves of their promised advantages.

I would now like to say a few words on the question of the methods of division of profits adopted by Australian companies. In England a very great diversity of practice in this respect prevails, and the methods adopted by the majority of offices can hardly be said to aim at allotting the surplus to policies of different kinds in proportion to the degree in which each of them has contributed towards that surplus. Many of the plans of division are almost entirely arbitrary, and few of them make any attempt at discriminating accurately between whole-life policies and endowment assurances as to the respective rates of bonus to which each of these classes is relatively entitled. The method adopted in 1873 by the leading Australian society, and followed since then with slight modification by all the others of any importance, seems to me to be as nearly as possible perfect. It first allots to each policy, in proportion to its value at the next preceding investigation, the surplus derived from the reserves having been accumulated during the valuation period at a rate of interest in excess of that calculated upon in the investigation, and then the whole remaining profit is allotted to each policy in proportion to the *loading on the premiums paid* during the period, some deduction being made in practice from the profit which would thus accrue to policies of very recent date as a set-off against the expense incurred in obtaining them. The cash bonus to which each policy is entitled being thus found, it is then converted into the equivalent reversion payable along with the sum assured, and the result declared to the individual members in that form. The method just described is a modification of that which was, I believe, described for the first time in an editorial note appended to a reprint of a paper by the late Professor Pell, of Sydney, on the subject of the "Distribution of Profits in Mutual Assurance Societies" (*J.I.A.*, xiv, 396). In this note the editor recommends as the most equitable method one which is identical with that above described, except that after allotting the interest profit the remainder is to be divided in proportion to the *premiums paid* on each policy in place of in proportion to the *loading thereon*. Of course, in the case of an office all whose premiums are loaded with a uniform percentage the two methods would be identical, but this is very seldom the case, the rate of loading at different ages under the same table sometimes ranging from 25 to 50 per-cent of the net premium, and the average rate of loading being sometimes very different under the various tables. The superior equity of the method I

have first described is therefore. I think, beyond dispute. The question as to the proper method of allotting bonuses to endowment assurances as against whole-life policies has recently evoked considerable discussion among English actuaries, and was the subject of a paper read before the Institute on 23 December 1889 by Mr. A. W. Sunderland. The plan adopted by the Australian companies, as just described, seems to me to completely solve the difficulty, although it must be admitted that it is more laborious than the rough-and-ready method of giving every kind of policy an equal rate of reversionary bonus, regardless of the amount of loading actually involved in the premium payable, of the age at which the policy was effected, or of the amount of reserve held against it by the company. In America the plan of division of profits most generally adopted is that known as the "contribution method", which was the invention of Mr. Sheppard Homans, at one time actuary of the Mutual Life Insurance Company of New York. This method, like that generally in use in Australia, aims at returning to each policyholder his contribution towards the surplus earned in the investigation period. The necessary formula is extremely complicated as applied to quinquennial divisions, and even for annual divisions is still of an intricate character. The principal difference between the two methods seems to be that in estimating the year's bonus to each policy the rate of mortality *actually experienced* during the year at each age among the lives on the company's books enters into the calculation in the case of the American offices—a rule the strict application of which would seem likely to cause inconvenient fluctuations between the bonuses declared to policies existing on lives of different ages in the same calendar year, as well as between those declared in successive years to the same policy.

Another matter to which I should like to refer is the question of the simplification of policy-conditions. Great advances have of late years been made in this direction by many of the English companies; but in America, progressive as the people are in many respects, in this they are lamentably behind. The policies issued by most companies there are encumbered, both in the body of the document and on the back, with a multitude of conditions, which are nearly as numerous and as perplexing to construe as those with which our friends in the fire branch of the profession are so familiar. In Australia, on the other hand, I think we can claim to have almost attained the *ne plus ultra* of simplicity in this respect, for, with the exception of the usual

suicide clause, the policies now issued by our leading companies contain no restrictive conditions of any sort as to residence, occupation, or anything else.

The want of an Act whatever in this colony regulating assurance companies is a standing disgrace to our Legislature, and it seems almost incredible that in this alone of all the Australasian colonies no such legislation has ever existed. Under the superintendence of a committee of this Institute a bill for the purpose was drafted a good many years ago, but has never yet been introduced to Parliament. I have recently heard, however, that the present Government have in preparation a measure dealing with financial institutions of various kinds, including insurance companies of all sorts. I am afraid, however, that an Act embracing within its scope so many diverse matters may stand a considerably less chance of becoming law than one dealing with life companies alone, and I think that this Institute should again press on the Government the desirability of losing no time in removing this reproach from the mother colony of the group. The question as to the principles on which such legislation should proceed is one which has of late been much debated both in England and America, and only a few months ago a highly interesting discussion took place at the Institute of Actuaries on a paper upon this subject, read by Mr. George King. In this paper the writer maintained that the general principles which have heretofore regulated the law relating to life assurance companies in the United Kingdom, as well as in the Australian colonies, namely, *freedom* and *publicity*, were preferable to the system of *State intervention* which prevails in the United States and in Canada. In this contention I am disposed to agree with him, believing, as I do, that too much State interference is apt to be hurtful to any business enterprise, and, moreover, that the American plan has not been successful in the past in averting widespread insolvencies and grave mismanagement. The various Australian Acts are much more exacting in many particulars than that enacted in England in 1870, especially in the direction of requiring detailed statements as to the new and void business of each company, the latest Act, that of Western Australia, requiring a statement of the business in each of the seven separate colonies. One of the amendments of the existing law, suggested by Mr. King, is that in this respect the colonial Acts should be followed, and detailed statements required as to the new and void business every year.

Before concluding, I would like briefly to refer to the inauguration of the system of holding examinations in connection with the Institute of Actuaries in various colonial centres, including, among others, Sydney and Melbourne, and I am glad to know that a fair number of applicants intend to present themselves for the first examination at the end of the current month. Hitherto there has been a want of sufficient stimulus to induce the young men employed in our Australian life offices to make themselves masters of the theoretical foundation, as well as the practical working of the business in which they are engaged; but with the incentive which is now supplied by the possibility of their being able to achieve an honourable place among the candidates from all parts of the world, we may cherish the hope that our Australian youth will not be left behind in the race for distinction.

Provision for Old Age.

The following interesting remarks upon some of the methods that have been recently suggested for providing assistance in old age are extracted from the Report of the Chief Registrar of Friendly Societies (Mr. E. W. Brabrook), for the year ending 31 December 1891, part A, page 23:

It is provided by "The Friendly Societies Act, 1875," in continuation of an enactment which has, in one form or other, been in force since 1819, that a society granting a certain annuity, or certain superannuation, to its members can only do so under such guarantee as is afforded by the certificate of an actuary possessing a stated qualification that the Tables of Contribution for such insurance are sufficient for the purpose. Few societies care to comply with this requirement, and accordingly very few societies are registered for the purpose of granting pensions in old age. Relief of an uncertain character in old age, and relief during sickness whenever it may occur, even after old age has disabled the member from working, can be insured without any such actuarial certificate; and accordingly many societies are still registered for the purpose of granting such relief, though long experience has shown that it cannot safely be undertaken, except upon the condition of payment of premiums much higher than such societies usually charge.

Actuarial certificates, even, are sometimes given improvidently; and in one particular case of a society in Glasgow, where it appeared to the actuary of this office and to myself that the English actuary employed had been induced to sanction tables at too high a rate of interest, I was enabled to take note of it by the circumstance that

the gentleman in question was also a public valuer. In general, however, the actuarial certificate is final.

That provision for old age by way of pension ought to be more generally adopted than it is, is a statement that no one will dispute. The difficulties in the way are more financial and practical than otherwise. Let it be once ascertained precisely what provision ought to be made, and precisely what it will cost, and the good sense and foresight of the great body of working men may be trusted to devise means for raising the necessary funds by their own exertions.

By an almost general consent, the provision that ought to be made has been assumed to be a certain weekly allowance commencing at the age of 65. It may be convenient for friendly societies to fix an age beyond which they will not pay sickness allowance, and thus avoid such difficulties as arose in the case of dispute before referred to; but inability to work through old age may commence long before 65, and may be deferred till much later. "The Friendly Societies Act, 1875," defines "old age" very happily as "any age after 50." There is no physiological change which necessarily makes a man unable to work the day after his 65th birthday, who was able to work the day before. Surely, therefore, the provision required to be made is for an allowance to commence when inability to work through old age commences, and not before or after. If a man becomes unable to work through old age at 50, he should be entitled to cease his payments and claim the annuity which his past payments have purchased, which would, of course, be less than if he continued to work and to pay until 65. If a man is able to continue working until 70 or 75, or even longer, he should be entitled to continue accumulating the fund so as to be able to claim, when at last he gives up working, a much larger annuity than the one fixed to commence at age 65. It would have to be borne in mind that when old age comes early, it is not improbable that death will come early too; and that, therefore, annuity tables calculated upon the experience of selected lives should not be used in estimating the equivalent pensions; but that is a problem which actuarial science should be capable of solving. The rapid increase of the pension, after passing the standard age, would probably induce many men to delay claiming it. A man who is capable of earning £1 a week would not give it up for the sake of a pension of 5s.; but if a few years delay enabled him to gain a pension of 7s. or 10s. or more, he might be trusted to seek his own time for retirement. By a plan of this kind, no man would be left without provision. If infirmities gathered round him at a comparatively early time of life, he would still have some provision, which need not be very much less than that allowed to a man whose chances of life have not been impaired; if his life and health were prolonged he would be assured of a provision that would grow every year more and more ample for the comforts that ought to "accompany old age."

An arrangement of this kind necessarily implies nonforfeiture; and here arises the crucial difficulty of the question of insuring pensions in old age. If the money is not to be forfeited in the event of death before entering upon the pension, then the amount of pension insured must be small, and the amount of contribution exacted must

be relatively large. On the other hand, no man will willingly enter into a contract by which, after he has pinched himself to save year after year, for a long succession of years, his own death, or his inability to continue the contributions, may incur the forfeiture of every penny of his savings. Nor will he willingly lock them up beyond possible recall in the case of some present and pressing emergency. This consideration may be carried even further; the unpopularity of pensions assurance is due largely to the idea that when, in the long distant future, the workman breaks down and claims his pension, he may only live a short time to receive it, and the painful savings of years be sunk for the sake of a few weeks or months' allowance. This latter difficulty cannot be removed by any scheme of pensions, and can only be met by allowing some option to the subscriber when his time for claiming comes.

Upon the whole, I am convinced that the line of least resistance will be found in a small pension, claimable at the option of the owner, increasing the more it is deferred, without any forfeiture in the event of previous death. I would, therefore, reverse the usual form in which the question is put, and get the working man to ask not, what amount of pension in old age ought he to subscribe for? but, what amount of present money ought he to put aside to provide a pension for old age? Let it be supposed that working men generally, say, for example, the members at large of one of the great affiliated orders, were prepared to answer this question by saying we will each set aside £1 a year for that purpose, the machinery would really be very simple. A printed card might be prepared for each year of age, showing the fractional weekly sum, commencing at every age after 50, that the £1 paid in that year would insure. The £1 might be paid in any convenient instalments, and as soon as completed the card handed to the member. If he paid more than £1 in any year, an extra card might be given him for each extra £1 paid. A man who wanted to know what he could claim at any given age would have to add together the figures opposite that age on all his cards. The necessary calculation would not be beyond the competence of any board school scholar; and the liability of the society would be merely the amounts received and the interest upon them.

So far, I have discussed the question upon the assumption that the friendly societies are ready and willing to undertake its solution by their own efforts; but it has lately taken a wider range, and much consideration has been given to the position the State might occupy in the matter. That branch of the subject lies outside my province in the present report, as it depends upon the effect of the Poor Laws, with the administration of which I have nothing to do, but a passing allusion to it may be permitted. Three theories of State pensions have been put forward:

1st. That the State ought to compel every citizen to buy a pension. (The Rev. Canon Blackley.)

2nd. That the State ought to assist every citizen who wishes to buy a pension. (The Right Hon. Joseph Chamberlain, M.P.)

3rd. That the State ought to grant a free pension to every citizen. (Mr. Charles Booth.)

As to the first, it is undoubtedly within the competence of the Legislature, if it thinks fit, to convert every moral duty into a legal obligation, and the providing for old age is a moral duty; but there are others paramount to it, as, for example, the providing for a man's widow and children in the event of his early death; and the Legislature should surely begin with the more urgent obligation. As to the second, its success would depend upon the extent to which its liberality acted as an inducement to persons to buy pensions who do not now provide in any way for old age; if the proportion of such persons remained anything like the same as at present, the result would necessarily be an increase (if that be possible) rather than a diminution of the severity of the existing Poor Law. As to the third, though there is much to be said for its simplicity and thoroughness, it might in practice result merely in a general substitution of out-door relief for in-door relief in the case of old age; for, although the pension is to be granted in theory to every citizen who asks for it, it will not be expected that in fact anyone will ask for it who does not want it; and the ingenuity of the keeper of the public purse may be trusted to devise such conditions of making the application as will ensure the restriction of the grant to those who do need it. This remark applies even though it should be proposed to deduct the pension from the amount to be paid in taxation by persons whose liability to taxation exceeds it; for all those whose liability to taxation was less than the amount of pension would have to apply for the money, to prove age and identity, and so forth.

Whether the best remedy for the evil would not, after all, be a more kindly, generous and considerate treatment of the destitute aged, is a question the discussion of which is beyond the scope of this report.

Others, besides those named, have made suggestions on the question which are worthy of consideration. Mr. Louis Tylor urges the substitution of a plan of acquiring a capital sum as property for that of assuring an annuity as pension. The Rev. F. W. Fowle proposes a scheme for subsidizing the friendly societies, but he does not (as it seems to me) meet the difficulty of inducing them to consent to the control and interference which must be the conditions of it.

Leading men in connection with friendly societies have expressed opinions strongly adverse to the acceptance of any scheme involving State aid to the societies, upon the intelligible ground that State aid necessarily implies State control and State interference. This has been forcibly put both by Mr. Campkin, the retiring grand master of the Manchester Unity, and by Mr. C. J. Radley, the retiring high chief ranger of the Ancient Order of Foresters. I cannot mention the name of the latter without expressing my sense of the great loss the cause of friendly societies has sustained in the unexpected death of a man whose ability, energy and eloquence had placed him in the front rank among its advocates.

Whether State aid can be rendered to the individual instead of to the society, or whether it can be so presented as to meet the difficulties raised, is a matter for the future to determine. At present, if I may be permitted to say so, I remain of the opinion "that

there is no other way of providing for old age than by thrift, self-denial and forethought in youth." I do not understand how any plan for relieving the working man of that which ought to be a charge upon his wages can be other than a disadvantage to him, by leading him to refrain from claiming and enforcing his right to such wages as will enable him to meet the charge. It is for his friendly society to fix what he ought to pay, and for his trade union to see that he has the means of paying it. It is better for the State, that is, the general body of taxpayers, that he should be paid suitable wages for such service as he renders, than that it should make up for a deficiency of wages by doles of any kind.

If State aid should be rendered to friendly societies, I confess I think one of the least objectionable forms of aid is that of granting a rate of interest somewhat higher than that paid to the public creditor. The plan was adopted in the early years of friendly societies, and did much good; it is at this moment enabling many societies to fulfil contracts entered into more than 50 years ago; but it has been abandoned on account of the cost. If, however, the extra interest had been voted every year, instead of being allowed to grow into a large deficiency of capital, it would not have been an unreasonably heavy burden.

In this connection it may be mentioned that Mr. Thomas Fatkin, the secretary of the Leeds Permanent Building Society, has contributed to the *Leeds Mercury* a series of articles in which he ingeniously argues that, in consequence of the very low rate of mortality assumed in the tables under which Government Deferred Annuities are now granted, the actual rate of interest is sometimes even less than 2 per-cent, and suggests that the borrowing powers of municipalities and other local authorities should be employed for the investment of the funds subscribed in the purchase of deferred annuities, in order at once to secure a better return for the money, and to avoid the embarrassment that must arise from the centralizing of funds so enormous as would be accumulated if any general system of State pensions was successfully launched. His views appear to be well worthy of consideration.

Mr. Young, in a valuable paper read before the Institute of Actuaries, and published in the journal of that body, has given a searching analysis of the German law of pensions.

On Mr. Makeham's Theory of Inverse Probabilities. By
EDWARD L. STABLER, *Actuary of the Manhattan Life*
Insurance Company, New York.

IN the *Journal* for July and October 1891 (*J.I.A.*, xxix, 242 and 444), Mr. William Matthew Makeham published two papers "On the Theory of Inverse Probabilities." In the first he obtained a new formula, and in the second he pointed out some applications of

the formula. I desire to present some considerations which I think will show that this formula is not suitable for any application.

Mr. Makeham supposes a number of urns to be filled from an unlimited mass of white and black balls, from which the chance of drawing a white ball is always p . If the result of $m+n$ drawings from one urn (each ball drawn being replaced) is that m white and n black balls have been drawn, the probability ($p_{m,n}$) that a white ball will be drawn in the next trial is evidently between $\frac{m}{m+n}$ and p . Mr. Makeham's new formula,

$p_{m,n} = \frac{m+rp}{m+n+r}$, is therefore true, as by giving r different positive

values this expression takes every value from $\frac{m}{m+n}$ to p . But in

this form the formula gives no more information as to the probability desired than was already evident from the nature of the case. He then attempts to show that r is "some undetermined constant independent of $m+n$ ", and either independent of p or "not affected by the substitution of q , or $1-p$, for p ."

From a comparison with Laplace's much misapplied formula, $\frac{m+1}{m+n+2}$,* which is the same as his new formula if $p=\frac{1}{2}$ and $r=2$, he decides "for the present, to assume a mean value of p , " for which purpose it is evident that the mean value in all cases " for the function r , whatever may be the value of p , for which " purpose it is evident that the mean value in question must be " taken $=2$."

The reasoning used to show that r is independent of $m+n$, &c., does not seem conclusive. To test the accuracy of the result let us take a simple numerical example. Suppose four balls placed in an urn form a mass in which $p=\frac{1}{3}$; after drawing and replacing successively two balls, both of which prove to be white, required the probability of drawing a white ball in the next trial. The following table shows the correct method of solving the problem:

* It should be remembered that this is deduced on the supposition that all proportions of white balls from 0 to 1, in the urn, are *a priori* equally probable—a very different case from the one under consideration, whatever the value of p .

(1)	(2)	(3)	(4)	(5)
Possible Number of White Balls in Urn	<i>A priori</i> Probability of each Number	In each Case, Probability of Drawing Two White Balls	<i>A posteriori</i> Probability of each Number	
4	$\left(\frac{1}{3}\right)^4$	$\left(\frac{4}{4}\right)^2$	$\frac{4^2}{D} = \frac{2}{27}$	$\frac{2}{27} \times \frac{1}{4}$
3	$4 \times \left(\frac{1}{3}\right)^3 \times \frac{2}{3}$	$\left(\frac{3}{4}\right)^2$	$\frac{4 \times 2 \times 3^2}{D} = \frac{9}{27}$	$\frac{9}{27} \times \frac{3}{4}$
2	$6 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)^2$	$\left(\frac{2}{4}\right)^2$	$\frac{6 \times 2^2 \times 2^2}{D} = \frac{12}{27}$	$\frac{12}{27} \times \frac{2}{4}$
1	$4 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^3$	$\left(\frac{1}{4}\right)^2$	$\frac{4 \times 2^3}{D} = \frac{4}{27}$	$\frac{4}{27} \times \frac{1}{4}$
0	$\left(\frac{2}{3}\right)^4$	$\left(\frac{0}{4}\right)^2$	0	0
Sum,				$\frac{63}{108} = \frac{7}{12}$

Column 4 shows the probability, after drawing the two white balls, of each possible combination of balls in the urn. For brevity, D is used to represent the sum of the numerators of the fractions of which it is the denominator, being here equal to 216. Each fraction in this column is proportional to the product of the corresponding quantities in columns 2 and 3. The probability of drawing a white ball in the next trial is equal to the sum of column 5, *i.e.*, the sum of the products of the probability of each hypothesis multiplied by the probability of drawing a white ball on that hypothesis. To produce this probability ($\frac{7}{12}$) requires the value $3\frac{1}{3}$ for r in Mr. Makeham's formula.

Solving the same problem, except with $p = \frac{2}{3}$, we find the probability $\frac{29}{36}$, which requires r to be equal to $2\frac{1}{3}$.

(1) Possible Number of White Balls in Urn	(2) <i>A priori</i> Probability of each Number	(3) In each Case, Probability of Drawing Two White Balls	(4) <i>A posteriori</i> Probability of each Number	(5)
4	$\left(\frac{2}{3}\right)^4$	$\left(\frac{1}{4}\right)^2$	$\frac{2^4 \times 4^2}{D} = \frac{32}{81}$	$\frac{32}{81} \times \frac{1}{4}$
3	$4 \times \left(\frac{2}{3}\right)^3 \times \frac{1}{3}$	$\left(\frac{3}{4}\right)^2$	$\frac{4 \times 2^3 \times 3^2}{D} = \frac{36}{81}$	$\frac{36}{81} \times \frac{3}{4}$
2	$6 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^2$	$\left(\frac{2}{4}\right)^2$	$\frac{6 \times 2^2 \times 2^2}{D} = \frac{12}{81}$	$\frac{12}{81} \times \frac{2}{4}$
1	$\times \frac{2}{3} \times \left(\frac{1}{3}\right)^3$	$\left(\frac{1}{4}\right)^2$	$\frac{4 \times 2}{D} = \frac{1}{81}$	$\frac{1}{81} \times \frac{1}{4}$
0	$\left(\frac{1}{3}\right)^4$	$\left(\frac{0}{4}\right)^2$	0	0
			Sum,	$\frac{261}{324} = \frac{29}{36}$

It is thus seen that r is "affected by the substitution of q , or $1-p$, for p ." That r is not "independent of $m+n$ " is shown in the following table of results, which are very different from those of Mr. Makeham's Table, p. 251:

FROM AN URN CONTAINING FOUR BALLS, m WHITE BALLS HAVING BEEN SUCCESSIVELY DRAWN AND REPLACED IN m TRIALS.

m	Probability of Drawing a White Ball in the $(m+1)$ th Trial		Value of r in Mr. Makeham's Formula	
	$p = .01$	$p = .99$	$p = .01$	$p = .99$
0	.01	.99	$\frac{0}{0}$	$\frac{0}{0}$
1	.2575	.9925	3	3
2	.2617	.9944	5.8	2.6
3	.2782	.9958	8.1	2.2
4	.3018	.9968	9.6	1.9
5	.3386	.9976	10.1	1.6
6	.3871	.9982	9.8	1.3
7	.4393	.9987	9.1	1.0
8	.4867	.9990	8.6	.9

In the same way it may be shown that in the general case in which the number of balls in the urn is N ,

$$\begin{aligned}
 p_{m,n} = & \left[p^N N^{m+1} . 0^n + N p^{N-1} q (N-1)^{m+1} 1^n + \dots \right. \\
 & \left. + \frac{N}{N-s} p^{N-s} q^s (N-s)^{m+1} s^m + \dots + q^N . 0^{m+1} . N^n \right] \\
 & \div N \left[p^N N^m . 0^n + N p^{N-1} (N-1)^m 1^n + \dots \right. \\
 & \left. + \frac{N}{N-s} p^{N-s} q^s (N-s)^m s^n + \dots + q^N . 0^m N^n \right] \\
 \text{or} \quad p_{m,n} = & \frac{\sum_{s=0}^s p^{N-s} q^s \frac{(N-s)^{m+1} s^n}{N-s}}{\sum_{s=0}^s p^{N-s} q^s \frac{(N-s)^m s^n}{N-s}} .
 \end{aligned}$$

For the case $m=1$ and $n=0$ this reduces to $\frac{1+(N-1)p}{N}$, which makes r equal to $N-1$.

The value of $p_{m,n}$ is completely determined if p , m , n , and N are known. If N is unknown $p_{m,n}$ is unknown, and cannot be ascertained from an "assumed mean value of r ", as r varies between zero and infinity, as p , m , n , and N change.

It may be interesting to note that in Bertrand's *Calcul des Probabilités* (Paris, 1889, p. 152), a somewhat similar problem is presented. Given $p = \frac{1}{2}$, it is required to find the most probable composition of the urn after m white and n black balls have been successively drawn and replaced. On the supposition that N is large and that the divergence of the number of white balls in the urn from $\frac{N}{2}$ is small, it is found that the most probable proportion of white balls in the urn is $\frac{N+2m}{2(N+m+n)}$, or $\frac{m+N \times \frac{1}{2}}{m+n+N}$.

The Cape of Good Hope Life Assurance Act, 1891.

No. 13—1891.]

[Promulgated 21 August 1891.]

ACT to Amend the Law relating to Life Assurance Companies, with a view to encouraging persons to Insure, and to protecting persons Assured.

Preamble.

BE it enacted by the Governor of the Cape of Good Hope, with the advice and consent of the Legislative Council and House of Assembly thereof, as follows:

Short title.

1. This Act may be cited as "The Life Assurance Act, 1891."

Interpretation of terms.

2. In the interpretation of this Act, unless inconsistent with the context, the term "Company" means any person or persons or association, who issue or are liable under policies of assurance, upon human life within this Colony, or who grant annuities upon human life within this Colony, whether the head office or principal place of business of such person or persons or association is in this Colony or elsewhere, and includes associations on the mutual principle.

The term "Foreign Company" means a company established or having its head office or place of business out of this Colony.

The term "Policy" includes a contract for securing a life assurance, endowment or annuity.

The term "The Court" means the Supreme Court of the Cape of Good Hope.

3. Every company transacting life assurance business only, or such business concurrently with any other kind of assurance or other business, shall, at the expiration of each financial year of such company, prepare in the forms prescribed a revenue account for such year and of its balance sheet at the close of such year.

Company to prepare revenue account and balance sheet annually.

4. Every company shall at the expiration of each financial year of such company, in addition to any other statements required by this Act, prepare in the form prescribed in the schedule to this Act a statement of all its policies in force at the close of such year.

Company to prepare annual statement of policies in force at close of year.

5. Every company shall once in five years, or at such interval as may be prescribed by the deed constituting the company or by its articles of association, regulations or bye-laws, cause an investigation to be made into its financial condition by a duly qualified actuary, and shall cause an abstract of the report of such actuary (in the prescribed form) to be deposited as by this Act is provided.

Investigation into financial condition of company to be made every five years.

6. The Governor may from time to time prescribe or vary the forms of statements, abstracts, accounts or returns to be made by any company, and may direct with whom or at which office all statements, abstracts, accounts or returns to be prepared under the provisions of this Act shall be deposited.

Forms of statements, &c., can be prescribed and altered by Governor.

The Governor may also make rules prescribing the duties of such person as may be charged with the custody of such documents as aforesaid, and for the better administration of this Act.

Certain rules to be made by Governor for custody of documents.

7. Every statement, abstract, account or other document by this Act required to be made by any company shall be in such form as the Governor shall prescribe, shall be signed by the principal officer thereof managing the life assurance business in this Colony, and shall be printed; and the original so signed as aforesaid, together with three printed copies, shall be deposited with such officer, or at such office as the Governor may direct, within six months of the dates respectively prescribed as the dates at which the same are to be prepared.

Statements, abstracts, &c., of company: how to be dealt with.

8. Every company which shall not be incorporated by an Act of the Colonial Legislature, or registered in this Colony under "The Joint Stock Companies Limited Liability Act, 1861", shall lodge at such office as the Governor may in terms of this Act appoint, a copy of its charter of incorporation, articles of association or other deed under which it is constituted, and of any amendment thereof, as follows:

Company not incorporated in Colony or registered under Joint Stock Act, 1861, to lodge copy of deed under which constituted.

1. If a company already carrying on business in this Colony, within three months after the passing of this Act.
2. If a company hereafter established before such company shall commence business.

3. In case of an amendment of any such charter, articles or other deed as aforesaid within six months after such amending instrument shall come into operation.

Foreign company to notify in *Gazette* name of principal officer or agent, and address of office in the Colony.

9. Every foreign company transacting life assurance business in this Colony shall within three months after the coming into operation of this Act, or after commencing business, as the case may be, notify in the *Gazette* the name of its principal officer, manager, secretary or agent, and the address of the principal office of the company in this Colony.

The person whose name is so notified can sue and be sued.

10. Every such principal officer, manager, secretary or agent, of whose appointment notice shall be given in the *Gazette* as aforesaid, may sue and be sued on behalf of the company he represents, and service of all notices, summonses or other legal process, shall be effectual against such company if served upon such principal officer, manager, secretary or agent, or left at such principal office with any officer or servant of the company.

Penalty for false statement, abstract, account, &c.

11. If any statement, abstract, account or other document required by or under the provisions of this Act is false in any material particular, the person who signed or deposited the same shall be liable on conviction to a penalty not exceeding one hundred pounds for each offence, or to imprisonment with hard labour for a term not exceeding twelve months.

Penalty for default in furnishing returns.

12. Every company which makes default in the furnishing of its statements or returns, or otherwise complying with any of the provisions of this Act for a period of three months or upwards may, upon the application of the Attorney-General, be interdicted by the Court from transacting business within this Colony either absolutely or for a time as the Court may see fit to order.

Statements, &c., open to inspection on payment of fees.

13. Any person may, upon payment of such fees as the Governor may direct, inspect any statement or other document deposited as required by this Act, and procure copies thereof.

Statements and certified copies of statements, &c., receivable in evidence.

14. Every statement, abstract, account or other document, deposited in terms of this Act, shall be receivable in evidence, and every document purporting to be certified by the officer or person having the custody thereof to be a true copy of such deposited document shall be deemed to be such copy, and receivable in evidence as such, unless some variation between it and the original shall be proved.

Minister to lay statements, &c., before Parliament.

15. It shall be the duty of the Minister to whom the Governor shall assign the management of this Act, to secure the due observance by every company of its provisions, and such Minister shall lay annually before Parliament the statements and abstracts of reports deposited in accordance with the provisions thereof during the preceding year.

16. The property and interest of every person under any policy of assurance upon his own life, and which shall have endured for not less than three years from the date of the payment of the first premium thereon, or in any moneys payable thereunder, shall not be seized or taken in execution under any process of Court, or in the event of the sequestration of the estate of such person as insolvent shall not vest in the trustee, or otherwise, for the benefit of the creditors of his estate, subject, however, to the conditions or limitations following:

Under certain conditions the interest under policy of assurance which shall have endured for not less than three years, not liable to execution.

1. If such assurance shall have endured for three years or upwards, it shall be protected as aforesaid to the extent of three hundred pounds; and in addition thereto one hundred pounds for each additional year or part of a year, not exceeding in any case the sum of two thousand pounds.
2. The sums of money aforesaid shall mean sums assured by one policy or several policies, irrespective of any bonus or other additions thereto.
3. If the assured can by the terms of his policy surrender the same to the company, and agrees with the company to surrender the policy, then the money which shall be payable upon such surrender shall not be protected as hereinbefore provided; but nothing herein contained shall be construed to prevent the assured from agreeing with the company, for the surrender or exchange of his existing policy, for a fully paid-up policy, equivalent to its value, which latter policy shall have the protection afforded by this section.

Conditions.

17. A woman married in community of property may effect a policy of assurance upon her own life or upon the life of her husband, and every such policy, and every policy effected by any woman upon her own life before marriage in community, and the moneys payable in respect of every such policy, shall be deemed to be her sole and separate property, excluded from community, and shall not be subject to the control of her husband, nor after three years from the date of the payment of the first premium thereon, be liable for his debts, subject, however, to the provisions, and the conditions and the limitations contained in the last preceding section.

Woman married in community may insure her own or her husband's life.

18. Notwithstanding any law to the contrary, a husband being the holder of a policy of assurance upon his own life or upon the life of his wife, may lawfully cede such policy to his wife, whether married in community of property or not, or to some person in trust for her, for her benefit, or for the benefit of herself and her children, or some or one of them, subject to the following conditions or limitations:

Husband may cede policy on own or wife's life to wife.

Proviso.

1. Such policy shall have the protection provided by the sixteenth section of this Act, to the extent therein mentioned, but not further or otherwise.
2. Any such policy ceded as aforesaid to a woman married in community of property, or in trust for her benefit, shall be deemed to be her sole and separate property excluded from community.

Policy ceded to intended wife, to whom he is afterwards married in community of property, not to fall in community.

When policy so protected exceeds amount for which protected, substituted policies may be issued.

19. A policy of assurance effected or ceded by an intended husband for the benefit of an intended wife, to whom he shall afterwards be married in community of property, shall be deemed to be the sole and separate property of such wife excluded from community.

20. When any policy so protected as aforesaid exceeds the amount for which protection is provided, the person who, but for such protection, would be entitled to the policy, or to the disposal thereof, may require the company by whom such policy was issued to cancel the same, and to issue substituted policies of equal standing with the former policy; and such company shall thereupon issue such substituted policies, at the expense of the persons applying for the same, one for the amount protected for the benefit of the assured, or whoever shall be thereto entitled, and the other for the unprotected residue.

Moneys payable in respect of protected policies not available for debts before certain claims are met.

21. The moneys payable in respect of any policy by this Act, protected or exempted from execution or sequestration to the extent to which such protection or exemption is granted, during the life of the assured, shall not upon his death be available for the payment of his debts as against the claims of—

1. The widow of the deceased in respect of any interest acquired by her by virtue of a marriage in community of property, or devolving upon her by testamentary or other lawful disposition.
2. Any child of the deceased claiming by succession, *ab intestato*, or under any testamentary disposition.

If policy effected with intent to defraud, Court may order sum equal to amount of premiums paid, to be a charge on policy.

22. Notwithstanding anything contained in this Act, if in any case it shall be proved that any policy was effected, or the premiums upon any policy were paid, with intent to defraud the creditors of the assured, or in the case of a policy effected by a husband upon the life of his wife, or by a wife upon the life of her husband, with intent to defraud the creditors of either of them, the Court (or the Court of the Eastern Districts, the High Court of Griqualand, or any Circuit Court within the jurisdiction of such Courts respectively) may order a sum equal to the premiums so paid, with interest thereon, to be a charge upon the policy, and to be payable out of the sum assured.

Provision in case a policy lawfully ceded to wife, cannot be kept up.

23. If any person who has effected, or who, by marriage settlement or otherwise, has lawfully ceded a policy of assurance for the benefit of his wife, or of his wife and his or her children or some of them, or if any

trustee holding any policy under any such settlement or otherwise is unable to continue to provide for the payment of the premiums, such person or trustee may either—

1. Agree with the company which granted the policy to surrender the same, and to accept in lieu thereof a paid-up policy for such sum as, according to the rules of the company, may be its equivalent in value, payable at the time and in the manner, and for the benefit of the persons entitled to the sum assured by the original policy; or,
2. Borrow such sum as may be necessary to keep the policy in force, and any sum so borrowed and applied for keeping up the policy shall be a first charge upon such policy, and be repaid with interest out of the sum assured; or,
3. Agree with the company to apply any accumulated profits in reduction of future premiums.

Agreement may be made with company.

Or money may be borrowed to be first charge on policy.

Profits may be applied to reduction of premiums.

Court may order winding-up of company.

24. The Court may order the winding-up of any company, in accordance with "The Winding-up Act, 1868", upon the petition of twenty or more policyholders or shareholders, upon its being proved to the satisfaction of the Court that the company is insolvent; and in determining whether or not the company is insolvent, the Court shall take into account its contingent or prospective liability under policies or annuity, or other existing contracts.

25. The Court may, in the case of a company which has been proved to be insolvent, reduce the amount of the contracts of such company upon such terms and subject to such conditions as to the Court may seem fit, instead of granting a winding-up order.

Proceedings in lieu of winding-up may be ordered by Court.

26. Where application is made to the Court to wind-up a company, or in the case of a company ordered by the Court to be wound-up, the rules for estimating the value of a policy or life annuity, and generally the law as administered by the High Court of Justice in England, and the practice of the said High Court, shall respectively be the rules to be adopted, the law to be administered and the practice to be observed by the Court, so far as the same shall not be repugnant to or inconsistent with any law in force in this Colony; provided that the rate of mortality shall be computed on the basis of the tables known as the Institute of Actuaries' Experience H^M Tables.

Law and rules to prevail when company ordered to be wound-up.

Proviso.

27. No assignment of a policy shall confer on the assignee therein named, his executors, administrators or assigns, any right to sue the company for the amount of such policy, or the moneys assured or secured thereby until a written notice of the date and purport of such assignment shall have been given to the company at its principal place of business, and the date on which such notice shall be received shall regulate the priority of all claims under any assignment, and any advance or payment

Notice of assignment of policy to be given before company can be sued by assignee.

bonâ fide made upon or in respect of any policy by the company before the date upon which such notice shall have been received shall be valid and effectual against the assignee giving such notice.

Receipt of
executor or
administrator
of estate,
sufficient
discharge.

28. The receipt of the executor, testamentary or dative, or of the administrator duly authorized or appointed to administer the estate and effects of a deceased policyholder, by any Court or lawful authority exercising jurisdiction in the country where such deceased policyholder was domiciled at the time of his death, shall be a valid discharge for any moneys payable under any policy held by him.

Saving clause
with regard to
claim existing
at date of
coming into
effect of Act.

29. Nothing in this Act contained shall be deemed to affect or prejudice the rights of any creditor whose claim or demand shall exist at the date of the coming into operation thereof, which rights shall be judged and determined as if this Act had not been passed.

Schedule L.

SCHEDULE.

I. New Policies issued by the _____ during the year ending _____, 189 _____.

	No. of Policies	Sum Assured	Single Premium	Annual Premiums
Assurance . . .				
Endowment . . .		per annum		
Annuity . . .				
Total . . .				

II. Policies discontinued in the _____ during the year ending the _____, 189 ____.

[illegible]

III. Policies of the
existing on the , 189 .

	IN THE CAPE COLONY		ELSEWHERE	
	No. of Policies	Sum Assured	No. of Policies	Sum Assured
Assurance				
Endowment				
Annuity				
Total				

GOVERNMENT NOTICE.—No. 1044, 1891.

Colonial Secretary's Office, Cape Town.

Cape of Good Hope,

14 December 1891.

HIS Excellency the Governor, with the advice of the Executive Council, has been pleased, under the provisions of "The Life Assurance Act, 1891", to make the following Regulation in connection with Section 13 thereof, and to prescribe the subjoined Forms of Statements and Accounts in connection with Sections 3 and 5.

J. W. SAUER,

Colonial Secretary.

Regulation with respect to Section 13 of "The Life Assurance Act, 1891":

The fee payable for the inspection of any statement or other document deposited as required by the Act shall be Two Shillings and Sixpence.

Forms of Statements and Accounts prescribed in connection with Sections 3 and 5:

Forms prescribed in accordance with Section 3 of "The Life Assurance Act, 1891."

The following Schedules (First and Second) to apply to Companies transacting Life Assurance business only:

FIRST SCHEDULE.

Revenue Account of the *for the Year ending*

18 (Date)	£ s. d.	18 (Date)	£ s. d.
Amount of Funds at beginning of the year		Claims under Policies (after deduction of Sums Re-	
Renewal Premiums after deduction of Re-assurance		assured)	
Premiums { Single £		Surrenders	
New Premiums { Other £		Annuities	
of Re-assurance Premiums) on new Policies,		Commission on New Premiums	
assuring £, and yielding an Annual Revenue		Renewals	
of £, after deduction of Re-assurances		Expenses of Management	
Consideration For Annuities granted		Dividends and Bonuses to Shareholders (if any)	
Interest and Dividends		Other Payments (Accounts to be specified)	
Other Receipts (Accounts to be specified)		Amount of Funds at the end of the year, as per Second	
	£	or Fourth Schedule	£

Note 1.—Companies having separate accounts for annuities to return particulars of their annuity business in a separate statement.

Note 2.—Items in this and in the accounts in the Third and Fifth Schedules should be the net amounts after deduction of the amounts paid and received in respect of re-assurance.

SECOND SCHEDULE.

Balance Sheet of the

on the , 18

LIABILITIES.		£ s. d.	£ s. d.	£ s. d.
Shareholders' Capital paid-up (if any)				
Assurance Fund				
Annuity Fund (if any)				
Other Funds (if any), to be specified				
Total Funds as per Schedule No. 1				
Claims admitted but not paid*				
Other Sums owing by the Company (Accounts to be specified)*				
ASSETS.				
<i>In Cape Colony :</i>				
Cape Government Securities				
Other Government Securities (particulars to be specified)				
Mortgages				
Loans on the Company's Policies				
Loans on Personal Security				
House Property				
Other Investments (to be specified)				
Agents' Balances				
Outstanding Premiums				
Outstanding Interest				
Cash—On Deposit				
In hand on Current Account				
Other Assets (to be specified)				
Total Assets in Cape Colony				
<i>Elsewhere than Cape Colony :</i>				
British Government Securities				
Indian and Colonial Government Securities				
Foreign Government Securities				
Mortgages (stating where)				
Loans on the Company's Policies				
Loans upon Personal Security				
Railway and other Debentures and Debeture Stocks				
Railway Shares (Preferential and Ordinary)				
House Property (stating where)				
Other Investments (to be specified)				
Agents' Balances				
Outstanding Premiums				
Outstanding Interest				
Cash—On Deposit				
In hand and on Current Account				
Other Assets (to be specified)				
Total Assets elsewhere than in the Cape Colony				
Total Assets				

* These items are included in the corresponding items in the First Schedule.

The following Schedules (Third and Fourth) shall apply to Companies which transact Life Assurance business concurrently with any other kind of Assurance or other business.

THIRD SCHEDULE.

Revenue Account of the
for the Year ending
No. 1.—LIFE ASSURANCE ACCOUNT.

(Date)	£ s. d.	(Date)	£ s. d.
Amount of Life Assurance Fund at the beginning of the year		Claims under Life Policies (after deduction of Sums Re-assured)	
Renewal Premiums, after deduction of Re-assurance Premiums		Survivors	
New Premiums { Single £ (after deduction of Re-assurance Premiums) on New Policies, assuring £ , and yielding an Annual Income of £ , after deduction of Re-assurances .		Annuities	
Consideration for Annuities granted		Commission on New Premiums	
Interest and Dividends		Renewals	
Other Receipts (Accounts to be specified)		Expenses of Management	
		Other Payments (Accounts to be specified)	
		Amount of Life Assurance Fund at end of the year, as per Fourth Schedule	
£		£	

Note.—Companies having separate accounts for annuities to return the particulars of their annuity business in separate statement.

No. 2. — FIRE ACCOUNT.

(Date)	£	s.	d.	(Date)	£	s.	d.
Amount of Fire Insurance Fund at the beginning of the year				Losses by Fire, after deduction of Re-assurance			
Premiums received, after deduction of Re-assurances				Expenses of Management			
Other Receipts (to be specified)				Commission			
				Other Payments (to be specified)			
				Amount of Fire Insurance Fund at the end of the year,			
				as per Fourth Schedule			
	£						£

Note. When marine or any other branch of business is carried on, the income and expenditure thereof to be in like manner stated in a separate account.

No. 3. — PROFIT AND LOSS ACCOUNT.

(Date)	£	s.	d.	(Date)	£	s.	d.
Balance of last year's Accounts				Dividends and Bonuses to Shareholders			
Interest and Dividends not carried to other Accounts				Expenses not charged to other Accounts			
Profits realized (Accounts to be specified)				Loss realized (Accounts to be specified)			
Other Receipts				Other Payments			
				Balance as per Fourth Schedule			
	£						£

Note. This account is not required if the items have been incorporated in the other accounts of this Schedule.

FOURTH SCHEDULE.

Balance Sheet of the

on the

, 18

LIABILITIES.

Shareholders' Capital
 General Reserve Fund (if any)
 Life Assurance Fund*
 Annuity Fund (if any)*
 Fire Fund
 Marine Fund
 Profit and Loss (if any)
 Other Funds (if any to be specified)

£
 Claims under Life Policies, admitted but not yet paid*
 Outstanding Fire Losses
 Outstanding Marine Losses
 Other sums owing by the Company (Accounts to be specified)

£ s. d.

ASSETS.

In Cape Colony:
 Cape Government Securities
 Other Government Securities (particulars to be specified)
 Mortgages
 Loans on Company's Policies
 Loans upon Personal Security
 House Property
 Other Investments (to be specified)
 Agents' Balances
 Outstanding Premiums
 Outstanding Interest
 Cash—On Deposit
 In hand and on Current Account
 Other Assets (to be specified)
 Total Assets in Cape Colony
Elsewhere than in Cape Colony:
 British Government Securities
 Indian and Colonial Government Securities
 Foreign Government Securities
 Mortgages (stating where)
 Loans on the Company's Policies
 Loans upon Personal Security
 Railway and other Debentures and Debenture Stocks
 Railway Shares (Preferential and Ordinary)
 House Property (stating where)
 Other Investments (to be specified)
 Agents' Balances
 Outstanding Premiums
 Outstanding interest
 Cash—On Deposit
 In hand and on Current Account
 Other Assets (to be specified)
 Total Assets elsewhere than in Cape Colony

£

£ s. d.

* If the Life Assurance Fund is a separate trust fund for the sole security of the life policyholders, a separate balance sheet for the life branch may be given in the form contained in the Second Schedule. In other respects the company is to observe the above form. See also note to Second Schedule.

[Forms prescribed in accordance with Section 5 of "The Life Assurance Act, 1891."]

FIFTH SCHEDULE.

Statement respecting the Valuation of the Liabilities under Life Policies and Annuities of the _____, to be made by the Actuary.

(The answers should be numbered to accord with the numbers of the corresponding questions.)

1. The date up to which the valuation is made.

2. The principles upon which the valuation and distribution of profits among the policyholders are made, and whether these principles were determined by the instrument constituting the company, or by its regulations or bye-laws or otherwise:

(a) Were the policies valued individually or in classes?

(b) If in classes, how was the valuation age determined?

(c) What portion (if any) of a year's premium was assumed to be due?

(d) Were lives assured at increased rates assumed to be of the age at entry, corresponding to the premium charged? If not, how were they dealt with?

3. The table or tables of mortality used in the valuation.

4. The rate or rates of interest assumed in the calculations.

5. By what table of mortality, and according to what rate of interest have the net premiums valued been computed? Give specimens of such premiums for a whole-life and an endowment assurance policy of £100, for ages at entry 20, 25, 30, 35, 40, 45, 50, 55, 60.

6. The proportion of the annual premium income (if any) reserved as a provision for future expenses and profits. [If none, state how this provision is made.] In cases where assurances have been effected

by single or limited premiums, state what provision has been made for expenses when the premiums have been paid up.

7. The consolidated revenue account since the last valuation, or in the case of a company which has made no valuation, since the commencement of the business. [This return must be made in the form annexed.]

8. The liabilities of the company under life policies and annuities at the date of valuation, showing the number of policies, the amount assured, and the amount of premiums payable annually under each class of policies, both with and without participation of profits, and also the net liabilities and assets of the company with the amount of surplus or deficiency. [These returns must be made in the form annexed.]

9. The principles upon which the distributions of profits are made.

10. The time during which a policy must be in force to enable it to share in the profits.

11. The result of the valuation showing—

- (a) The total amount of profit made by the company.
- (b) The amount of profit divided among the policyholders, and the number and amount of policies which participated.
- (c) Specimens of bonuses allotted to whole-life and endowment assurance policies for £100 effected at the respective age of 20, 30, 40, 50, and having been respectively in force five years, ten years, and upwards, at intervals of five years respectively, together with the amount appropriated under the various modes in which the bonus might be received.

[Form referred to under heading No. 7 in the Fifth Schedule.]

Consolidated Revenue Account of the *and ending* *for* *years, commencing*

	£	s.	d.		£	s.	d.
Amount of Funds on 18, the beginning of the year				Claims under Policies after deduction of Sums Re-assured			
Premiums after deduction of Re-assurance Premiums				Surrenders			
Consideration of Annuities granted				Amitties			
Interest and Dividends				Commission			
Other Receipts (Accounts to be specified)				Expenses of Management			
				Dividends and Bonuses to Shareholders (if any)			
				Other Payments (Accounts to be specified)			
				Amount of Funds on 18, the end of the period as per First (or Third) Schedule			
							£

[Form referred to under heading No. 8 in Fifth Schedule.]

Summary and Valuation of the Policies of the

as at

DESCRIPTION OF TRANSACTIONS	PARTICULARS OF THE POLICIES FOR VALUATION						VALUATION					
	No. of Policies	Sum Assured	Bonus Additions	Office Yearly Premiums	Net Yearly Premiums	Loading	Sum Assured	Bonus Additions	Office Yearly Premiums	Net Yearly Premiums	Loading	percent
<i>Assurances</i>												
1. With participation in Profits—												
For Whole Term of Life . . .												
Other Classes (to be specified) . .												
Extra Premiums payable . . .												
Total Assurances with Profits												
2. Without participation in Profits—												
For Whole Term of Life . . .												
Other Classes (to be specified) . .												
Extra Premiums payable . . .												
Total Assurances without Profits												
Total Assurances . . .												
Deduct Re-assurances . . .												
Net Amount of Assurances . . .												
Adjustments (if any) . . .												
<i>Annuities—</i>												
Immediate . . .												
Other Classes (to be specified) . .												
Total of the Results . . .												

The item "Extra Premium" shall be taken to mean the charge for any risk not provided for in the minimum contract premium. If policies are issued in or for any country, at the rates of premium deduced from tables other than the European mortality tables adopted by the company, separate Schedules, similar in form to the above, must be furnished.

SIXTH SCHEDULE.

Statement of the Life Assurance and Annuity Business of the
on the , 189 .

(The answers should be numbered to accord with the numbers of the corresponding questions. Statements of re-assurances corresponding to the statements in respect of assurances, under headings 2, 3, 4, 5 and 6, are to be given.)

1. The published table or tables of premiums for assurances for the whole term of life which are in use at the date above-mentioned.

2. The total amount assured on lives for the whole term of life, which are in existence at the date above-mentioned, distinguishing the portions assured with and without profits, stating separately the total reversionary bonuses and specifying the sums assured for each year of life, from the youngest to the oldest ages.

3. The amount of premiums receivable annually for each year of life, after deducting the abatements made by the application of bonuses, in respect of the respective assurances mentioned under heading No. 2, distinguishing ordinary from extra premiums; also the amount of the respective net premiums valued at each year of life.

4. The total number of policies and amounts assured under classes of assurance business, other than for the whole term of life, distinguishing the number of policies and the sums assured under each class, and stating separately the amount assured with and without profits, and the total amount of reversionary bonuses.

5. The amount of premiums receivable annually in respect of each such special class of assurances mentioned under heading No. 4, distinguishing ordinary from extra premiums.

6. The total amount of premiums which have been received from the commencement upon all policies under each special class mentioned under heading No. 4, which are in force at the date above-mentioned.

7. The total amount of immediate annuities on lives, distinguishing the amount for each year of life.

8. The amount of all annuities other than those specified under heading No. 7, distinguishing the amount of annuities payable under each class, the amount of premiums annually receivable, and the amount of consideration-money received in respect of each such class, and the total amount of premiums received from the commencement upon all deferred annuities.

9. The average rate of interest at which the life assurance fund of the company was invested at the close of each year during the period since the last investigation, together with a statement of the manner in which such average has been computed.

10. A table of minimum values, if any, allowed for the surrender of policies for the whole term of life, and for endowments and endowment assurances, or a statement of the method pursued in calculating such surrender-values, with instances of its application to policies of different standing and taken out at various interval ages from the youngest to the oldest.

11. Separate statements to be furnished for business at other than European rates, together with a statement of the manner in which policies on unhealthy lives are dealt with.

CORRESPONDENCE.

A LONG-LIVED FAMILY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the last number of the *Journal*, Mr. C. D. Higham contributes a statistical record of the longevity of a family of eleven brothers and sisters, in which the average age at death was 68·7 years. I am tempted to send you a somewhat similar record which, however, contains two generations. The average age at death of the six children of the first family was 79·1 years, and that of the thirteen of the second family was 75. The parents in the first family averaged 63·5 years, and in the second 84·5 years.

Mr. Higham further interests us, by noting the fact that the head of the family in his illustration was a founder of a great life office.

I will add that one of the last new members of the Council of the Institute is a grandson of the individual who closes my tabular statement.

I am, Sir,

Your obedient Servant,

21 Sept. 1892.

ARCHIBALD DAY.

	Sex	Year of Birth	Year of Death	Age at Death
R. H.	M.	1730	1772	42
Wife.	F.	1724	1809	85
CHILDREN				
1	M.	1753	1833	80
*2	F.	1756	1850	94
3	F.	1759	1843	84
4	F.	1764	1852	88
5	M.	1766	1814	48
6	F.	1769	1850	81
J. B.	M.	1749	1824	75
*Wife.	F.	1756	1850	94
CHILDREN				
1	F.	1776	1864	88
2	M.	1777	1777	0
3	M.	1778	1857	79
4	F.	1780	1869	89
5	M.	1782	1861	79
6	M.	1783	1866	83
7	M.	1785	1866	81
8	M.	1786	1845	59
9	F.	1788	1866	78
10	F.	1790	1875	85
11	M.	1791	1872	81
12	M.	1793	1882	89
13	F.	1796	1877	81

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President, AUGUSTUS HENDRIKS, Esq.

[Delivered 28 November 1892.]

ELECTED by your goodwill to the high office of President of the Institute of Actuaries, I find myself facing the fact that although it is not so provided for by any of our bye-laws, custom has consecrated into an unwritten, but equally stringent, obligation, that following in the footsteps of my predecessors, I shall, in virtue of my office, deliver to you an inaugural address. In pursuance, therefore, of that duty, I will venture to make to you a few observations upon some of the subject matters which will be the most likely to press themselves upon our consideration during the term of my holding the leading position in our Institute, and which position you have conferred upon me, with all its honours, and with all its responsibilities. It is, however, well understood that my individual views may or may not commend themselves to your adhesion, and that although it is not customary upon these occasions to question the views of your President, this does not imply that silence gives assent, or that his views are unquestionable and may not be questioned hereafter.

It may be thought not inappropriate that I should commence my remarks with a consideration of the basis of those calculations

upon which our edifice is built, the corner-stone upon which we rely—the *Tables deduced from the Mortality Experience of Life Assurance Companies*, and as a prelude to this consideration I will give a short resumé of our practice in the times antecedent to the adoption of the *Institute of Actuaries' Tables*.

In the early days of life assurance, the table of mortality the most generally adopted was the Northampton. Its formation and its origin are so well known to all of those present that I need not enlarge thereon, nor, indeed, upon the nature or construction of any of those sundry tables which subsequently supplemented the Northampton, such as that of the Seventeen Offices, or of the Carlisle, or of the English and other tables in vogue from time to time; but it will be remembered that the Northampton Table was, by subsequent tests, found to be defective, in the sense that it gave erroneous results especially when applied to the mortality prevalent amongst assured lives, and unless possessing that identification with our current business, no table can be permanently accepted for our main purpose, namely, to determine the correct mortality of lives for which we have undertaken to make payment when the deaths shall emerge. Fortunately for life assurance companies, the errors of the Northampton Table were on the side of safety, the fact having been surely ascertained that it represented the expectation of life below the real expectation, so that the premiums deduced therefrom, and treated as assets, were consequently under estimated, whilst the liability to pay the sum assured was over estimated, with the result that companies thus valuing not only were on the safe side in their estimates of liability, but by the action of their table, by light expenditure and by the high interest-earning powers of the period, were able to accumulate large profits and materially increase their sums assured by way of bonuses. The Northampton Table, however, with its self-evident faults, was supplanted by other tables, such as the Carlisle, whilst, even in the earlier part of this century, the attention of actuaries was given to the desirability of the employment of a table representing the true mortality experience amongst assured lives, the outcome of which was the formation of tables derived from the experience of individual offices, such as the Amicable and the Equitable, whilst the combined experience of the Seventeen Offices, in the principles of its formation, came as the forerunner of the Institute Tables now in use. It is, however, worthy of note how few companies, comparatively speaking, adopted the Seventeen Offices

experience as against the large number of offices which pinned their faith on the Carlisle, and, in looking back to that fact, one cannot blame either our predecessors or ourselves, there being even now much to admire in that compilation, for the Carlisle Table, if now dead, has died harder than any table yet adopted by the profession, as, even to the present time, there are actuaries of the old school who use it for the valuation of reversions, and who still cling to it with lingering affection, in order to test, and compare with it, results deduced by the application of its more modern supplinters.

Slowly, but surely, the conviction ever present in the minds of actuaries, that the true table for our basis should be the mortality experience of the assured themselves, found its expression with increased emphasis in the task subsequently undertaken and brought to a practical issue by the formation of the tables of the Institute of Actuaries. We now, once again, cannot ignore the duty fast approaching, if, indeed, we are not already face to face with it, when we must bring down to the most modern times, the results of the intervening mortality since the last investigation so many years ago, for there cannot be any doubt that the subject of mortality experience of assured lives is one of those matters which must press now or ere long for solution, by the research of this Institute in unison with that of our professional brethren of the Faculty of Actuaries in Scotland. The majority of the leading workers in the compilation of the existing tables of the Institute have passed away to that bourn from which no traveller returns, and where tables of mortality are not: but some of those eminent men who took the initiative and leading part in the task then carried out, such as Mr. Bailey, Mr. Sprague and Mr. Day, are still amongst us as our mentors and friends, and will be spared, I trust, for further usefulness in our contemplated undertaking, guiding us by their matured knowledge and by their ripe experience for the undertaking which presents itself for early solution, whilst to those veterans must be added those who like myself were juniors at that time and took a subsidiary part, under the direction of my revered friend and instructor, the late Mr. Samuel Brown, and last, but not least, some younger volunteers of the present period, for the task before us promises to be a work of time, and time is an integral proportion of a perpetuity in this world, although it may be disregarded in a future existence in which the only sign left to us of our notation will be that true perpetuity which knows no end, and whose value

is unaffected by any given rate of interest. Therefore, in starting on our task, we should by relying on the assiduity of our younger members, at least do our best to avoid any such untoward prospect as that of contributing to the new table the emerged mortality of even a small proportion of the members of the acting committee, before the work is closed, though it is to be hoped our undertaking is not fated to occupy as many years of labour as it did on the last occasion of the formation of our table.

Looking to the instruction afforded by the past, it will be found that in the year 1862, after the question had been discussed on several occasions, both in England and Scotland, it was brought before the Council of this Institute, and a resolution was adopted, setting forth—

“ That it is very desirable to collect and combine, as far
“ as possible, the experience of the Life Insurance
“ Companies of the United Kingdom to the present
“ time ” (1862),

and, further, a committee was appointed to “ report on the best
“ means of collecting such experience, with power to communicate
“ with any other committee or actuaries, or managers of com-
“ panies who may be disposed to assist in obtaining the required
“ information.”

The further details relating to the labours of that committee, in conjunction with the Committee of Managers of the Association of Scottish Companies, and of the Faculty of Actuaries in Scotland, are fully and clearly set forth in the instructive preface written in May 1869 by our then eminent and indefatigable President, the late Mr. Samuel Brown, in the volume published at that time intituled *The Mortality Experience of Life Assurance Companies collected by the Institute of Actuaries*. Thus, there was an interval of seven years between the date of the appointment of the working committee and the date of the publication of the first results of its labours, and even then was published only the general results of the valuable data then collected, it being stated further in the report that “ the preparation of monetary
“ and other tables, for official purposes, is a work involving so
“ much time, labour, and expense, that the Council of the
“ Institute of Actuaries have decided not further to delay the
“ publication of the original facts on which they will have to be
“ computed.”

This anticipation was fully realized, for it was not until the

year 1872 that the all-important crowning of the edifice took place, by the publication of the *Institute of Actuaries' Tables* and the adoption of the designations now so familiar to us of H^M , H^{M3} , and H^F . During the 20 years which have since ensued, gradually, and one by one, a large majority of our British companies have measured their liability by the application of those tables to their valuations; but it will be observed that, although it is but 20 years since those tables were published, it is 30 years since the last life of that period was included in the data upon which that mortality experience was assessed, so that we find that those of the adolescents of the period of that table who are still living are now old actuarial hands, and a generation has passed away since those tables were compiled.

Unless, therefore, we believe that the days allotted to man change not with the changes of this world, can we, as the Institute of Actuaries, continue indefinitely to take our stand upon a table which may be materially inapplicable, if tested by the result of modern mortality? It must further be borne in mind that, although some of the lives were admitted to the existing tables at, comparatively speaking, the later date of 30 years ago, the great majority of the lives subjected to observation were of the generation of the grandfathers of the insuring public of to-day.

If we believe that it is likely to be found that the mortality up to the present date differs essentially from former experience, it becomes our bounden duty to face the enquiry whatever may be the labour involved; and if, as the results of our investigation, we shall find that the mortality does not differ materially from that on which we now rely, the labour will not be incurred in vain, for it will confirm the past, engender confidence in the future, and remove all existing doubts, for notwithstanding that our doubts tend to the anticipation that should the present Institute of Actuaries' Tables err, they err on the side of safety, yet it is better that all doubt of whatever kind should be put to rest, by the formation of a table giving effect to the mortality experience relating to assured lives up to date. The labour to be incurred is evidently very considerable, and although it is likely that, when once fairly commenced, the work may be completed in a less number of years than on the former occasion, 1862-1872, the very fact that when commenced there must be a long interval before the final results can be arrived at, ought to be a strong incentive to the Institute to commence their research

now, or at no distant date, for even with the greatest speed at our disposal it bids to be a *fin de siècle* table. In the opening address of my predecessor in office, Mr. Newbatt, in November 1890 (*J.I.A.*, xxix, 1), he stated "a fitting task is seen to be waiting for" the Institute in the compilation of a new table of mortality "amongst assured lives." The fitting task was waiting then. It still waits. It may be that, feeling convinced that the current mortality is less severe than that provided for by our existing tables, some of us may consider that our strength is to sit still with such a safety-valve to counteract the adverse effect of a decrease in the earning power of money, and an increase in the cost of obtaining new assurances; but so long as we continue to boast that our science is an approximation to a true science, we are bound, in my opinion, to approach to facts and to veracity as nearly as may be, and at whatever cost and at whatever labour, to move with the times, and proceed with the quest for a *true table of mortality based on assured lives*.

I take it that when, and if, the waiting time has a termination, a committee will have to decide on several very important points connected with the compilation of the tables, for example—

- (1) Whether to limit the offices contributing their experience to the classic 20 of the present tables, and to include the lives placed on their books since the last investigation.
- (2) Whether also to form, separately, a very interesting continuation of the existing table, bringing it down to the present date, based on the mortality experience of those lives only which were included in the former research, thus, ultimately, by continuous investigation, tracing the mortality appertaining to assured lives to the bitter end, with "*respice finem*" for its motto, and for its boast the finality of a true table of mortality amongst assured lives.
- (3) Whether to form a subsidiary table confined to modern mortality, based on the experience of those lives exclusively accepted during the 30 years which have elapsed since the last life entered into the formation of the existing table.
- (4) Whether to bring into the new table the experience of companies other than those included in the tables now in use?

- (5) Further, to determine whether to perpetuate or not the theory of $H^{M^{50}}$ —and, in this connection, if a table (2) were to be formed carrying on the existing table to the present date, it would help materially to the solution of the problem of whether the effect of the medical and official selection of lives is worn out entirely according to the suggestion of $H^{M^{50}}$, or whether it is merely a temporary check to the advantages obtained by the careful selection of lives by insurance companies.

These and other momentous preliminary points would have to be considered by your committee.

It seems to me that the *modus operandi* will be for a committee first to decide upon the principles to be adopted, then that it should appoint some one of tried ability, as a paid officer, to superintend the work, and that the work be then performed by duly qualified younger members of the Institute, all expenses to be defrayed by insurance companies as a body, the assessment upon each company to be in proportion to the magnitude of its respective business, and to be borne *pro rata* by all companies, whether they supply their own data to the formation of the tables or not.

The result of the investigation upon the threshold of which we now seem to be standing cannot be predicated with safety, but on the whole there is a somewhat general impression that the mortality has of late years continued to improve, as evidenced by the almost stereotyped phrase in the reports of insurance companies "that the deaths have been less than the mortality expected by the Institute Tables." This improvement is believed to be mainly attributable to improved sanitation, the outcome of modern science practically applied to the prevention of disease; but the struggle of humanity for a prolongation of existence may be likened to the conflict of guns and fortifications, in which one or the other prevails alternately. We combat disease by sanitation and medical science, but whilst we stamp out or diminish epidemics in one form, some new and undefined or scarcely understood germ, such as that which we term "influenza", fixes its "grippe" upon the human race, and ruthlessly stops the improvement in our rate of mortality. And even as I now speak the grim probability of a return of cholera tends to prove that epidemics are scotched, but not killed, by the science of sanitation.

Therefore it is incumbent on us, as actuaries, to be watchful so that we may not be caught on the ebb when the flowing tide of mortality is against us through the action of such epidemics, or even more so by a mortality from future warfare—probably far in excess of the past experience of long periods of peace—or of wars of small dimensions, fought out in distant countries, unfaced by an European foe, and limited to a force of combatants representing only a small percentage of assured lives.

Another question which is gradually but surely coming to the front is the necessity for providing against the effect of a diminution in the interest-earning powers of our investments. Insurance companies in quiet times are like ships with all sails set, and the actuary is the pilot. Some pilots take in all sail as soon as they see a cloud on the horizon no bigger than a man's hand. Other pilots furl their sails one by one as the breeze comes up, and all pilots must make the ship trim before the storm bursts. Already several companies, desirous of making assurance doubly sure, have recently reduced the assumed rate of interest for their liability valuations from 3 to $2\frac{1}{2}$ per-cent, whilst other companies hitherto valuing at higher interest than 3 per-cent, have now valued at rates ranging from 3 to $3\frac{1}{2}$ per-cent, thus giving proof of the common desire to strengthen the reserves, so as to meet and anticipate the diminution in the recuperative power of investment improved at compound interest. In this connection actuaries have to be careful to study on the one hand the safety of the institutions committed to their charge, whilst not neglecting, on the other hand, to avoid doing an injustice to present claimants, by postponing the distribution of profits fairly earned to the undue advantage of the still living policyholders as well as to the future entrants. The point has not only to be considered in regard to its effect in the future, but with reference to its aspect in its present phase. Also, it may be admissible for one company, upon a careful revision of all the facts connected with a present investigation, coupled with the prospects of the future, now to reduce its valuation to a $2\frac{1}{2}$ per-cent basis without doing an injustice to any of the interests, present or future, confided to its care; yet it may be equally open to other companies differently circumstanced as regards their

mode of distribution of surplus, or from other legitimate cause, to defer taking the step of valuing at a rate of interest so far below its present earning powers, or of those in probability for the near future. To illustrate my meaning, I would say that one of the main reasons assigned for valuing at $2\frac{1}{2}$ per-cent is that within a measurable distance consols will be reduced to that rate, therefore, it is argued, if it was right and proper to have valued hitherto at 3 per-cent, it follows, *a fortiori*, that it is right and proper now to value at $2\frac{1}{2}$ per-cent; against that view it may be said that the measurable distance for such a diminution of earning power is not yet, nor in the near future. It is not the ground on which we now stand, and all the indications thus far point to the probability of a $2\frac{1}{2}$ per-cent Government Stock not reaching an average price of £100 sterling for £100 of stock for many years to come; and it is also to be observed that although it is true that the earning power of other investments takes its tone from the price of Consols, nevertheless, the amount invested by insurance companies in British Government Securities being a very small fraction compared with their total investments, the fact of Consols paying only $2\frac{1}{2}$ per-cent cannot be taken as a true criterion of the likelihood of that rate of interest being the average rate of the future to be earned by their funds, taken as a whole. Thus I venture to express an opinion that companies are perfectly justified in carrying into effect a reduction in the rate of interest assumed in their valuations, *festina lente*—by a gradual and cautious process—and with due deliberation, and this whilst still retaining all the elements of strength possessed by those companies which take that step in greater haste.

It will be remembered that, in the earliest days of this century, the foresight of our predecessors fixed, by prediction, the future average earning powers of interest at 3 per-cent, and the prudent men amongst them consistently based their premiums at the same rate as they estimated their liabilities, and that, although we are all these years nearer to the accomplishment of their prophecy of a reduction to 3 per-cent, the wider field for investment continuing to spread, as it does, over safe securities in this country of a nature not then extant, or even dreamt of by our predecessors, and also to the modern transmission of British capital to pastures new in the colonies and foreign countries, formerly unknown to investment, enables insurance companies to earn a rate, less, it is true, than in the days of the past, but yet appreciably in excess of the rate still assumed by the great

majority of our societies, both in the formation of our premiums and in the present estimate of our liabilities, based, as they should be, on the probabilities of the interest-earning power of capital for years to come. The indications, however, point more to an assimilation in the rate of interest all over the world than to a sudden drop in this country. Capital, as well as man, is becoming cosmopolitan, and the difference between the earning power of money by safe investments in this country and in its Colonies and America is being narrowed year by year.

Taken in connection with the fact of the fall in the average rate of interest being probably a slow and gradual fall, I would refer to its effect on endowment assurances, and to the injustice which might be done to policyholders insured under that system, for terms of short duration, by too rapid effect being given in valuations to a slowly-approaching fact. The recipient of the sums assured 10 years hence may well exclaim—"Why do I receive smaller bonuses now because the interest-earning power may possibly be reduced to $2\frac{1}{2}$ per-cent 30 years hence"? And this same question might be asked by old men assured for the short remainder of their whole lives, and during which the higher rate of interest will continue to be earned.

The only way to answer such pertinent questions will be to take great care that the mode of distribution of profits is arranged so as not to act unduly on insurances of those descriptions, and, this being accomplished, then, after the passing of the rubicon, by a reduction in the rate of interest assumed in any one valuation, at the next ensuing, and at all subsequent periods of allotment of bonuses, the short-term endowments, as well as whole-term insurances on aged lives, still extant, will enjoy in common with all other policies the full advantage of the enhanced profits to be derived from the wider margin thus created between the reduced rate assumed and the actual interest earned by the reserve fund, so that the disadvantage, if any, is confined to the one period only of valuation embraced in the change of rate.

Whilst on the subject of endowment assurances, I would refer to the interesting study now commencing as to the extent, if any, to which the mortality experience on whole-term-of-life assurances will be deteriorated by the marked increase now taking place in the number of the assured by policies taking effect on the attainment of fixed periods of years or at previous death. On the one hand, the endowed evince a belief in being good lives by backing

themselves to live to the end of the limited term, and thus, by withdrawing themselves from the whole-term-of-life votaries, may possibly, by their own selection in favour of the company, leave a larger leaven, in that latter class, of those who have selected against it; but, on the other hand, and in mitigation of the power of selection from opposite standpoints, may be taken the growing custom of relegating lives with a bad family history, or of deteriorated constitution, to endowment assurances, as a haven for easier extra premiums.

Second only in importance to the formation of a true table of the mortality amongst assured lives of this period, is the formation of a like table applicable to the suspended mortality amongst annuitants, until they have arrived at ages far beyond those assigned to them if measured by the standard of ordinary mortals who have not retired from the care of management of their own capital, or of that still larger class of beings who have no capital to manage. Great assistance has always been given to the elucidation of the mortality experience of annuity lives by the publication of the Government statistics, and of the tables deduced therefrom by the successive actuaries of the National Debt Office; but there is a growing doubt in the minds of actuaries attached to insurance companies granting annuities, whether those tables, however stringent, fully meet the conditions of annuitants of the modern type. I myself hold that having regard to the fact that a preponderating proportion of those who sink their money do so in old age, it is necessary in order to arrive at the modern rate of mortality to base the experience on the results obtained not more than 20 to 25 years back, and thus discard the experience of a period now passed away, and possibly inapplicable to the problem now before us of ascertaining whether modern hygiene and sanitation, or other potent causes, have led to an increase in the number of lives attaining to extreme old age, and if so, whether that intensity of power of existence will not be found in all its fulness of strain as regards annuitants, proverbial at all times for their staying powers. That the matter is of great importance to insurance companies is evidenced by the fact mentioned by one of our Vice-Presidents, Mr. Young, in a discussion which recently took place in these rooms. He brought to our attention, that 10 years ago the annuity payments made

by British Offices in one year were £439,000, but that last year the Board of Trade Returns showed that they had increased to £797,000.

In my opinion this increase is likely to be progressive—all the indications point that way. I ask, therefore, whether it is not time that those of us who are so largely interested in the subject, should not combine the experience of our respective offices in the production of a true table of mortality, applicable to the annuity lives of insurance companies, as found by modern tests of modern experience.

In regard to the granting of annuities, it has seemed to me a moot point whether the Government and life companies are altogether wise in allowing the selection against themselves in its fullest force without question, and without any attempt at alleviation, the acceptance of all proposals turning upon the mere proof of age as the sole information required upon which to base the rate of annuity. In life assurance, if a man's father and mother have died of consumption we decline the case, or accept only at an increased rate of premium. Why, then, might we not ask for the family history of a would-be annuitant? and if there should be a marked tendency to family "staying power", such as a father and mother and grandfathers and grandmothers, having lived in their generations to the mature ages of 80 or 90, why not decline to cover the risk of the proposer following the sad example of his ancestors? Or, if in a more limited sense, his progenitors—or his own brothers and sisters have shown a less, but still in a well accentuated degree a predisposition to extreme longevity—why not follow the example of life assurance in an inverse ratio, and accept only at a rate corresponding to a less age than the actual age attained? Competition more than any other cause prevents these safeguards from coming within the possibility of practical politics.

It is already within your knowledge that the Board of Trade some months since addressed a communication to this Institute, suggesting that the time had arrived for contemplating amendments of the Life Assurance Companies Acts of Parliament, and asking for our advice thereon.

It seemed desirable on our part to enquire of the Board of Trade upon what points relating to those Acts they were desirous

of receiving advice, so that our attention might be directed more especially to giving our views in the direction in which amendments might be deemed necessary or desirable. At this juncture, however, the Board of Trade decided not to proceed further in their research, and it can well be understood that coming as it did in the last day of a moribund Parliament the question should be temporarily shelved. It was, however, fully understood that the question was to be revived on the advent of the now re-elected House of Commons. It is believed that the Board of Trade is not likely to desire any grave or heavy changes, either in the schedules, or in the statistics, or in the information supplied under the existing Act, which Act, taken as a whole, has been admirable in its working during the 22 years of its existence. It has been practically taken as a model, with unimportant variations, by nearly all of the Colonies, with the sole exception to the contrary of Canada, which, guided no doubt by its propinquity, pronounced in favour of the system of supervision adopted in the various States of North America. It would seem, so far as one can gather, that the attention of the Board of Trade is directed more particularly to the defects in the Act which give such unrestricted scope to waste and spoliation when a company is being wound up, or its contracts diminished, by the provisions of the Act; and it seems almost incredible, but it is too true, that we should still be able to witness the disgrace of the possibility of one captious shareholder, if, indeed, he ever was a shareholder, being able to inflict in abortive law proceedings, an expenditure of upwards of £30,000 upon a company already wrecked by malversation, but struggling to give as good salvage as possible to its unfortunate policyholders. Words cannot be too strong to condemn any system which can possibly give a loophole to such cruel wrong, and one can well imagine the desire of the Board of Trade to protect the public against any recurrence of such a grave scandal as the one now commented on.

In regard to principles of the Act, as well as to the less, but still important, point of statistical information set forth in the schedules, the Institute has already had before it the excellent paper read by Mr. G. King (*J.I.A.*, xxix, 481). Some of his suggestions might be adopted, whilst others of them, useful, no doubt, to the statist and expert, do not seem to make for the real purpose of the Act, which is to place the public, as succinctly as possible, in possession of sufficient facts to enable it to understand

the position and prospects of life assurance companies, and the principles upon which they are conducting their business. The public demands no elaboration of statistics confusing to the ordinary mind. There is, however, no doubt that there are some statistics which are not only interesting but desirable from the public standpoint. Thus it has been suggested by a former President that the American and other foreign companies should give information in their returns as to the amount of the business transacted by them in the United Kingdom, and it is clear that this should be so, as the absence of such a return prevents a proper knowledge of the total amount of life assurance transacted in this country; but, to be complete, British companies should also be called upon to state the amount of business they transact out of the United Kingdom, otherwise we should get the credit side of the account without the debit.

When the time arrives for a general enquiry into the working of the Act, this Institute, and the other bodies referred to by the Board of Trade, will give their advice and their cordial support to the Government authorities in their endeavour to perfect a system which has already conferred great benefit both to the public and to insurance companies.

It is worthy of note that the Board of Trade, in addition to a like communication to the Faculty of Actuaries in Scotland, has sought also the advice on the same subject of the two other organizations in London—the Actuaries' Club and the Life Offices Association—both of which bodies share with this Institute the desire to respond thereto with utility.

Whilst on the subject of those kindred institutions working in the same field, and often with the same object and at the same time, I would express the desire, the constant and fervent desire, I have always had to see those three bodies acting in unison for the common good.

Your late President, Mr. Newbatt, employed all the strength of his clear intellect to bring this consummation about, but he has told us in his opening address in 1890, already referred to, that he came to the conclusion, reluctantly but clearly, that by no device of which he was cognizant could practical effect be given to the notion. I also did my best to accomplish this result by a plan which I submitted to the Institute at the time of the inception of the Life Managers' Association. My views thereon had already obtained in principle the adherence of the Actuaries' Club, and my efforts then came so near to success, that I do not despair of

the future, even now, and in face of Mr. Newbatt's reluctant conclusion. Opinions, however, differ as to the desirability of a fusion even if it could be attained, for another of our eminent Presidents, Mr. Sutton, whose views at all times command our respectful consideration, in his opening address in 1888, in wishing success to the then gestating Life Managers' Association, "hailed its approaching birth with enthusiasm, because it would "reduce to an absolute minimum the possibility of the Institute "of Actuaries ever being diverted from its original objects which "may be briefly described as educational and professional." Thus Mr. Sutton relegated all questions of practice, which he termed "burning questions", to that association which has not inappropriately chosen for its place of deliberation and discussion the committee room of the Associated Fire Offices; but I would submit that these current and practical questions, said to be "burning questions", are merely the incandescient lights of life assurance, and I would fain switch them on to this hall in Staple Inn. They are home questions, and not foreign to the spirit of this Institute.

It has been pointed out by that sound actuary in practice as well as in theory, Mr. Bailey, that this Institute was established "as a scientific and practical association because on a scientific "basis alone could its investigations be properly conducted, "and practical because it was to concern itself with the "monetary affairs of actual life, and at the same time to "carefully avoid interference with the conduct of any existing "institutions."

Now these are precisely the lines upon which are conducted the proceedings both of the Actuaries' Club and of the Life Offices Association. Our Charter has clearly defined the duties of this Institute, and has confined it within its educational and scientific limit; but I do not find any bye-law prohibiting the Institute from an intimate relationship with other societies devoted to the practical application of our science, or preventing the Institute, as the *alma mater*, to affiliate to itself the now scattered energies of the three several bodies. All the members of the Actuaries' Club are Fellows of the Institute, and if with the members of the Life Offices Association, whether Fellows or not, and of the Council of the Institute or other working committee thereof, there could be formed a concentrated body, taking the shape of a standing working committee, which, with its unanimity of action, could be grafted as an affiliated branch upon this

good old Institute tree, it would, I feel confident, bear good fruit to be enjoyed in the seasons to come, and there would thus be accomplished a unity of purpose and a unity of action to check the centrifugal effect of modern competition, and bring us into that true accord which would be good for us and beneficial to the insuring public. It has been said with much weight that scientific theory should not meddle with the details of practice. My proposition is that theory and practice should combine so as to discountenance malpractice.

We had this summer the pleasure of welcoming to these shores that accomplished actuary, Mr. D. P. Fackler, the lately-elected President of the Actuarial Society of America, in succession to Mr. Sheppard Homans, whose able life has been devoted to the solution of the problems which engage our profession. Mr. Fackler, at a recent meeting of the National Insurance Convention held in America, read a paper on the "Limitation of the Business of Life Insurance Companies", in which he held that "many have grave fears of bad results if the largest companies are allowed to continue to grow indefinitely, and if any fair, open-minded person thinks these fears groundless, he must certainly admit that he can conceive that some companies might by some possibility, grow to such vast size that their assets and affairs could not be perfectly managed by their officers, supervised by their directors, and examined by state insurance officials . . . so that they would no longer yield the best attainable results for their own policyholders." And Mr. Fackler states that "an esteemed friend of his, an officer of one of the largest companies, once remarked to him privately that life insurance is the only business in which competition has not benefited the public." Further, Mr. Fackler states: "Our life companies are becoming vast financial corporations, and may become a source of danger to the commonwealth by reason of the vast money power lodged in the hands of a few men, probably one man in each company, and be held that the power might be used to influence legislature or to affect the money market. It is easy to imagine very startling possibilities when our life companies shall attain their probable future size", and his conclusion is: "As it has been shown that any great further increase in the size of our largest life companies will—to put it mildly—benefit neither policy-

“holders nor the public, and that such increase serve only the “ambitions of some managers, and the pecuniary interests of “certain agents who could not do so well with some other “companies”; and he asks “should not every disinterested person “favour the needed legislation”? The suggestion which he makes, and the legislation which he recommends to the several states is, that “when any company shall have attained assets of “200 millions (dollars) it will be liable to get into a condition of “unwieldiness if allowed longer to push for new business, and “shall therefore no longer be licensed to employ agents or “solicitors, leaving the citizens, however, free to obtain insurance “in such a company if they wish to do so.”

Much of this reasoning is, no doubt, inapplicable to this country, as our State officials do not enjoy the freedom which the official class has in America to control the freedom of the citizen; but who can say, from the American point of view, that Mr. Fackler is wrong in his conclusions?

It has been experienced in that country that individual plutocrats have, by the unlimited wealth which they command, been able to control railway and other finance, and it may possibly be dreaded that insurance companies also, having enormous accumulations, may, by the effect of that power wrongly wielded, control and overshadow the investment of capital, and it may follow *a fortiori* that such overgrown institutions should be checked by State limitation to unthreatening dimensions.

It does occur to me also, in the inner consciousness of my logical reasoning, that if some of those institutions are already in their own more immediate domain too large, why do precisely those companies, whose bulk is commented on by Mr. Fackler, seek for further increase in these small islands, and continue to employ, so far as I know, agents who *are* remunerated? And the thought comes unbidden of that tale taught us in our youth of the ancient Briton made captive, who, seeing outspread the glory and power of Rome, enquired, “Why does a people “possessed of such magnificence at home envy me my humble “cottage in Britain?”

Far from decrying healthy competition, we welcome fellow labourers in the cause, whether from the east or from the west; and it may well be that when those leviathans who are now sporting themselves in these latitudes feel too plethoric and are in a fit state to gracefully retire from this field which has no

ring-fence, they may send to these pastures some of their smaller institutions for honourable competition, at an equal ratio of expenditure, with companies of their own size and of their own aspirations; and if they could throw more assurance and less tontine into their methods, it would tend to a perpetuation of the true principle of life assurance as apart from thrift in the shape of investment.

If it may be admitted that, however unlikely in this country, in America, companies may possibly exceed their natural growth, may it not also be remarked that it is possible in these islands for our companies to remain too stunted in themselves, separately, and that it may be right for them, of their own free will, and apart from Government interference, to seek expansion by combination with other of our institutions possessing more vigour and power of expansion.

For very many years, and arising from the experience engendered by a right principle wrongly directed, there was a strongly-felt prejudice against amalgamations; but time and common sense have brought us back to the conviction that it is not amalgamation itself, but the neglect of the standard principles which should guide such transactions that in former days brought amalgamations into temporary condemnation, for what can be truer than the abundantly-proved fact that the fusion of separate companies, when honestly carried out and perpetuated, has provided a haven of safety and of good results to the assured, arising out of the economization of expenditure and the concentration of energy upon a condensed field of operation.

I feel constrained, however, to remark that the question of amalgamation, with its balance of advantages and disadvantages, has to be approached with the greatest care and circumspection; for unless the objective point of the fusion be economy, coupled with efficiency, far better for the smaller company to plod on than to sacrifice the prospects of its policyholders to the bewildering dazzle of big figures, wrongly obtained by abnormal expenditure. Looking back 30 years, there were five of the tritons of assurance which attracted to themselves the minnows who were swimming around them. Three of those tritons have been caught into the meshes of bankruptcy because they represented all that is evil in amalgamation—extravagance, mismanagement and, in one case, malversation; the two remaining have taken rank with the best of our institutions because they have represented all that

is good in amalgamation—economy, good administration and honesty of purpose.

Also coming to us from America, and with much sounding of the trumpet, have arrived those offsprings of the west, born of our likeness and yet unlike, the semblance of us but not with us, appealing to the public on the common ground of thrift and provision for the generation to come, promising in the words of the tempter a like result at one-half of the cost, endeavouring to supplant a fixed mathematical quantity by an annual assessment, and leading the proposer to infer that all claims will be met without increasing his original annual premium, instead of pointing out that the only prospect an assessment company has of meeting the claims at death of all those on its register will be by being able to assess an increasing premium ultimately, far in excess of the level premium applicable to ordinary life assurance; but such increasing assessments are unbinding on the policyholder and not susceptible of enforcement, therefore, to be responded to by the strong and healthy just so long as the assessment note shall not be *crescendo*, and leaving to the weak and to the dying the cold comfort of empty, unreplenishable coffers.

And what do such companies say of us?—the old companies of the time-tried system of level premiums—I quote from one of their prospectuses.

“*Reserve Funds.*—If additional evidence were needed to
“ prove that the charges made by existing offices, are
“ out of all proportion to the risk incurred, it need
“ only to be stated that the British life offices have
“ now accumulated out of surplus premiums more than
“ £150,000,000.”

Thus they hold that it is wrong for us to have in hand a good and sufficient reserve to accumulate in order to meet our liabilities. Like traders, we issue promissory notes—for what are life policies but promissory notes—subject to a due performance of the contract. *At death* we promise to pay the sum assured and *we provide therefor*.

If we were engaged as those institutions are in that transparent travesty of assurance for life, called assessmentism—in all its happy-go-luckiness—then, no doubt, we should proceed from year

to year, and from hand to mouth, with nothing in reserve but disappointment and ultimate disaster. Assessmentism at its best, may be likened to a magic lantern. The oil to lighten it up is the annual assessment. The lantern throws a shadow on a white sheet, which takes the shape of a policy. The supply of oil lasts for one year certain, and the shadow remains on the sheet, and is equal to a palpable form for that space of time—and from year to year—but the oil becomes dearer and dearer, and when too dear, *le jeu ne vaut plus la chandelle* and the supply stops. There is no tank containing an appreciable reserve of oil—the lantern is extinguished—the blank sheet is there, but where is the shadow? The shadow was a dissolving view: the policy on the slide. And yet a certain section of an unthinking public are lured from the firm ground of life assurance to the quicksand of assessmentism.

Apart from the main questions which at the present time more immediately affect the business of life assurance, there is the cognate topic of old age pensions, with all its ramifications of proposed state aid, with all its official and non-official expressions of opinion, with the criticisms thereon of men political, of men statistical, and of men of many grades and positions, each of them anxious for the well-being of his suffering fellow creatures, all of them anxious to suggest his own specific panacea against all evil, and none of them apparently able with entire success to grapple the immense difficulties which beset the path of the philanthropist who takes as his task to controvert the truism “that the poor shall be always in our midst.” To alleviate the misfortune of the aged poor is worthy of the noble effort which is being made in this and other countries, and it may well be hoped that some plan may commend itself, whereby encouragement may be given to self-help in a form not repugnant to self-reliance.

The question having been already disposed of to some extent in Germany, in Denmark, in Russia and other countries with more or less success, it may be hoped that in our own land proverbial safety may be found in the multitude of councillors, and that the Government may prove itself equal to the solution of the problem of ameliorating the condition of the poor and helpless, as well as in affording encouragement to those who are

able and willing to provide for the rainy day out of their own hard savings, and this without loss to their own self-respect.

The task of the Government is, however, by no means an easy one. If it were itself to endeavour to establish a Government State Help Insurance Department, it is more than likely that history would repeat itself, and it would fall as flat upon the country as the already existing Insurance Department of the Post Office, which apparently has been called into being to clearly demonstrate the inability of a State department, however well administered internally, to compete, passively, with the active efforts of joint-stock and mutual enterprise.

It would thus appear that unless at this juncture the Government were to be prepared to enter into an active and far-spreading propaganda of State-provided old age pensions, their effort would meet a like inglorious fate to that of Government-provided assurance hidden in this country under a bushel. State insurance, or State pensions, to be effective would have to be taken to the people—the people towards whom those blessings are intended to flow are too burdened with work, or too apathetic, to seek for any hidden light. The workman reposing from his day of toil will not trudge to deposit his hard-earned penny, even to find it in conjunction with another subsidized penny from the State—he must be waited on, and be persuaded thereto, by the ever active and ever watchful agent of the industrial company or benefit society.

Thus, unless the Government were to establish a vast department, with ramifications all through the country, and a standing army of propagandists and other satellites, its efforts would be nugatory to that enormous section of the public which lives from hand to mouth.

The difficulties which the Government of any country has in carrying out State aid are immense, even in a country like Germany, where "Laws of Insurance against Invalidity and Old Age" can be passed and made compulsory, and as so ably commented on in our *Journal* by Mr. T. E. Young (*J.I.A.*, xxix, 269); but they become almost insuperable and practicable only to a limited extent in a country like England, in which, to use the forcible language of Mr. Ralph P. Hardy, "we have not been drilled into obedience, or dragooned into civility." This hypothesis is quite borne out by the advanced wing of socialism, and with even greater intensity by the still more advanced wing of anarchism, which, believing more in helping itself than in self-help, will

accept no Government aid even to its own well-being: whilst to these fervid objectors must be added many of the well-conditioned members of existing benefit societies, who, strong in the manly independence of genuine self-help and self-reliance, view, not without apprehension, any approach to relief, State or otherwise, unearned by their own right hands.

To these latter would be the reply—It is to your own right hands you do owe such aid, and it is in recognition of your own merit that the State seeks you out for reward. There are many to whom this axiom would be acceptable, but there are others, and probably a majority, who would prefer to plod on in the independence generated by the individual confidence of self-support, and who holding these views enter their protests against State interference.

In a recent discussion which took place in this hall, I ventured to express an opinion that self-help and thrift would continue to do more than any scheme which could be devised either in this room or in the higher tribunal of the House of Commons, and that history would be likely to repeat itself, in that these discussions, however interesting, would be still within the range and not outside the range of academic discussions as to what would be best for the aged indigent members of the British public.

Subsequent events have still further confirmed me in that view. I am far from saying or thinking that nothing will be done, or that some progress will not be made, but the final solution of the main problem of what is best to be done for its accomplishment will remain over for a period about midway between the Greek kalends and the millennium—no scheme, however well ordered, will, until that period be attained, ever be so far-seeing as to provide for all the mutations of suffering humanity, and it will be well for this country when it re-opens its eyes to the fact that it is already provided with all the machinery of well-devised Poor Laws, and that some of the superfluous energy and talent which are now running into the vortex of pension schemes, might, with propriety, be better employed in seeing to the proper administration of those Laws, and to an enlightened revision thereof when found to be defective or inapplicable to modern requirements.* Personally, I believe more

* Since these remarks were in type, it has been announced that a Royal Commission will be appointed shortly to consider whether any alterations in the system of Poor-Law relief are desirable "in the cases of persons whose destitution is occasioned by incapacity for work resulting from old age."

in such a reform, in the direction of help to the helpless, than in the universal applicability of these present attempts at State aid to those who are able and willing to help themselves; but it is fervently to be wished that whatever scheme may ultimately become law it will deal comprehensively with other misfortunes of humanity besides that of attaining the age of 65.

There are such things as sickness, and incapacity for work, and want, and misery, even before attaining to that mystic age. At a recent meeting of the British Association, Sir Charles Freemantle, the able President of the Section on Economic Science and Statistics, questioned doubtingly as regards the attainment of a pension, "whether a man under such circumstances would feel a glow of moral superiority and of conscious pride in his manhood", and he thought that the motto of such an one would only vary the epicurean expression "Let us eat and drink for to-morrow we die", to "Let us eat and drink for to-morrow (that is when we complete our 65th year) we shall get our pension of five shillings a week."

To this aphorism I myself would add, how will it be if I have nothing to eat or drink on the eve of that to-morrow when I shall attain the age of 65? Starving, and without a roof to cover my head, knocking at the workhouse door in search of skilly and rest, of what avail will it be to tell me through a grating that I shall have five shillings a week if, and when, I shall attain the age of 65? In that connection I trust that some modification might be made in the age to be attained; the struggling workman may find difficulty to struggle on to 65—which for a mechanic is an advanced age—without assistance.

The Old Age Relief Bill, in Denmark, fixes the age at the 60th year, at which age, when the individual is unable to provide himself and those immediately dependent upon him with the necessities of life, he may, under certain conditions, have old age relief granted to him—whilst in Russia, one of the rules for regulating the position of workmen employed in the mines belonging to the Crown qualifies him for a full old age pension when he has had 35 years of service, irrespective of age attained, and even for lesser periods of service he becomes entitled to a pension according to a graduated scale.

Following on to all that has already been uttered and written by statesmen and statists on this subject, it may still be open to some fortunate member of this Institute to devise a scheme embracing all that is good, and discarding all that is evil in the

attempt to supplement self aid and thrift by Government assistance; but to me, from this chair, and on the present occasion, it devolves only to offer to you these few observations on what may be termed the great social question of the day, and I will only further advert to the fact that in our capacity of actuaries, our interest lies not so much in devising schemes as in counting their cost—and in that connection we are all under great obligation to our learned coadjutor, Mr. R. P. Hardy, for his paper “On the Formulæ for determining the “Value of Benefits according to the Principle of Collective “Assurance” (*J.I.A.*, xxx, 79), a contribution which will be of much help to us in such calculations.

Whilst, however, we may be usefully and scientifically employed in counting the cost, we ought never to dismiss from our minds that, however great may be the obligation entailed on the country by such onerous engagements, it is not primarily a question of having a sufficient reserve in hand, as it would be if undertaken by mutual or joint-stock enterprise. On the contrary, it is a question of taxation, in which a strong Government of a wealthy country can enforce a true assessment, instead of the puny assessmentism of those joint-stock companies or mutual societies, which, aping assessment, have no power to assess.

Although a reserve fund, in the ordinary acceptance of that term, would not be required by the Government, I am fully alive to the fact that however rich and prosperous this country is, it behoves its legislators to reckon up the cost of the sacrifice and not incur too heavy a burden, for State aid will not remove the poor from us, and these new obligations will come as an additional tax to that required for the administration of the present Poor Laws, as it will be merely aid and subsidy to those who at present, however inadequately, are already self-aiding, and already mainly independent of parish relief, and will therefore not appreciably diminish existing charges in that respect.

Therefore there should be the greatest care exercised in legislating, so that the new obligations do not fall too heavily on those classes of the public who deserve as well of their country by working with their heads, as do those who work with their hands, for their own support and for the commonwealth.

Returning now to our own avocation, I would remind you that it was brought prominently to our attention some years ago, by one of our leading authorities, that we were *traders*. If there could have been any doubt on that point in those pristine days, all such lingering feeling must have been since dispelled, for there are daily and increasingly manifest signs that we are following other traders in the means adopted to catch on to the public, so as to dispose of our goods. Thus, we see offered "double benefits", for which double premiums have to be paid for the double enjoyment to be derived therefrom, and again, instead of granting good old "deferred annuities", the public are invited to make their old age easy by securing "perfected pensions." Some of us, too, are good enough to give them the goods they purchase at "prime cost" or at "cost price", that is to say, the goods are delivered for just what they cost the public. Then there are others of us who profess to part with our wares "under cost price." These modes of expressing ourselves may be all very right and proper and technical in the language of actuaries, but in our capacity of traders, in our legitimate occupation of catching-on to supply a public want, it may occur to the public that our language should be that understood of the people. If a trader, having no other means, vends his goods at cost price, or at prime cost, he may find himself incontinently enjoying his *otium sine dignitate* in a house with no work to do, and at the cost of an ungrateful country; or, if bent on disposing of his wares "under cost price", then, by one of those strange evolutions of bimetallism, his sovereign becomes evolved into an exchangeable commodity at a ratio of less than twenty shillings in the pound.

These and sundry other modern modes of obtaining an influx of business of varied kinds may be good, bad or indifferent, but in its grand simplicity commend me to the old-fashioned whole-term-of-life policy in all its unselfishness and self-denial; and I always derive comfort from a perusal of the Board of Trade Returns, showing that although other just modes of thrift, such as providing for self-comfort in old age, are gaining ground, the greater proportion of the insuring public still tread the path of provision for their families, as it did in the days of our early teaching, by effecting policies payable at death unsubjected to the chance run by the recipients of endowment assurances, who may lose in their own lifetime, by injudicious investment, the provision which they intended for their surviving dependents.

There is nothing derogatory in our trading propensities or in being honest traders, but there is also the higher and brighter side of our vocation, and which entitles us, as actuaries, to take our rank as a profession. Trained by education to a mathematical and practical knowledge of our science, we claim to take rank with other bodies larger in number, and more prominently before the public than ourselves, but not more zealous than we are to carry out the respective work of our professions for the welfare of our fellow men. To us is confided the responsibility of seeing to the safety of the principles upon which are based the calculations on which life assurance is conducted, and to making the necessary calculations with mathematical precision. When we reflect that upon the truth of those principles, and upon the accuracy of those calculations, there is dependent the very stability of the companies which employ us, as well as the bread of the widow and the orphan, it is a responsibility which we assume with no lightness of heart, except that arising from the consciousness of faithfully using our technical knowledge to safeguarding the great interests confided to us.

Let us also take pride in the still higher and still brighter mission of the manager or actuary, or other official or agent—that of preaching the gospel of life assurance. I make use of this expression in all humility, but in what consists the gospel? Does it not consist in doing one's duty to God and towards our neighbour? And I ask what duty to one's neighbour is more unselfish or more self-denying than to make a provision for the helpless, by life assurance, and thus to snatch the widow and the fatherless from the cold mercies of the world.

And now before I conclude these personal views which I have ventured to enunciate on this, the first occasion, of my occupying the chair of our general meeting, I would say a few words of encouragement to my younger friends whom I see around me, students, associates and also fellow-actuaries, though unattached in that capacity to their respective companies.

In the old days gone by it was held by some of our profession that this Institute, however well meaning, was on a wrong path in educating too large a number of its members to the point at which they became actuaries by examination, for it was foreseen with that mathematical precision inherent to our craft, that there would in course of time be more actuaries than there are institutions to confer appointments of that standing upon them. This safe and certain prophecy has long since been fulfilled, but fulfilled only as

a self-evident fact, and as a fact which does not impugn the wisdom of the originators of the Institute, nor shake the confidence of their successors in the immense benefit which has been, and will be conferred on its members who by study, become worthy of its fellowship, and who by judicious education are perpetuating the best traditions of the Institute, so as to transmit all that is good and worthy of preservation to succeeding generations.

The educated mathematical mind of the unbeneficed actuary will not fail to grasp the truth that, in his temporary condition of hope deferred, there need not be sickness of heart, for his profession does not stand alone in the condition of expectancy as a fore-runner of success. How many curates are fit to be vicars?—how many vicars are qualified to be bishops? And yet we cannot all be bishops amongst actuaries. Such positions of dignity, coupled with the cares of responsibility, are attained by patient waiting, by perseverance, by opportunity, but above all by talent and merit, and the brighter day may be at hand at any moment for the qualified actuary, to whom the success which he has deserved, is slowly, but I trust surely, on its way.

In conclusion, I would say that I am now at the outset of the one or two years prescribed by our rules, for me to occupy the presidential chair. My earnest endeavour will be so to conduct our deliberations and discussions, that they shall not deteriorate from the high standard to which they have attained under the auspicious leadership of my predecessors in office.

On Family Annuities. By GEORGE KING, F.I.A., F.F.A., one of the Honorary Secretaries of the Institute of Actuaries, and Actuary of the Atlas Assurance Company.

[Read before the Institute, 19 December 1892.]

A VERY important and difficult element in some of the State insurance schemes which have lately been put forward is, the provision of a small annuity to each child of a deceased father until that child attains the age of, say, 12 or 14 years. These may, not inappropriately, be called “family annuities”, and the object of the present paper is to investigate how the premiums for such family annuities may be calculated.

The factors in the calculation are (1) the probability that at the moment of death of a male he is a “married man”, that is, a husband or widower; (2) the value at the moment of death of a

married man of an annuity to each of his children; (3) the probability of death at a given moment of age; and (4) the rate of interest. If by B_x we represent the single premium to secure the family annuity, by $(pm)_{x+t}$ the probability that at the moment of age $x+t$ a male will be a married man, and by $(fa)_{x+t}$ the value at age $x+t$, the moment of death of a married man, of an annuity to each of his children, we shall have

$$B_x = \frac{1}{l_x} \int_0^x v^t \cdot l_{x+t} \cdot \mu_{x+t} \cdot (pm)_{x+t} \cdot (fa)_{x+t} \cdot dt,$$

because the factors $\frac{l_{x+t} \mu_{x+t} dt}{l_x}$ give the probability that a man now aged x will die at the precise moment of age $x+t$, and $(pm)_{x+t}$ is the probability that at that moment he will be a married man, while $(fa)_{x+t}$ is the value at that moment of the family annuity, and v^t discounts that value down to the present time. If the annuity is to become payable only if the father die before, say, age 65, then age 65 must be taken as the upper limit of the integral.

The benefit in question is here put in the form of an integral, because it was by means of that form that I first arrived at the way to find the value; but it will presently appear that there is no need to resort to the calculus. The calculations take upon themselves a very simple character, and can readily be thrown into the form of commutation columns.

The integral shows that, in order to value the benefit, we must have means of assigning values to $(pm)_x$ and $(fa)_x$, and therefore this paper divides itself naturally into three parts, namely, A, an investigation into the matrimonial condition of male adults; B, an investigation into the orphanhood of children; and C, methods for the numerical calculation of the values of benefits depending on A and B.

A.—MATRIMONIAL CONDITION OF MALE ADULTS.

The detailed Census Report for 1891 is not yet available, and recourse must be had to that for 1881, the information contained in which is nearly 12 years old. An examination of the censuses of the United Kingdom, which have been taken decennially during the present century, renders it at once apparent that the condition of the community alters materially from decade to decade, and

the first of the possible improvements upon existing arrangements that forces itself into prominence is that of more frequent enumerations. Probably a quinquennial census would for some time to come serve every necessary purpose, but, certainly, the intervals should not be of greater length than five years.

On page v of vol. iii of the *Census of England and Wales*, 1881, there appears a table giving for certain age intervals the number of bachelors at that time living, the number of husbands, and the number of widowers respectively. The following are the figures :

TABLE A.
England and Wales. Census 1881.

Ages	Bachelors	Husbands	Widowers
0-15	4,728,466
15-20	1,262,311	5,860	98
20-25	864,402	245,166	2,186
25-35	577,349	1,217,714	26,174
35-45	195,427	1,170,779	51,689
45-55	99,498	859,840	73,928
55-65	59,865	562,693	99,995
65 & over	40,892	314,546	180,124
Total	7,828,210	4,376,898	434,794

The Registrar-General deals with age intervals and not with each individual age; and were he to tabulate the numbers actually enumerated at each age, the information would not in itself be of much additional value; but it would be well that the figures for each age should be published, so that in any particular enquiry those groupings might be selected which would best serve the purposes in hand.

In Table A above, the number of bachelors, of husbands, and of widowers, respectively, living in England and Wales in 1881, is given for certain age intervals; and by Milne's graphical process these numbers might be distributed over all the ages of life. But it will be more convenient to have not the actual, but the relative, number in each class in each year of age, because the object more particularly in view is, to ascertain the probability of an adult male being at the time of his death a bachelor, a husband, or a widower. In Table B, therefore, the facts given in Table A are repeated in modified form; the numbers in Table B

showing how many bachelors, husbands, and widowers, respectively, there would be in a population of 1,000 males living in each of the age intervals.

TABLE B.

England and Wales. Census 1881.—Number of Bachelors, Husbands, and Widowers, respectively, per Thousand of Male population in each Age Interval.

Ages	Bachelors	Husbands	Widowers
0-15	1,000
15-20	995	5	...
20-25	777	221	2
25-35	317	669	14
35-45	138	826	36
45-55	96	832	72
55-65	83	779	138
65 & over	76	587	337

From the numbers in the age intervals in Table B, it is desired to find how many bachelors, husbands, and widowers, respectively, there would be in a population of 1,000 males living in each year of age. The process is displayed in the three following diagrams. It is adapted from that pursued by Milne in constructing the Carlisle Table; but, as it is not absolutely identical, no harm will be done by explaining it somewhat fully.

Taking the curve for bachelors by way of illustration—the age intervals 15-20 and 20-25 are set out on cross-ruled paper in equal sections along the abscissa axis, and the intervals 25-35, 35-45, &c., in sections of double length. Rectangles are then set up on these sections, of the respective altitudes shown by the numbers in Table B. Thus, the altitude of the rectangle for the age interval 15-20 is 995, while that for the age interval 35-45 is 138. The areas of these rectangles then represent the number of bachelors in a population of 1,000 males living in each year of age in each of the respective age intervals. Thus, the area of the rectangle for the age interval 15-20 is 4,975, being the number of bachelors in a population of 5,000 males living between ages 15 and 20, of whom 1,000 are in the year of age 15 to 16, 1,000 in the year of age 16 to 17, and so on; and the area of the rectangle for age interval 35-45 is 1,380, being the number of bachelors in a population of 10,000 males living between ages 35 and 45, of whom 1,000 are in the year of age 35 to 36, 1,000 in the year of age 36 to 37, and so on.

By drawing a continuous curve through the tops of the rectangles in such a way that the area of the curve is in each case equal to the area of the corresponding rectangle, and by reading the lengths of the ordinates of the curve at the points representing the middle of each year of age, we immediately obtain the number of bachelors in a population of 1,000 males living in each year of age.

As a mechanical expedient it is convenient to use two, or some other even number, of the ruled spaces for each year of age, so as to be able to read accurately the ordinate which is central to the year.

For the distribution thus effected to be satisfactory, two conditions must be fulfilled. The sum of the numbers at the several ages comprised in any particular age interval must equal the area of the corresponding rectangle, and the distributed numbers throughout the table must succeed each other in regular sequence, the differences, which follow some unknown law, not showing any violent irregularities; for it may be assumed that, while the number of bachelors at, for instance, age 25 differs much from that at age 45, yet the latter is connected with the former by regular gradations.

It sometimes happens that, owing to fractions being discarded, it is not possible absolutely to reconcile these two conditions, and it becomes unavoidable to alter very slightly the sums of the distributed numbers from the number in the corresponding age interval; but such alterations should be exceedingly small, and should be so made that, as far as possible, in each pair of adjacent age intervals they may neutralize each other.

A similar process having been employed to distribute the number of husbands and of widowers, it should be noted that, by comparing the three sets of distributed numbers, a complete check on the accuracy of the drawing of the curves, and on the correctness of the reading off of the ordinates, is obtained, because the sum of the numbers in the three columns must in every case be exactly 1,000. The curves are given in diagrams Nos. 1 to 3.

In Table C are given the numbers thus distributed for bachelors, for husbands, and for widowers, respectively; and a column has been added for "married men", being the sum of the husbands and widowers. In this sense the term "married men" is used throughout this paper.

TABLE C.

England and Wales. Census 1881.—Number of Bachelors, Husbands, and Widowers, respectively, per Thousand Males living in each year of Age; also the number of Married Men.

Age next Birthday	Bachelors	Husbands	Widowers	Married Men being Husbands and Widowers
15	1,000	0	0	0
16	1,000	0	0	0
17	1,000	0	0	0
18	999	1	0	1
19	996	4	0	4
20	980	20	0	20
21	950	50	0	50
22	897	102	1	103
23	776	222	2	224
24	673	324	3	327
25	584	412	4	416
26	509	485	6	491
27	449	544	7	551
28	397	594	9	603
29	352	637	11	648
30	313	675	12	687
31	279	707	14	721
32	250	734	16	750
33	226	756	18	774
34	207	773	20	793
35	192	786	22	808
36	180	796	24	820
37	168	805	27	832
38	156	815	29	844
39	146	823	31	854
40	138	828	34	862
41	130	833	37	870
42	123	837	40	877
43	118	839	43	882
44	114	841	45	886
45	111	841	48	889
46	108	841	51	892
47	105	840	55	895
48	102	839	59	898
49	99	837	64	901
50	96	835	69	904
51	93	833	74	907
52	91	830	79	909
53	90	826	84	910
54	89	822	89	911
55	88	817	95	912
56	87	811	102	913
57	86	805	109	914
58	85	799	116	915
59	84	793	123	916
60	83	787	130	917
61	82	779	139	918
62	81	770	149	919
63	80	760	160	920
64	79	749	172	921
65	78	737	185	922

The bachelors commence at the highest possible number, namely 1,000, but the number is reduced steadily, though after about age 25 in diminishing ratio, by marriages. At the older ages very few marriages of bachelors take place, and at last, after about age 70, the number of bachelors per thousand of male population at each age is practically constant.

Below age 20 there are very few husbands, but after that age they rapidly increase in number, until a maximum is reached at about age 45. The number then, at first slowly, but presently rapidly, diminishes, and probably at about age 85 there would be very few, if any, left; but the statistics as given by the Registrar-General do not permit of this point being elucidated. After about age 85, the population of 1,000 males living at each age would consist of about 70 bachelors and 930 widowers. The class of husbands is recruited by the marriages of bachelors and the re-marriages of widowers, and is depleted by the deaths of wives.

There are very few widowers at the younger ages, but the number steadily increases, and it will reach its maximum of about 930 at about age 85, at which figure it will thereafter remain constant. This class is recruited by the deaths of wives, and is depleted by the re-marriages of widowers.

The Registrar-General ceases to tabulate the numbers in intervals above age 65, and simply gives the number for "65 and upwards." It is unfortunate that he has not given the subdivisions 65-75, 75-85, 85-95, and 95 and upwards; and it is to be hoped that this omission will be rectified in his reports of future censuses. Perhaps it is now too late to suggest this improvement for the census of 1891. While there are sufficient indications to show what the curve for bachelors would be above age 65, it is impossible under present circumstances to come to any sound conclusions about the curves for husbands and widowers.

It has been shown by several investigations into the mortality prevailing among different restricted classes of the community (see, for instance, Mr. Sprague's paper on combined Marriage and Mortality Tables, *J.I.A.*, xxi, 406), that the rate of mortality among husbands is very different from that among bachelors, and also from that among widowers—that among husbands being, as a rule, much the lightest. It does not follow that marriage produces a favourable effect upon health, although to a certain extent that may be the case; but it is more probable that a certain selection takes place at marriage, the healthy man being more likely to

marry than the unhealthy. Whatever the explanation of the fact may be, the difference is so marked that it is important to ascertain for the general community the rates of mortality among bachelors, husbands, and widowers, separately; but the Registrar-General's annual reports do not enable us to do so. In the census returns the numbers of bachelors, of husbands, and of widowers, are, as we have seen, given, although not in sufficient detail; but in the annual returns of deaths no distinction of matrimonial state is made, and I would suggest that this deficiency should be remedied. In the Registrar-General's annual returns there should be given, not only the number of deaths in each age interval, but these should be subdivided into the deaths of bachelors, of husbands, and of widowers, respectively. Similarly for females. For want of this information I have been obliged to assume that the average rates of mortality in the community prevail irrespective of matrimonial state.

While Table C shows the numbers per thousand in the population, of bachelors, of husbands, and of widowers, in each year of age, it also gives, on the above assumption as to mortality, the probability of a man at the moment of death being a bachelor, a husband, or a widower. Thus, in the year of age 34-35 there are among each 1,000 men living, 192 bachelors, 786 husbands, and 22 widowers. Therefore, the probability of a man who dies in that year of age being a bachelor is .192; of his being a husband .786; or of his being a widower .022. It is in this way that Table C will be used in the further investigations of this paper.

B.—ORPHANHOOD OF CHILDREN.

The statistics collected in the United Kingdom by the Registrar-General are utterly silent as to the orphanhood of children. At the decennial censuses, no record is taken as to whether or not the children enumerated have their fathers or their mothers alive; and when the death of a man is registered, no question is asked as to the children he may have left behind him. The absence of this information is a bar to the investigation of many problems, important from both the social and the economic standpoints. More particularly in the National Pension schemes which have recently engaged so much attention, statistics of orphanhood have been urgently required, and have not been forthcoming. The wildest guesses have had to be made as to the numbers and ages of children left orphans by the deaths of their fathers, and

the financial superstructure, which has been built upon these guesses in the absence of a surer foundation, is of very doubtful stability. The estimates may be near the truth, or they may err widely either in excess or defect; and if the Legislature were practically considering any pension schemes, there would be no means, so far as statistics of the United Kingdom are available, of arriving certainly at even an approximate idea of their cost. Some of the Australasian Colonies, however, are far ahead of the mother country in this respect. When Mr. F. W. Frankland was Government Actuary of the Colony of New Zealand, he perceived the immense importance of trustworthy information, and he prevailed upon the Cabinet to introduce a measure to the Legislature, which was passed, imposing the duty upon the registrars of ascertaining and recording the numbers, sex, and ages of children left by deceased males. Through the courtesy of Mr. Ballance, the present Colonial Premier, and of Mr. Brown, the Colonial Registrar-General, I have been furnished with the statistics of the Colony for the year 1890, and I have been informed that similar figures could be obtained at small cost for each year since the enactment came into operation. I believe that certain others of the Australasian Colonies have followed suit, and there is therefore in process of collection at the Antipodes, a vast body of most valuable information which some day may be turned to good account.

In Table D will be found the New Zealand figures for 1890, exactly as furnished, excepting that the male and female children have been grouped together, as for the present purpose there is no need to distinguish the sexes, and the paucity of the facts renders such a distinction unadvisable. It will be seen that in column 2 are the total number of male deaths in each year of age from 20 to 65. Column 3 shows the number of married men, including both husbands and widowers, who died childless, and column 4 the number who died leaving children. In the succeeding columns the numbers of children in each year of age are shown. There are a few children whose ages are not specified, but as these occur for the most part where the majority of the children are somewhat advanced in age, it is probable that they were mostly of 15 years or over. In any case, the number with ages unspecified is not sufficient to affect seriously the use of the figures. The New Zealand returns do not distinguish husbands from widowers, and are to that extent defective. Also they do not show the number of children in each family.

TABLE D.—RETURNS OF THE REGISTRAR-GENERAL OF NEW ZEALAND.

Table showing, for the Year 1890, the Total Number of Males who Died at each Year of Age from 20 to 65; the Number of Married Males who Died (a) leaving no Children; and (b) leaving Children; and the Number and Ages of the Children living at the time of the Father's Death.

Age at Death	Total No. of Male Deaths	NO. OF MARRIED MALES WHO DIED		No. of CHILDREN, AND AGES																							Total
		Leaving no Children	Leaving Children	0-	1-	2-	3-	4-	5-	6-	7-	8-	9-	10-	11-	12-	13-	14-	15-	16-	17-	18-	19-	20-	21 & over	Ages not specified	
20-	34
21-	38
22-	27	1
23-	30
24-	30
25-	27
26-	30
27-	26
28-	27
29-	17
30-	22
31-	19
32-	27
33-	21
34-	19
35-	36
36-	26
37-	30
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
38-	33	3	18	21	4	4	5	5	2	2	1	5	5	6	4	2	2	1	1	1	1	21	20
39-	27	5	13	1	5	4	2	3	3	3	2	5	3	7	2	4	1	3	3	2	21	54
40-	32	2	14	...	3	2	6	3	2	2	5	3	5	3	4	3	1	12	54
41-	17	2	9	...	3	3	...	3	6	2	3	2	5	5	6	1	7	3	3	2	1	2	1	11	32
42-	35	4	20	3	3	3	...	3	3	6	5	3	3	3	5	3	3	3	3	1	4	2	1	73
43-	26	...	15	3	2	3	3	3	3	4	6	2	5	3	5	3	6	3	3	1	3	4	10	80
44-	27	5	20	3	4	4	4	3	3	4	4	3	5	3	5	3	6	3	3	2	2	2	6	76
45-	51	5	6	97
46-	37	4	20	3	3	3	6	4	3	4	4	5	4	5	4	3	4	3	3	2	2	5	3	3	13	10	92
47-	30	3	19	3	3	3	4	5	3	5	3	3	3	7	7	2	3	3	3	2	4	5	6	2	13	12	116
48-	40	4	22	3	3	3	100
49-	40	5	23	141
50-	69	2	23	239
51-	37	5	25	1	3	3	160
52-	43	4	16	1	3	3	3	3	3	3	6	1	2	6	4	8	3	5	5	3	7	5	6	10	39	30	130
53-	29	6	...	1	21
54-	42	4	28	34
55-	61	5	23	5
56-	53	4	31	36
57-	50	7	25	7
58-	46	7	21	127
59-	35	4	18	154
60-	71	3	29	21
61-	39	1	26	17
62-	41	5	26	4
63-	46	4	27	41
64-	27	2	21	32
65-	27	9
66-	3
67-	21
68-	112
69-	3
70-	71
71-	21
72-
Total	1,573	129	690	57	69	91	89	95	87	110	103	110	110	126	107	117	126	119	109	83	112	101	121	106	868	412	3,328

As might have been expected from the paucity of adult females in the Colony, the proportion of bachelors to married men is much larger than at home; but I do not think that a great error will be committed, if it be assumed that the number of children to each deceased married man is much the same in New Zealand as in Great Britain. By combining the New Zealand statistics of children with the ratios of Table C, and with the home rates of mortality for adults, a good approximation can be made towards the measure of orphanhood in this country. At all events, the results will be far more trustworthy than those founded upon mere surmise, and if it should be found that the proportionate number of children to each marriage in the United Kingdom is different from that in the Colony, it will be easy to make suitable corrections.

C.—VALUATION OF BENEFITS.

When the annuity of any particular child falls into possession by the death of the father, its value will be simply that of a continuous term annuity ceasing at the given age: and for purposes of reference the values of such annuities by the English Life Table No. 3, at $2\frac{1}{2}$ and 3 per-cent interest, and up to ages 12 and 14 respectively, are given in the following Table E. They are obtained from the formula—

$$\begin{aligned} \bar{a}_{x+\frac{1}{2}:n-x-\frac{1}{2}} &= \frac{1}{2} \{ a_{x+\frac{1}{2}:n-x-\frac{1}{2}} + 1 + a_{x+\frac{1}{2}:n-x-1\frac{1}{2}} \} \\ &= \frac{1}{2} + \frac{1}{4} \{ a_{x:n-x} + a_{x+1:n-x-1} + a_{x:n-x-1} + a_{x+1:n-x-2\frac{1}{2}} \}. \\ \text{or } &= \frac{1}{2} \left(a_{x:n-x} + 1 + a_{x+1:n-x-1} - \frac{D_n}{D_{x+\frac{1}{2}}} \right). \end{aligned}$$

TABLE E.
Value of a Continuous Temporary Annuity.

Year of Age	To AGE 12		To AGE 14	
	$2\frac{1}{2}$ per-cent	3 per-cent	$2\frac{1}{2}$ per-cent	3 per-cent
0-1	8.110	7.906	9.216	8.947
1-2	8.234	8.041	9.500	9.238
2-3	7.842	7.674	9.206	8.970
3-4	7.258	7.117	8.699	8.493
4-5	6.576	6.462	8.084	7.910
5-6	5.827	5.739	7.398	7.254
6-7	5.028	4.962	6.657	6.542
7-8	4.188	4.143	5.875	5.785
8-9	3.312	3.284	5.056	4.990
9-10	2.403	2.388	4.203	4.158
10-11	1.463	1.457	3.320	3.292
11-12	.493	.491	2.406	2.391
12-13	1.464	1.458
13-14493	.492

The value at the death of a man of the family annuity then falling into possession, will be the sum of the values of the annuities to all his children; and the average values of family annuities might be found for each age at death of married men, by multiplying the average numbers of children in each year of their age left by married men, into the corresponding annuity-values, and taking the sum. This would be a satisfactory method were the statistics so extensive as to produce regularity in the results, or were they sufficient to permit of their being themselves successfully graduated. The graduation would, however, require to be effected on a figure of three dimensions, and would be very difficult; and, with the scanty data which alone are available, it could not with advantage be adopted. In preparing the following Table F, therefore, a different plan was followed. For each age at death of married men, the number of children in each year of age was multiplied into the corresponding value of the temporary annuity given in Table E, and the sum taken and divided by the number of married men dying. The results were then plotted out in a curve on cross-ruled paper, but proved to be so irregular that very little indication was afforded of the real form of the curve, and successful graduation, even by a graphical process, was scarcely possible. As a second step, therefore, the original observations were gathered into quinary groups, so that for any particular age at death of married men, the statistics of five ages (namely, the given age and two on each side of it) were collected. The grouped numbers of married men and children thus obtained were then dealt with as before. This having been done for each age at death of married men, these results were also plotted out in a curve on the same sheet of paper as those derived from the single ages of married men. With the guidance of this second curve a continuous curve was then drawn, which, on being read off, was found to be very satisfactory, and which required only slight adjustment in order to make the sum of all the values included in it exactly equal to the sum of the ungraduated results. The final figures are given in Table F. A separate graduation had to be effected for each annuity term, and for each rate of interest. As an illustration of the process I submit diagram No. 4, being that for annuities to cease at age 14, at $2\frac{1}{2}$ per-cent interest.

TABLE F.

Value at the moment of Death of a Married Man of a Family Annuity of 1, payable in respect of each Child whom he may leave.

Age next birthday (at Death) of Married Man	To AGE 12		To AGE 14	
	2½ per-cent	3 per-cent	2½ per-cent	3 per-cent
21	7.43	7.25	8.56	8.33
22	7.48	7.29	8.62	8.38
23	7.75	7.55	8.93	8.68
24	8.13	7.93	9.43	9.17
25	8.62	8.42	10.06	9.79
26	9.17	8.97	10.77	10.49
27	9.75	9.55	11.52	11.23
28	10.38	10.17	12.34	12.03
29	11.12	10.90	13.30	12.97
30	12.14	11.90	14.60	14.25
31	13.51	13.25	16.44	16.07
32	14.97	14.70	18.59	18.20
33	15.61	15.33	19.71	19.30
34	15.85	15.56	20.03	19.60
35	15.61	15.33	19.89	19.48
36	15.07	14.80	19.39	19.00
37	14.51	14.25	18.74	18.37
38	13.92	13.68	18.08	17.73
39	13.30	13.08	17.41	17.08
40	12.65	12.45	16.73	16.42
41	11.99	11.80	16.04	15.75
42	11.31	11.13	15.34	15.07
43	10.63	10.46	14.63	14.38
44	9.95	9.79	13.92	13.68
45	9.29	9.14	13.21	12.98
46	8.64	8.50	12.50	12.28
47	8.03	7.90	11.79	11.58
48	7.46	7.34	11.08	10.88
49	6.92	6.81	10.37	10.18
50	6.40	6.30	9.66	9.48
51	5.90	5.81	8.95	8.78
52	5.41	5.33	8.24	8.08
53	4.93	4.86	7.52	7.38
54	4.47	4.40	6.80	6.68
55	4.01	3.95	6.09	5.98
56	3.56	3.51	5.39	5.29
57	3.12	3.08	4.72	4.63
58	2.70	2.66	4.08	4.00
59	2.29	2.25	3.48	3.41
60	1.88	1.85	2.92	2.86
61	1.49	1.46	2.39	2.34
62	1.10	1.08	1.88	1.84
63	.73	.71	1.39	1.36
64	.36	.35	.92	.89
6545	.43

The first question that is met with in connection with family annuities in State insurance schemes is, what sum must be paid in respect of each male when he reaches a given age, say 25, in order to provide a family annuity should he die before reaching the age of, say, 65. From Farr's English Life Table No. 3, combined with the probabilities of marriage given in Table C above, and with the New Zealand statistics of children in Table D, such an isolated value of the benefit can be calculated with great facility by means of formulas for the approximate evaluation of integrals; but the operation is much shortened by the help of such a table as is given in Table F. The following is an example worked by means of the formula of summation No. 39a, *Text-Book*, Part II, p. 188; namely,

$$\int_0^{\infty} u_x dx = n(28u_0 + 1.62u_n + 2.2u_{3n} + 1.62u_{5n} + .56u_{6n} + 1.62u_{7n}).$$

The age of the adult is taken as 25; the annuity to children is supposed to cease at 14; and interest is assumed at 3 per-cent. Instead of $v^t l_{x+t}$, the function $D_{x+t} = v^{x+t} l_{x+t}$ has been employed, because that function has been tabulated by Dr. Farr. Similarly, D_x has been used for l_x in the denominator.

Dr. Farr has not tabulated the force of mortality, but an approximation sufficiently close for the present purposes is obtained from the usual formula $\mu_x = \frac{l_{x-1} - l_{x+1}}{2l_x}$.

The values of the various factors under the sign of integration must be those for the precise age $x+t$, and not those for age next birthday as in the tables. Therefore $(pm)_{x+t}$ must be the mean of the values for ages $x+t$ and $x+t+1$ in Table C; and, similarly, for $(fa)_{x+t}$ in Table F.

The age of the life being 25, and $(fa)_x$ according to the New Zealand statistics permanently vanishing at age 65, $7n$ must be taken equal to 40, and n is approximately equal to 6. The value brought out by the formula of summation is 2.546, while the true value is 2.557.

t	$\log D_{x+t}$	$\log \mu_{x+t}$	$\log (pm)_{x+t}$	$\log (fa)_{x+2}$	$\log \text{Coef.}$	$\text{Colog } D_x$
0	5.1835	3.9596	1.6566	1.0060	1.4472	6.8165
6	5.0813	2.0099	1.8666	1.2340	0.2095	6.8165
18	4.8651	2.1519	1.9465	1.1470	0.3424	6.8165
30	4.6152	2.3845	1.9602	0.7513	0.2095	6.8165
36	4.4635	2.5333	1.9630	0.3201	1.7482	6.8165
	<u>24.2086</u>	<u>9.0392</u>	<u>1.3929</u>	<u>4.4584</u>	<u>1.9568</u>	<u>26.0825</u>

t	Σ	Antilog
0	2.0694	.0117
6	1.2178	.1651
18	1.2694	.1860
30	2.7372	.0546
36	3.8446	.0070
	<u>7.1384</u>	<u>.4244</u>
		6

Approximate value = 2.5464

The form of the integral, given on p. 292, suggests commutation columns, which may be used in combination with the usual commutation columns of the life table, for the purpose of giving numerical solutions of many interesting questions on the subject of family annuities.

In Table G such columns are given at $2\frac{1}{2}$ per-cent and 3 per-cent interest, and for family annuities ceasing at 12 and at 14 respectively; and also the values of B_x derived therefrom, being the values during the lifetime of a male of a family annuity, to be entered on by his children at his death.

The column D_x is the usual column under that symbol. The column F_x is constructed from the formula

$$F_x = i^{x+\frac{1}{2}}(l_x - l_{x+1})(fa)_{x+1}(pm)_{x+1}.$$

so that $F_{x+n} \div D_x$ is the value at age x of a family annuity to be entered on only if the life fail in the year of age $x+n$ to $x+n+1$. The column G_x is formed by summing F_x from the bottom of the table upwards; so that $G_x = F_x + F_{x+1} + F_{x+2} + \&c.$, and therefore the value of a family annuity to be entered on if the life fail at any time after age x is $B_x = G_x \div D_x$.

The column G_x renders simple the calculation of the values of temporary or deferred benefits, because $B_{x:n} = (G_x - G_{x+n}) \div D_x$ and ${}_nB_x = G_{x+n} \div D_x$. Moreover, if the N_x column of the life table were used, the annual or other periodical equivalents of the single premiums so far discussed could be at once obtained. We should have, when the annual premium is to be paid throughout life, $PB_x = G_x \div N_{x-1}$; or if the annual premium is to be paid for n years only, ${}_nPB_x = G_x \div (N_{x-1} - N_{x+n-1})$.

It may be well to enter the caution that, as Farr uses N_x of the initial form, when his tables are employed we must in the above formulas write N_x for N_{x-1} and N_{x+n} for N_{x+n-1} .

The foregoing investigations and tables apply only when the whole population is considered, or when at the initial age, x , a fair average is taken of bachelors, husbands, and widowers. The processes and statistics may, however, be used with caution under other conditions. For instance, if a man at age x is known to be married, then, in the integral, the factor $(pm)_{x+t}$ becomes unity. In this case we must, in the absence of data, assume that the value of $(fa)_{x+t}$ is unaltered. On account of the marriage having taken place earlier than the average, as is implied in the fact of the man being already married, it is probable that at age $x+t$ there will be more children than the average, and that therefore the average value of $(fa)_{x+t}$ will be too small; but this need not necessarily be so, because the probability of the man becoming a widower before age $x+t$ is increased, and also because, on account of the earlier marriage, the children at his death will be on the average older, and the value of the particular annuity of each of them will therefore be less.

From this it appears that the question of the birth-rate will not have so much effect on the values of family annuities as might be supposed. It is true that if a fall in the birth-rate should arise solely from self restraint on the part of married couples, there will be fewer young children at the death of the father, and the value of $(fa)_x$ will be less. But prudential motives lead to later marriages as well as to fewer births; and therefore those children that are born will be younger at the deaths of their fathers than would otherwise be the case, and the values of their particular annuities will consequently be greater. This will tend to increase the value of $(fa)_x$. On the other hand, under these conditions the value of $(pm)_x$ will be diminished. There being thus several divergent forces involved in a fall in the birth-rate, it is impossible

to say, except after a series of laborious investigations based on a variety of hypotheses, what the resultant would be.

A subject cognate to that discussed in this paper is, the value of an annuity, either temporary or for life, payable to the widow of a deceased husband. Similar processes might be employed as for family annuities. If we assume that the wives of husbands aged x are on the average aged y , the value of an annuity to the widow will be given by the integral

$$\frac{1}{l_x} \int_0^{\infty} v^t \cdot l_{x+t} \cdot \mu_{x+t} \cdot (ph)_{x+t} \cdot \bar{a}_{y+t} \cdot dt,$$

where $(ph)_{x+t}$ is the probability that at the moment of age $x+t$ a male will be a husband, and where the upper limit may be either a fixed age or infinity.

In the Registrar-General's Annual Reports information is given, though it is incomplete, as to the ages of men and women at the time of marriage; and the table on page v of vol. iii of the *Census of England and Wales, 1881*, already referred to at the beginning of this paper, supplies particulars from which the relative ages of the husbands and wives enumerated may be derived. But, to secure accuracy, the age y in the integral must be determined from statistics giving the ages of wives at the deaths of their husbands, and I do not know where such statistics are to be found. To take the average difference between the ages of bridegrooms and brides, or of husbands and wives enumerated at a census, would understate the values of annuities to widows, because the average difference between the ages of husbands who die and the ages of their wives is greater than in the case of bridegrooms and brides, or of enumerated husbands and wives; it being evident that those husbands who are much older than their wives are the most likely to leave them widows. It is to be hoped that some day changes will be introduced into our registration arrangements which will afford the means of pursuing with success the many interesting and important enquiries which present themselves to the investigator.

TABLE G.

Commutation Columns for Family Annuities to Age 12—Interest
 $2\frac{1}{2}$ per-cent.

Age x	D_x	F_x	G_x	$B_x = G_x : D_x$
20	203592	619·0	374455	1·839
21	196979	1269·1	373836	1·898
22	190546	2824·7	372567	1·955
23	184293	4267·9	369742	2·006
24	178213	5673·0	365474	2·051
25	172302	7018·8	359801	2·088
26	166564	8248·6	352782	2·118
27	160982	9461·7	344534	2·140
28	155561	10727·	335072	2·154
29	150300	12230·	324345	2·158
30	145184	14073·	312115	2·150
31	140217	15990·	298042	2·126
32	135391	16971·	282052	2·083
33	130698	17428·	265081	2·028
34	126142	17277·	247653	1·963
35	121711	16736·	230376	1·893
36	117406	16176·	213640	1·820
37	113219	15600·	197461	1·744
38	109147	14951·	181864	1·666
39	105184	14246·	166913	1·587
40	101330	13542·	152667	1·507
41	97575	12801·	139125	1·426
42	93920	12041·	126324	1·345
43	90361	11281·	114283	1·265
44	86894	10531·	103002	1·185
45	83514	9802·3	92471	1·107
46	80221	9121·2	82669	1·031
47	77011	8489·5	73548	·955
48	73881	7891·1	65058	·881
49	70829	7314·3	57167	·807
50	67853	6758·7	49853	·735
51	64950	6378·1	43094	·664
52	62085	5838·5	36716	·591
53	59286	5308·4	30878	·521
54	56551	4772·9	25569	·452
55	53883	4246·6	20796	·386
56	51278	3731·8	16550	·323
57	48735	3241·8	12818	·263
58	46251	2762·7	9576·1	·207
59	43821	2281·8	6813·4	·155
60	41446	1820·6	4531·6	·109
61	39119	1354·6	2711·0	·069
62	36842	906·1	1356·4	·037
63	34611	450·3	450·3	·013
64	32425
65	30284

TABLE G—continued.

Commutation Columns for Family Annuities to Age 12—Interest 3 per-cent.

Age x	D_x	F_x	G_x	$B_x = G_x \div D_x$
20	184711	546.6	304822	1.650
21	177845	1114.0	304276	1.711
22	171203	2466.4	303162	1.771
23	164780	3713.0	300695	1.825
24	158570	4918.6	296982	1.873
25	152567	6064.4	292064	1.914
26	146767	7101.8	285999	1.949
27	141162	8109.2	278897	1.976
28	135748	9153.1	270788	1.995
29	130517	10385.	261635	2.005
30	125464	11898.	251250	2.003
31	120583	13470.	239352	1.985
32	115867	14229.	225882	1.950
33	111310	14536.	211653	1.901
34	106907	14345.	197117	1.844
35	102653	13828.	182772	1.781
36	98540	13301.	168944	1.714
37	94561	12774.	155643	1.646
38	90720	12192.	142869	1.575
39	87092	11560.	130677	1.502
40	83406	10943.	119108	1.428
41	79926	10294.	108165	1.353
42	76559	9634.7	97871	1.278
43	73300	8981.7	88236	1.204
44	70145	8343.3	79255	1.130
45	67090	7728.2	70911	1.057
46	64132	7156.2	63183	.985
47	61267	6629.2	56027	.914
48	58491	6133.1	49398	.845
49	55803	5658.7	43265	.775
50	53198	5205.5	37606	.707
51	50676	4800.8	32400	.639
52	48205	4458.0	27510	.571
53	45808	4027.4	23052	.503
54	43483	3606.2	19024	.438
55	41230	3196.0	15418	.374
56	39047	2798.4	12222	.313
57	36930	2414.3	9423.6	.255
58	34877	2041.9	7009.3	.201
59	32885	1680.9	4967.4	.151
60	30951	1329.0	3286.5	.106
61	29072	985.9	1957.5	.067
62	27247	650.1	971.6	.036
63	25472	321.4	321.4	.013
64	23718
65	22072

TABLE G—continued.

Commutation Columns for Family Annuities to Age 14—Interest
2½ per-cent.

Age x	F_x	G_x	$B_x = G_x : D_x$
20	713·1	500770	2·460
21	1462·5	500057	2·539
22	3254·8	498594	2·617
23	4950·3	495340	2·688
24	6620·6	490389	2·752
25	8243·5	483769	2·808
26	9746·1	475525	2·855
27	11248·	465779	2·893
28	12830·	454531	2·922
29	14708·	441701	2·939
30	17125·	426993	2·941
31	19856·	409868	2·923
32	21429·	390012	2·881
33	22024·	368583	2·820
34	22014·	346559	2·717
35	21533·	324545	2·607
36	20892·	303012	2·581
37	20262·	282120	2·492
38	19572·	261858	2·399
39	18841·	242286	2·304
40	18115·	223445	2·205
41	17362·	205330	2·104
42	16572·	187968	2·001
43	15782·	171396	1·897
44	14974·	155614	1·791
45	14182·	140640	1·684
46	13392·	126458	1·576
47	12609·	113066	1·468
48	11825·	100157	1·360
49	11040·	88632	1·251
50	10253·	77592	1·144
51	9714·5	67339	1·037
52	8905·7	57625	·928
53	8075·3	48719	·822
54	7248·7	40644	·719
55	6429·5	33395	·620
56	5645·6	26965	·526
57	4898·8	21320	·437
58	4198·4	16421	·355
59	3544·0	12223	·279
60	2920·3	8678·5	·209
61	2315·1	5758·2	·147
62	1725·2	3443·1	·093
63	1150·8	1717·9	·050
64	567·1	567·1	·017
65

TABLE G—continued.

Commutation Columns for Family Annuities to Age 14—Interest
3 per-cent.

Age x	F_x	G_x	$B_x = G_x : D_x$
20	628.1	404904	2.192
21	1280.5	404276	2.273
22	2835.6	402995	2.354
23	4293.7	400160	2.429
24	5718.9	395866	2.497
25	7092.2	390147	2.557
26	8351.2	383055	2.610
27	9592.5	374704	2.654
28	10891.	365111	2.690
29	12435.	354220	2.714
30	14430.	341785	2.724
31	16677.	327355	2.715
32	17914.	310678	2.681
33	18310.	292764	2.630
34	18228.	274454	2.567
35	17753.	256226	2.496
36	17147.	238473	2.420
37	16556.	221326	2.341
38	15920.	204770	2.257
39	15258.	188850	2.171
40	14606.	173592	2.081
41	13937.	158986	1.989
42	13246.	145049	1.895
43	12551.	131803	1.798
44	11848.	119252	1.700
45	11165.	107404	1.601
46	10490.	96239	1.501
47	9826.3	85719	1.400
48	9168.1	75923	1.298
49	8515.1	66755	1.196
50	7866.3	58240	1.095
51	7414.1	50373	.994
52	6769.6	42959	.891
53	6114.5	36190	.790
54	5459.5	30075	.692
55	4816.8	24616	.597
56	4206.7	19799	.507
57	3630.5	15592	.422
58	3094.6	11962	.343
59	2598.6	8866.9	.270
60	2130.0	6268.3	.203
61	1679.8	4138.3	.142
62	1245.3	2458.5	.090
63	817.3	1213.2	.048
64	395.9	395.9	.017
65

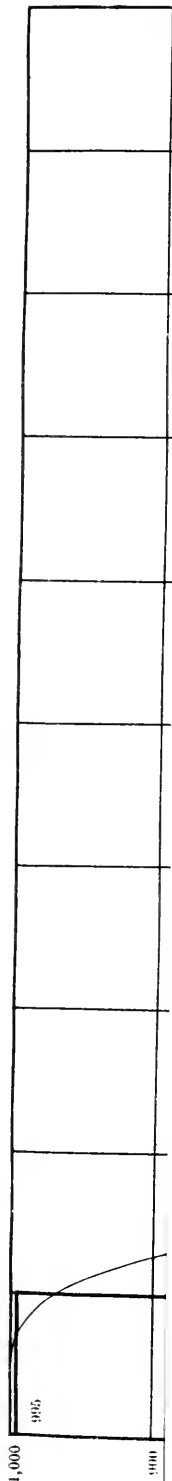
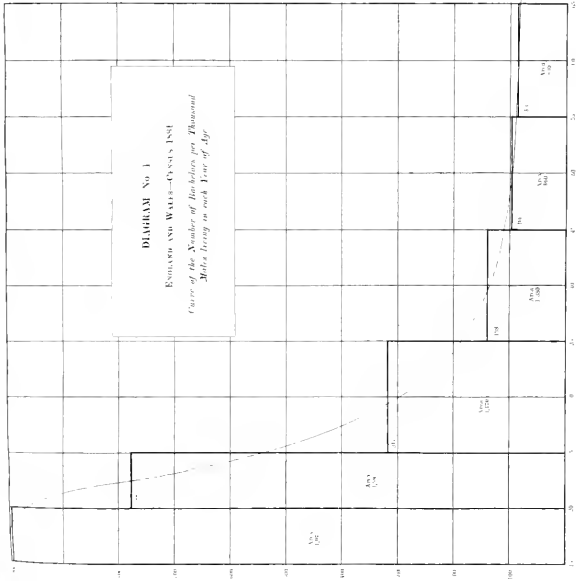


DIAGRAM No. 1

ESQUIMO AND WILLY—CIVIL 1884

*Curve of the Number of Barrels per Thousand
Males living in each Year of Age*



1000

800



20

25

30

35

40

45

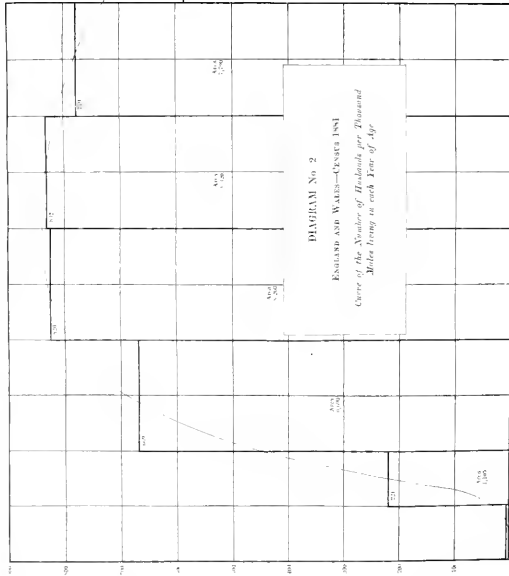
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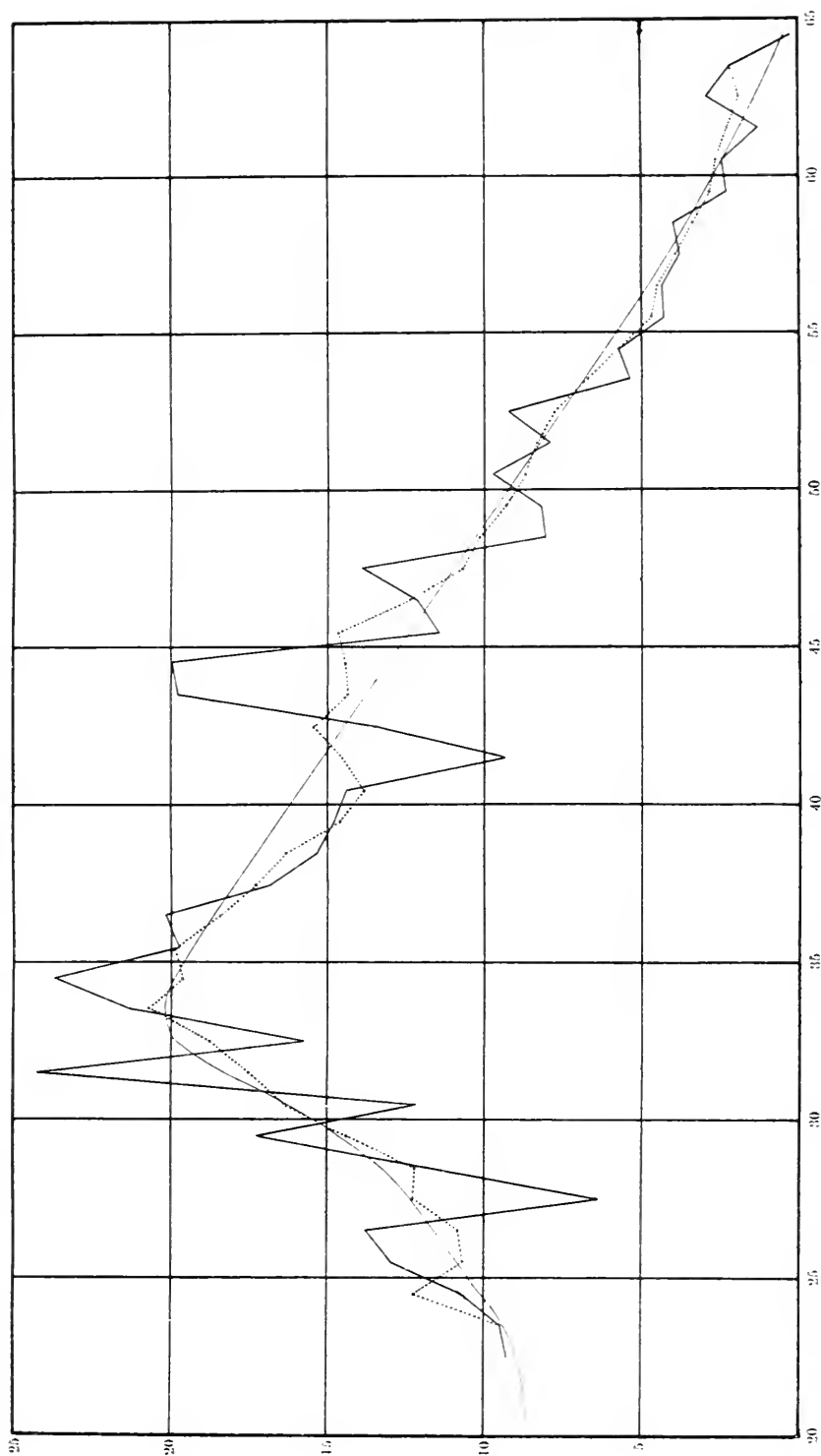
E'VALUATI'ONE W'ALFORD'S 1961

*Survival of the Number of Widowers per Thousand of Males
Living in each Year of Age*

N.B. In Diagram No. 3, vertically the scale is five times that of Diagrams Nos. 1 and 2, but horizontally all three Diagrams are on the same scale.

DIAGRAM No. 4.—GRADUATION OF TABLE F.—Annuities at $2\frac{1}{2}$ per-cent to Age 14.

N.B.—The abscissa represents the age at death of the parent, and the ordinate the value of the Family Annuity.



DISCUSSION.

The PRESIDENT (Mr. A. Hendriks) having called upon the referees, Mr. R. P. Hardy and Mr. Whittall, to open the discussion,

Mr. R. P. HARDY said that the paper tended towards the solution of a question which had the object of improving the well-being of almost the most helpless of the population—those deprived of the loving care and material support of husband and father. He would like to enlist Mr. King's name and pen in the service of a greater cause, namely, that of the toiling and scarcely-rewarded millions of this country, and the efforts that so many earnest men were now making, in the hope of securing for those most unhappy persons a modest provision in their old age, outside the depressing influences of the workhouse. If he might venture to give an opinion opposed to one which fell from the President's lips in his inaugural address, he would say that when he remembered the vast forces that stood behind this movement, it was his firm conviction that the consummation was far nearer than the date which the President so epigrammatically sketched. He felt that any scheme formulated on the lines of Mr. King's paper must stand back until the question of pensions was settled, but it was not too early to discuss its technical bearings, and to consider what suggestions could be made for improving the public registration of deaths, so as to supply data adequate to the solution of these social questions. Mr. King's method was very simple. It disregarded all the inner movements of the events, and looked to the final outcome as the real and eventual measure of the sum at risk. He (Mr. Hardy) had devised and employed the same principle 17 years ago in relation to a society where there were widows' benefits. He had given the method to Mr. James Meikle, who had adapted it to the valuation of one of the widows' funds in Scotland; but those anticipations in no way detracted from the originality of Mr. King's investigation. He did not propose to say much upon the data employed. Mr. King would no doubt tell them that the figures given were mainly for the purpose of numerical illustration and of suggesting the probable direction the rates would take. The birth rate derived from the figures of a vigorous and expanding country like New Zealand was hardly applicable to this older country, where the next enquirer would probably find a fall in the birth rate, certainly in the large towns and generally amongst the superior artisans. This fact alone would render any statistics drawn from past years open to considerable objection. He doubted whether the class of business dealt with in the paper could be safely undertaken by a trading assurance company; while so far as the State was concerned it was clear that it would be most imprudent to deal with individually selected cases, and if they adopted it they would probably have to employ the principle of collectivity and the odious one of compulsion.

Mr. W. J. H. WHITTALL said that the subject of annuities to fatherless children had been frequently mentioned in connection with State pension schemes, notably with that put forward by Mr. Chamberlain, and it had previously been considered in connection

with widows' funds for railway servants and miners. The subject had now derived increased importance. Mr. Sutton had said that from the point of view of a voluntary scheme, superannuation alone was a failure. On the other hand, mere assurance had been a great success. Did not this suggest that in the family annuity, which appealed to the best instincts of a man, just that modicum of insurance might be found, which would make superannuation palatable, and an object of desire where a voluntary scheme was concerned? For ascertaining the rates of marriage, death and orphanhood, Mr. King had recourse to three different sources of information. Two, namely, the census of 1881 and the English Life Table, were separated by a considerable interval in time—no less than a whole generation. The English Life Table No. III, was based upon the rate of mortality which prevailed in this country from 1838 to 1854. Dr. Ogle had calculated that, according to the experience of the decennium 1871-1880, a million males would live one million four hundred thousand more years than the "expected" according to the English Life Table. Of this large gain in longevity, which had taken place in recent years, 66 per-cent arose between the ages of 25 and 65, the ages that Mr. King had adopted as his limits of age. Above 65, the recent experience was worse than the older. Would the increased number of married men, by living a larger number of years, produce and leave behind them a larger number of children to share in the family annuities, or would they leave the children at a higher average age, and thus make a smaller charge on the annuity fund? Then there was the question of the recent decrease in the marriage rate. Mr. King was obliged to go back to the census of 1881 for his statistics. Better and more recent population statistics were greatly needed by a department which was rapidly growing in public importance—that of medical officers of health, whose zeal in vital statistics was damped by the impossibility of getting correct population returns with which to compare their death statistics. At present they were obliged to have recourse to the old-fashioned average death rate which involved many fallacies. With regard to population statistics he thought that if one great company could keep in its pigeon-holes separate cards for the whole of their assured, numbering nearly one-fourth of the population of these isles, the Government ought to be able to do the same thing for the whole population of England, and thus conduct something like a continuous census. Of course, there were difficulties in the way of such a scheme, questions of emigration and immigration, and the question of inquisitorial enquiries affecting the liberty of the subject; but he thought that something might be done on these lines which would give them data vastly better than those at present available.

Mr. F. W. FRANKLAND said that among the leading schemes for national pensions which had been before this country during the past two years, there was one which involved the provision of an annuity under circumstances which had not yet been submitted to exhaustive actuarial investigation. Hence, the analysis and the construction in the paper which had been read. Similarly, in New Zealand when a scheme for compulsory national insurance, almost

strictly on Canon Blackley's lines, was promulgated by the Colonial Treasurer, there was a provision, namely, for orphans, with regard to which the contingencies involved were such that the actuary had to break fresh ground. Being consulted by the New Zealand Government as to the pecuniary value of the annuities to orphan children which the Colonial Treasurer's scheme contemplated, he (Mr. Frankland) had adopted a method which, while confessedly rough, was adapted to produce such approximate estimates of pecuniary values as alone were then desired. For the purpose of discussion it was necessary for the Government to have a rough notion of the burden that would be imposed in the shape of either a premium or a tax. He hinted at the alternative of a tax, because the principle of collectivity was in the air as it was also here in connection with Mr. Charles Booth's scheme. The Colonial Treasurer was of opinion that there was nothing necessarily pauperizing in the idea that the pecuniary burden of the benefits should be borne by other persons than the beneficiaries. What he regarded as the essence of the scheme was that the benefit should be payable on the happening of well-defined contingencies that had no relation to the pecuniary situation of the beneficiaries: in fact, that it should not be the absence of capital saved or the presence of destitution which should constitute a claim upon the fund, but the attainment of a certain age, the being in ill-health, the leaving of widow or orphans, and so forth—contingencies which happened to rich and poor alike. In the definite integral used by Mr. King they had the sum of an infinite number of infinitesimal quantities, each being the product of an infinitesimal (dt) and several qualities of finite magnitude. This definite integral constituted the single premium which Mr. King called B_x . His (Mr. Frankland's) B_x , on the other hand, was the sum of a finite number of terms, each being the product of corresponding factors, all of finite magnitude. Each term of the series summed related to a particular quinquennium of age of adult males. The age at entry for which the value of the benefit had to be obtained was, he thought, 20. The first term then represented the value of an orphan benefit payable in the event of the adult male dying before reaching 25 and leaving a child or children under the age of 15, which was the age for the cessation of the annuity. The second term represented the present value of a similar annuity payable in the event of the adult male dying between 25 and 30, and leaving children similarly circumstanced. Each term of the series he took as being equal to the corresponding five-year temporary assurance value multiplied by the value of the family annuity when it came into possession, assuming that it did so at the central age of the quinquennial group. He made some arbitrary adjustment for the earliest and latest terms of the series. In regard to the second factor of the term, the value of the family annuity at the moment of its commencement, a difficulty as to statistical data presented itself. He knew of no data showing for males dying at various ages the number and ages of the children left, and therefore recommended a change in the Registration of Deaths Act, which would remedy the defect by gradually accumulating the required statistics.

Dr. W. A. HUXTER, M.P., said there were a few points in the

paper which were of interest to all Members of Parliament. The first was with regard to the defective statistics supplied by the Registrar-General. No doubt they were aware that before the last census was taken a Departmental Committee sat to enquire into any changes that might be made in taking that census, but he could not say to what extent the members of the Institute were consulted in the matter. It was quite clear that there was room for improvement in the direction pointed out by Mr. King, and he hoped the authorities would pay attention to his suggestions. With regard to the main part of the paper, it was, of course, one on which he was not in the least qualified to speak. He, however, took great interest in the general results of the examination, because it was owing to his pertinacity that the small Sub-Committee of Members of the House of Commons was formed, which elaborated the scheme generally known as the Chamberlain scheme. That scheme took the double form of an insurance for old age and for children. The arguments which induced the members of the Committee to adopt the alternatives for old age and children were two. In the first place, a voluntary pension scheme would encounter this great difficulty, that 65, or whatever age was taken, was a very remote period at the time when it was most easy to subscribe. The earlier the age the smaller the premium to be paid, and when old age was near at hand the premium became too heavy. The possibility of death was much more pressing on the mind of working men than the possibility of old age. Another thing was that while the middle classes made a substantial provision for the contingency of death, by means of insurance, the majority of working men, if they died at an early age, left their children unprovided for. Actuaries were familiar with the great difference in cost of a pension according as the premiums were returnable in the event of death or not, but persons who were not actuaries did not quite appreciate that point. The difficulty was serious, because premiums which were not returnable were not popular. Therefore, the Committee adopted the middle course of having premiums returnable only in the case in which it was of much importance to the working man, namely, on his death, he leaving children. The data on which Mr. King had been forced to proceed were narrow, and he hoped that the influence of the Institute might be sufficient to induce the authorities to secure statistics on a wider basis for the future.

MR. MORRIS FOX (New Zealand) thought that the birth rate and marriage rate were lower in New Zealand than in England. No doubt the lower classes in New Zealand, and perhaps every other colony, were in a better material condition than those in England, and the lower the material prosperity of a class of people the easier would they throw aside prudential motives of restraint. The better-off people would naturally be more prudential both in their marrying and in their having children. It might be interesting to state that an ex-Premier of New Zealand, Sir Robert Stout, had lately taken up the question of old age pensions, and he (Mr. Fox) should not be surprised if New Zealand were before long to add another to the list of experiments made on social and economic questions in the colonies for the benefit of the world in general.

Mr. A. F. WATSON said the Manchester Unity of Oddfellows had recently issued papers with a view of obtaining information in the direction pointed out by Mr. King. Several societies had been asked to give their experience during the past five years, and he thought it would be agreed that if a sufficient body of observations drawn from societies well and reliably managed could be collected, they would have more trustworthy statistics than those for which Mr. King had been compelled to go to New Zealand.

Mr. W. O. NASH thought the present might be an opportunity of eliciting from the Institute some opinion upon one point as to national pension schemes. He believed the opinion held in the Institute was that any scheme that would meet the burning question of old age pensions must be based, not on payment of premiums for the future benefit, but rather on what was known as the assessment scheme. As pointed out by Mr. Hardy last year, it was the emerged claims which should be dealt with, and assessments should be calculated to provide them: and in that case, of course, the problem mentioned by Dr. Hunter would not arise.

Mr. H. W. MASLY said with regard to Dr. Hunter's reference to the Departmental Committee that he thought it would be within the province of the Institute to make recommendations to the proper authorities, whoever they might be, for obtaining statistics which would be valuable for the elucidation of such problems as were now put before them, and to ask for their serious consideration before the next census was taken. The paper itself showed how deficient those statistics were for many questions that the actuary might be called upon to consider.

The PRESIDENT said that as regarded Mr. Hardy's reference to the sentiments expressed in the address which he (the President) recently delivered, if he relegated to a distant period the *finality* of the enquiry, it was that he felt that whatever scheme was brought forward, whether for pensions or family annuities or anything else, they would not reach that desideratum of a final solution of the problem of complete provision for the poor until the millennium. But even in that view Mr. Hardy and himself might be agreed, for there were some who thought that the millennium might arise during the coming session of Parliament. Mr. King did not tell them that the results which he had deduced from his formula were to be applied without discrimination to any questions which might hereafter arise out of Government schemes for pensions and family annuities. The tables were, however, of very great service to an actuary taking the formula and reasoning according to the class into which he was enquiring, but they could not as a whole be applied to any particular class without due revision.

Mr. KING, in acknowledging the usual vote of thanks, said that his efforts were of a tentative nature, and he did not presume to submit them as final. His intention rather was to indicate the direction in which research could best proceed. Mr. Hardy had referred to a fund valued by Mr. Meikle. Before preparing the paper he had not seen Mr. Meikle's report, but he had since had an opportunity of examining it. Mr. Meikle's problem was to find the value of an annuity payable to a family until the youngest child

reached the age of 18. The problem he (Mr. King) had tried to solve was to find the value of an annuity to each child until that child reached a given age. The two things were very different. Mr. Meikle took his ages in groups of 15 and 20 years and assumed an annuity-certain equivalent to the life annuity, so that although the idea of making the benefit an assurance on the father's life was very similar, his (Mr. King's) opportunities, although also defective, had enabled him to work it out more fully. With regard to the New Zealand statistics, the figures he used did not refer to the birth rate in the ordinary sense of the word. He did not take the number of births per 1,000 of the population, but the number of births to each family, and combining that with the English statistics of population and mortality, he obtained a closer approximate to the English birth rate than might at first be supposed. There were many complicated forces at work, and it was impossible to unravel them without a vast amount of labour. He had been at different times engaged on various investigations which had led him to examine the Registrar-General's Returns and Census Reports, and at every turn he was stopped by want of suitable information. More frequent enumerations were required, and for practical purposes it was necessary that the results should be published in improved forms and in greater detail. He could assure Mr. Hardy that his feelings were entirely with him on the question of old age pensions; but he saw so many difficulties in every proposal that had been submitted that he required to give a great deal more consideration to the subject before forming a definite opinion. He should not like it to go forth, as one speaker suggested, that the Institute as a body objected to what they might call the premium method. He could quite see that there were difficulties which might, perhaps, be found insuperable, but the question had not yet been sufficiently examined to enable the Institute to decide whether the assessment or the premium method should be applied to old age pensions.

The Liability of Life Assurance Companies to pay Income Tax upon Annuities, &c.

THE GRESHAM LIFE ASSURANCE SOCIETY v. STYLES (SURVEYOR OF TAXES).

THE point in the above-mentioned case, was the liability of Assurance Companies to be assessed in respect of their annuity payments, by the inclusion in the amount of their assessable profits of the sums paid away to life annuitants.

It was admitted that the *Gresham* had already paid tax, by way of deduction at its source, upon interest on investments of an amount in excess of its profits as actuarially ascertained, but the Surveyor sought to further include in the assessment the annuity

payments of the Society, mainly upon the ground that they were payable "out of profits and gains" in the sense of the Income Tax Acts (1842 and 1853).

We append :

1. The case as stated by the Income Tax Commissioners, dated 14 February 1889.
2. The judgment of the Divisional Court, dated 21 January 1890.
3. The judgment of the Court of Appeal, dated 10 June 1890.
4. The judgment of the House of Lords, dated 31 May 1892.

It will be seen that the decision of the House of Lords was entirely in favour of the Society, over-ruling the judgments of the inferior Courts. It has, however, been pointed out by Mr. T. G. Ackland, the Actuary and Manager of the *Gresham*, that the decision is apparently only effective up to the year 1888, owing to the provision of "The Customs and Inland Revenue Act, 1888", 51 Victoria, cap. 8, section 24, sub-section 3, wherein it is enacted, possibly in view of this particular case, that: "Upon payment of any interest "of money or annuities charged with Income Tax under schedule D, and "not payable, or not wholly payable, out of profits or gains brought "into charge to such tax, the person by or through whom such interest "or annuities shall be paid shall deduct thereout the rate of Income "Tax in force at the time of such payment, and shall forthwith render "an account to the Commissioners of Inland Revenue of the amount "so deducted, or of the amount deducted out of so much of the "interest or annuities as is not paid out of profits or gains brought "into charge, as the case may be; and such amount shall be a debt "from such person to Her Majesty, and recoverable as such "accordingly."

Where the annuity is payable to a British subject and the Company merely pays to the Revenue Authorities the amount deducted from the annuity payment, the Company need not complain of this enactment. In the *Gresham* office, however, the major portion of the annuity business is transacted in foreign countries, and in such cases it might be difficult, if not impossible, for the office to make the deduction from the annuity payments, and the loss would therefore fall upon them. We understand that consideration is being given to the question of the legal operation and effect of the Income Tax Acts generally (including the Act of 1888 above cited) as regards annuities payable to foreigners.

5. Annexed hereto, and forming part of this Case, is a print of the return furnished to the Board of Trade, dated January 1886, which correctly sets forth the income and expenditure of the said

Society for three years ended 30 June 1885, and the methods by which the profits or gains of the Society are actuarially ascertained.

6. By Account No. 7 of the Fifth Schedule, "The Valuation Balance-sheet" shows a surplus profit for three years of	£97,553	2	9
From this has to be deducted the surplus brought forward from the previous triennium. See note 9 (1)	1,196	6	1
					£93,056	16	8

The Society have during the triennium paid Income Tax on £120,722 chiefly by tax being deducted at its source on payment to them of interest and dividends.

7. The Society in arriving at the amount of profits and gains for three years, amounting as above shown to £93,056 16 8 had taken into account (among other items of expenditure) the following items, viz.—

The sum paid in discharge of its annuity contracts, as shown in the Consolidated Revenue Account of the Society, being Account 6 of the 5th Schedule in the same printed returns	253,252	5	10
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The sum which had been paid for Income Tax, as shown by the books of the Society, and being part of the sum of £21,377. 3s. 3d. in the said Account 6 of the 5th Schedule appearing	3,247	0	0
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The said two sums of £253,252. 5s. 10d. and £3,247 added to the said sum of £93,056. 16s. 8d. amount together to the sum of	£319,556	2	6
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From this amount the Surveyor of Taxes admitted by way of deduction the sum on which Income Tax had already been paid as aforesaid, viz.	120,722	0	0
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And claimed that the Appellants were chargeable for the three years with the difference of	£228,834	2	6
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One year's proportion, or one-third part of this sum, amounts to the sum of £76,278. 0s. 10d., for which amount the Society was assessed for the year ending the 5 April 1886.

8. The Surveyor of Taxes contended that the exception of the annuities and the Income Tax paid by the Society was by way of deduction from the gross income of the Society in arriving at the amount on which Income Tax should be charged, and that the same was not allowable under 5 and 6 Victoria, cap. 35, sections 100 and 159, and 16 and 17 Victoria, cap. 34, section 40; and further, that

the Society was entitled to deduct and should deduct the amount of the duty on payment of each annuity under 5 and 6 Victoria, cap. 35, section 102, and 16 and 17 Victoria, cap. 34, sec. 40.

9. The Society contended: First, that they had treated and were entitled to treat the payment of annuities in the same manner as the payment of sums payable on the death of the assured, and contended that the same cannot be treated as a deduction from or be brought into account so as to increase profits, or be treated as paid out of profits or gains either within the meaning of Schedule D, 4th Rule, 1st Case, section 100 and section 102 of the 5 and 6 Victoria, cap. 35, or as otherwise brought into charge by virtue of such Act, or as sums on which Income Tax is payable by the Society under any of the provisions of the 16 and 17 Victoria, cap. 34, and Secondly, that the amount of the Society's profits and gains for the triennium being £93,056. 16s. 8d., and they having paid tax on £120,722 as aforesaid, they were not liable for any further assessment.

10. The Commissioners were of opinion that the assessment should be confirmed, and confirmed the same accordingly, whereupon the Appellants declared their dissatisfaction with the decision of the Commissioners, and duly required them to state and sign a Case for the opinion of the High Court of Justice, which we have stated, and do sign accordingly.

The question for the opinion of the Court, is:

- 1st. Whether the Society is liable to be assessed in respect of the amount paid by them for annuities or any part thereof.
- 2nd. Whether the Society is in any event liable to be assessed for any sum in excess of the amount on which tax has already been paid by way of deduction at its source as aforesaid.

ADDINGTON, GEO. H. CHAMBERS, THOMSON HANKEY, H. H. HEATH, WM. CAVE FOWLER, D. P. SELLAR,	}	<i>Commissioners of Taxes for the City of London.</i>
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THE GUILDHALL BUILDINGS, E.C..

14 February 1889.

IN THE HIGH COURT OF JUSTICE, QUEEN'S BENCH
DIVISION.

DIVISIONAL COURT,

ROYAL COURTS,

Tuesday, 21 January 1890.

Before BARON POLLOCK and MR. JUSTICE HAWKINS.

Between THE GRESHAM LIFE ASSURANCE SOCIETY Appellants,
and
ROBERT HENRY POOLE STYLES (Surveyor of Taxes) Respondent.

JUDGMENT.

Pollock, B.—This case has been so fairly argued on behalf of the Appellants that we think it unnecessary to call upon the Respondent.

We cannot decide this appeal in favour of the Appellants without contravening not only the actual provisions of the Income Tax Acts, but the spirit of those Acts, as acted upon, not only by lawyers, but by commercial men, for a long series of years.

The real question in this case is whether this Society, who have, pursuant to their contracts and mode of business, to pay certain annuities, are the proper persons to be taxed in respect thereof, or whether the recipients of those annuities ought to be, and must be, taxed in the first instance.

In order to get at the profits or gains upon which the Appellants are taxable they must be treated in the same way as persons carrying on any other trade or business. The figures must be ascertained, first, by arriving at what all their receipts are, and then giving them credit, not for deductions, but for all sums which must be paid in the course of trade before the net profits are earned.

In the present case the Appellant Society have two classes of business—the one life assurance, and the other the sale of annuities. In the case of the first no question arises, because there is no payment of annual sums by the Society. In the case of the business of selling annuities the annuitant pays a lump sum down, and in return the Society pays him an annual sum during his life, which is commonly called an annuity. It is because this sum which they pay is an annual sum that it comes within the language of the Act.

The clear intention of the Income Tax Acts is to give the Revenue an opportunity of taxing all taxable sums at the earliest possible occasion, as in the case of annual payments made by the Bank of England, where the deduction of the tax is clearly provided for and made compulsory by section 89 of the Act of 1842, and by some following sections. There are also provisions dealing with other payments which are not in the nature of the payments made by the Bank of England, or by public companies, but are made in respect of some private arrangement. In these cases it is left optional, by section 40 of the Act of 1853, to the payers to deduct the Income Tax before payment. The present case, no doubt, is different from either of the foregoing, and the question is whether it comes within section 102 of the Act of 1842, which provides that "upon all

“annuities, yearly interest of money, or other annual payments, payable either as a charge on any property of the person paying the same by virtue of any deed or will, or otherwise, or as a reservation thereout, or as a personal debt or obligation by virtue of any contract”, there shall be charged the usual Income Tax; and further provides that “in every case where the same shall be payable out of profits or gains brought into charge by virtue of this Act no assessment shall be made upon the person entitled to such annuity, interest, or other annual payment, but the whole of such profits or gains shall be charged with duty on the person liable to such annual payment, without distinguishing such annual payment.”

It is said that these annuities are not payable out of profits or gains. I think that they are payable out of profits or gains, for, according to the principle of calculation which I have already stated, if these annuities were not payable, the lump sums received from annuitants would be wholly profits or gains. Although those lump sums may not be net profits, yet, being received in consideration of the granting of annuities, they are as much gains as any sum which is received by a merchant as a payment to him for goods sold, services rendered, or in respect of any special contract. In this particular case all the lump sums are not taxed as profits or gains, but they would be taxable as such if the payment of the annuities were not made out of them and out of what they produce, and therefore the annuities are paid out of profits or gains.

It seems to me clear that the intention of the legislature as expressed in sect. 102 is to do in such a case as the present the same as in other cases, and to find at its source a taxable item, and to tax it in the hands of the payer, that is, in this case, in the hands of the insurance company. Upon these grounds I think that this appeal must fail, and be dismissed with costs.

Hawkins, J.—I concur.

IN THE COURT OF APPEAL.

ROYAL COURTS OF JUSTICE,
10 June 1890.

Before THE MASTER OF THE ROLLS (LORD ESHER),
LORD JUSTICE LINDLEY, and LORD JUSTICE LOPES.

Between THE GRESHAM LIFE ASSURANCE SOCIETY . Appellants,
and

ROBERT HENRY POOLE STYLES (Surveyor of Taxes) Respondent.

JUDGMENT.

Lord Esher, M.R.—In my opinion this case is governed by the rule of construction laid down in *Alexandria Waterworks Co. v. Musgrave* (1). A rule of construction was there laid down and applied to the 4th rule of Case I. in Schedule D, which seems to me to be the rule governing the decision in this case.

(1) XI Q. B. D. 174.

This Society is carrying on business, and the part of their business with which we are concerned is that they sell annuities, or, if it is put in ordinary language, they grant annuities in consideration of money paid to them. What we have to see is what is the profit of that business upon which the Government is entitled to receive Income Tax. But for the 4th rule and the 102nd section, I should have no doubt but that the income on which the Government would be entitled to charge Income Tax, would be arrived at by taking the difference between what was received in any three years, and what it cost to obtain those receipts. Such a difference in any year is the annual income of a tradesman, and whether the way of getting at that is by means of an actuary or by some plain simple method seems to me to be immaterial. Then I have, further, no doubt that the receipts of this business would be the price paid by the proposed annuitants, and that one of the expenses would be the amount of the annuities which had to be paid in respect of the sums which had been received. But then comes the 4th rule, and it seems to me that rule lays down something different from that which would have been applicable if the rule had not been there. There would be hardship but for other clauses which get rid of it and produce this effect, that the amount which the Government receives by way of Income Tax is not increased, but the mode in which that amount is collected is altered so that it can be collected from the person who pays. Now what was the interpretation put on rule 4 in the case of *Alexandria Water Co. v. Musgrave*? It seems to me to be this—that for the purpose of the collection of Income Taxes the words “profits and gains” in the rule mean the credit side of the account. If so, that means the trade receipts over the period. Then, in making up the other side of the account for this purpose of taxation, no items on account of annual interest or an annuity or other annual payment are to be put on that side of the account, although, but for the rule, they would be properly placed in the account as expenses. That was held with regard to “annual interest” in the *Alexandria Water Co. Case*. Applying that holding to the next item in rule 4, that is to “annuities”, then, although these annuities would, but for the rule, have been expenses which would have to be deducted before you could get at the net profit, the Act has said, reading it according to the ordinary meaning of the English language, that these annuities are not to be taken into the account. That is the judgment of the Divisional Court, which I think was right, applying the rule of construction laid down in the *Alexandria Water Co. Case*, and therefore this appeal fails, and our judgment must be for the Crown.

Lindley, L.J.—I have entertained some doubt about this case in consequence of the difficulty of putting a construction on all the words of these sections; but I have come to the conclusion that the decision of the Court below is right. The case turns on the true construction of the 4th rule in Schedule D and on section 102, and upon the construction or interpretation which was put upon rule 4 in the case of *Alexandria Water Co. v. Musgrave*. It is quite obvious when we look at the rules in Schedule D that the rules laid down for the purpose of ascertaining the balance upon which Income Tax is to be assessed are very different from the rules ordinarily employed in trade

in ascertaining the net profits divisible among those who carry it on. That is made plain by running the eye down rule 3. There are all sorts of things which are not to be inserted which traders would insert in order to ascertain the net balance to divide among themselves, and the language of the rule itself is significant. I cannot help thinking that when the rule says that certain things are not to be deducted from, or set against profits or gains, that this latter expression means the items on the credit side of the account. The tax is upon the balance, and there is nothing expressly said about deducting any losses, the only authority for any deductions being the expression "balance of profits and gains", and it is only by implication that you get at how a balance is to be struck. It appears to me, therefore, that the annuities in this case are payable out of such profits or gains as are referred to in the rules to Schedule D or, to vary the expression, they are annuities payable out of the profits or gains brought into charge by virtue of this Act, which is the expression used in section 102. Having regard to the construction already placed on these phrases in the *Alexandria Water Co. Case*, it appears to me, though I had some doubts about it at first, that the decision appealed against is right.

Lopes, L.J.—It appears to me that the decision of this case depends on the 4th rule of case I of Schedule D of the Act of 1842, and on the 102nd section of that Act. If it were not for the rule and the section, I should have thought without doubt that the Appellants would have been entitled to make the deductions which they now claim to make—in point of fact, they would have been entitled to make such deductions as any other commercial undertaking would be entitled to make in respect of expenses incurred in carrying on their trade, deducting those expenses so as to get at the net result. But it appears to me that the legislature intended a different result, and a different machinery for collection, to be applicable to annuities or other annual payments. I think that in the 4th rule by "profits and gains" are intended receipts—trade receipts, that is, the credit side of the account; and I think this view is strengthened by the different phraseology used in the different rules. In the first rule of Case I, we have the expression "full amount of the balance of profits or gains", in the 3rd rule we have "the balance of profits and gains chargeable", and when we come to the 4th rule a different term is used, "the amount of the profits and gains arising as aforesaid." I think this difference of expression was in consequence of the different state of things applicable to the different rules. The view that "profits and gains" in rule 4 means the credit side of the trade account is strengthened by section 102 of this Act. It is to be observed that there is nothing harsh or unreasonable in this construction, which merely makes the Society liable primarily to pay the tax, they having full authority to deduct it from the annuity when they pay the annuitant.

I have only further to say that the view we now take of the construction of this rule is entirely consistent and in accordance with the *Alexandria Water Co. v. Musgrave*. I think, therefore, the judgment of the Court below was right, and that the appeal should be dismissed.

IN THE HOUSE OF LORDS.

*Tuesday, 31 May 1892.*Between THE GRESHAM LIFE ASSURANCE SOCIETY . Appellants,
andROBERT HENRY POOLE STYLES (Surveyor
of Taxes) Respondent.Lords present—THE LORD CHANCELLOR, LORD WATSON, LORD
HERSCHELL, LORD FIELD AND LORD MORRIS.

JUDGMENT.

The Lord Chancellor.—My Lords, the commercial concern in respect of which the question arises here, consists of a business which is carried on by receiving a sum of money as the purchase-money for the granting of annual sums. The sum so granted is certainly an annuity and an annual payment, and I believe it is the fortuitous use of those words, under circumstances that I shall presently discuss, which has given rise to what seems to me an erroneous construction of the Statute.

The question arises under the Income Tax Acts, and the Act itself, like all the Acts of the same class, purports to be "An Act for granting to Her Majesty duties on profits arising from property, professions, trades and offices." Now I leave out for the moment the question of land, and the object being to tax profits, certain Rules have been inserted in the Statute which have become, and are made in truth, parts of the Act itself. But the Rules are, as they themselves import, Rules for ascertaining the duties. "The amount of the profits and gains" was the expression all through the earlier Legislation on the subject down to the Act of 1805. In that year the expression was altered, as is pointed out by Mr. Dowell in his valuable book on the Income Tax Acts, page 114, and the observation appears to be well founded that the alteration of the language pointed rather to a more careful return or statement of income, an account in a Debtor and Creditor form, and intended to prevent deductions, which possibly had been up to that time claimed being permitted to reduce the amount of the profit which the Debtor and Creditor account ought to show. But except as indicating the form rather than the substance of what should be shown in the return as the balance of profits and gains, I am in the condition of Lord Blackburn, who said, in the case of the *Coltress Iron Company v. Black* (L. R. 6 Appeal Cases, 334), that he had been unable to discover any difference in the meaning of the two phrases.

Now if I understood the contention before your Lordships aright, it rested on the Fourth Rule relating to Assessment under Schedule D, which prohibited any deduction being made on account of any annual interest or any annuity or any other annual payment, and as those sums were annuities or annual payments, it was said that they were not to be deducted in estimating the amount of the profits and gains arising as aforesaid. And if the Rule had stopped at the point up to which I have quoted it, I should have concurred

with the contention of the Surveyor. But the Rule does not stop there. In order to be a prohibited reduction it must be sought to be made on an annuity or other annual payment payable "out of such profits or gains."

Now to my mind it is very clear what is the *intuitus* both of the enactment and of the Rules under which the duties are to be ascertained. The thing to be taxed is the amount of profits and gains. The word "profits", I think, is to be understood in its natural and proper sense, in a sense which no commercial man would misunderstand. But when once an individual or a Company has in that proper sense ascertained what are the profits of his business or his trade, the destination of those profits or the charge which has been made on those profits by previous agreement or otherwise is perfectly immaterial. The tax is payable upon the profits realised, and the meaning to my mind is rendered plain by the words "payable out of profits."

It would be an extraordinary thing to suggest that where a business consists of granting annuities it is to be taxed upon a different principle from any other commercial concern, and no one, I suppose, could doubt that in any other commercial concern the cost of the thing sold to the trader is one of the expenses incident to the carrying on of the trade. If an annuity seller is to be treated differently from a seller of any ordinary article of commerce—coals or corn or the like—one would have expected to find some words in the Statute rendering him obnoxious to a different system of taxation, and enforcing a different mode of ascertaining profits, whereas it seems to me that the application of the general words, "profits and gains", or "balance of profits and gains", are equally applicable whatever the commercial concern carried on may be.

I think one gets a very fair notion of what was in the mind of the Legislature by a variety of cases put in the course of the argument which it is not necessary to repeat here. The Court of Appeal seems to have been impressed by the decision in the *Alexandria Water Company Limited v. Musgrave* (L. R. XI Q. B. D. 174). With the decision in the case I am not disposed to disagree, though I am not quite certain I am able to adopt the reasoning which to some extent appears to depend upon a distinction between the words "gains" and "profits", a distinction which I cannot assent to; but upon the facts I think I should have come to the same conclusion, since, to put it very plainly, in that case it was a claim to deduct the Company's debts for borrowed capital, and diminish the amount of the profits of the trading.

The whole point seems to me to depend upon the words "out of profits and gains." Profits and gains must be ascertained on ordinary principles of commercial trading, and I cannot think that the framers of the Act could be guilty of such confusion of thought as to assume that the cost of the article sold to the trader, which he in turn makes his profit by selling, was not to be taken into account before you arrived at what was intended to be the taxable profit.

As I have said, the confusion has arisen from the use of the words "annuity or annual sums payable" without considering that

this particular commercial adventure consists in selling annuities, and that which they pay, therefore, is to them the cost of the article supplied. You can no more refuse to take that cost into your consideration when ascertaining the balance of profits and gains, than you could the cost of the coals or the corn to the coal merchant and to the corn merchant in ascertaining what are the profits from his trade.

I am, therefore, of opinion that the judgment appealed from ought to be reversed.

Lord Watson.—My Lords, the business carried on by the Appellant Society is that of Life Insurance, and one branch of it consists in the sale of annuities to the public. The consideration for an immediate annuity is a lump sum instantly paid, and for a deferred or contingent annuity may either be a lump sum or a series of periodical payments. The question raised for decision in this Appeal relates to an assessment under Schedule D in respect of the annual profits and gains arising from the Appellants' business for the fiscal year ending 5 April 1886.

In accordance with the provisions of Section 100, Case First, Rule 1 of the Act of 1842 (5 and 6 Victoria, cap. 35) the amount upon which the Appellants became liable to pay Income Tax for the year in question, was one-third of the assessable profits or gains which had accrued to them during the three years immediately preceding the 30 June 1885, the date at which their books were last balanced. The Appellants had in the course of these three years paid upwards of £250,000 in discharge of their contract obligations to their annuitants. The Respondent in estimating the triennial average included all these payments in the balance of assessable profits or gains, disregarding the contention of the Appellants that they ought to be treated as items of debit, for the purpose of ascertaining the balance. On Appeal, the Commissioners for General Purposes of the Income Tax Acts for the City of London confirmed the assessment and stated a special case in terms of 43 and 44 Victoria, cap. 19, section 59. The decision of the Commissioners has been upheld by a Divisional Bench as well as by the Court of Appeal.

Rule 1 of Section 100, Case First, prescribes that the duty to be charged shall be computed on a sum not less than "the full amount of the balance of the profits or gains of such trades, manufacture, adventure or concern." It plainly contemplates the preparation of a balance sheet in which proper trading disbursements and liabilities are to be set against trade assets, so that the surplus of the latter, if any, will represent the assessable profits or gains of the concern. All the other rules applicable to Schedule D are framed upon the same footing. Rule 3 of Case First specifies various items which a trader might naturally enough, for his own private purposes, insert on the debit side of the sheet, and enacts that these shall not be allowed as deductions in estimating his profits for the purposes of the Income Tax. Rule 4 of Case First provides "that in estimating the amount of the profits and gains arising as aforesaid, no deduction shall be made on account of any annual interest or any annuity or other annual payment payable out of such profits or gains."

The first of the Rules applicable to Cases 1 and 2 of Section 100 provides that in estimating the balance of the profits or gains to be charged according to either case no sum shall be set against or deducted from such profits or gains for "any disbursements or expenses whatever, not being money wholly and exclusively laid out or expended for the purposes of such trade, manufacture, adventure, or concern." It seems tolerably clear, and it was hardly disputed by the Respondent, that all payments made by the Appellants on account of the annuities which they have contracted to pay are in the strictest sense of the words disbursements made wholly and exclusively for the purposes of their business. The Appellants trade in annuities, and these disbursements are the consideration, or part of the consideration, which they are bound to give in return for the annuitants' purchase-money already received by them and carried to their credit in the balance-sheet. Unless the Statute forbids it, the Appellants must have the same right to deduct such payments in estimating their assessable profits, as a grocer has to deduct the price paid by him for the tea or sugar which he retails. If there be one point free from obscurity in the Act of 1842 it is this, that the Legislature intended all traders, whether in groceries, annuities, or other articles of commerce, to be assessed upon the same footing.

The learned Counsel for the Respondent were constrained to admit that, were it not for the terms of Rule 4 of Case First, the Appellants would be entitled to the deductions which they claim. But they argued that the expression "profits and gains" which occurs twice in that Rule, in each instance signifies the gross receipts of the trader, or, in other words, the whole items standing on the credit side of the balance-sheet. Upon that construction of the Rule no deduction could be allowed in estimating profits of "any annual interest or any annuity or annual payment", and one of many startling results would be that the yearly rent and taxes paid by a trader for his business premises, and yearly wages paid to his servants, would not be dealt with as payments out of trading capital, but would be included in the profits upon which he pays Income duty. Although it appears to have been accepted by both Courts below, I cannot assent to the construction put upon Rule 4 by the Respondents, which is, in my opinion, at variance with its context. The Rule begins thus: "In estimating the amount of the profits and gains arising as aforesaid." Does that mean "in estimating the amount of the trader's assets?" I venture to think not. I think the words refer, not to total assets, but to the balance of profits and gains to be ascertained by virtue of the preceding rules for the purpose of being charged with Income duty. The Rule then goes on to prescribe that there shall be no deduction made on account of any annual interest or any annuity or other annual payment "payable out of such profits or gains"; these last words refer back to profits and gains as described in the commencement of the Rule, and must therefore be taken to mean, not gross receipts, but profits and gains the amount of which is to be estimated for the purposes of the Act. Had it been the intention of the Legislature to enact that no annual interest, annuity, or other annual

payment should be inserted on the debit side of the balance-sheet, it appears to me that Rule 4 would have been expressed in very different terms. The Rule, as it stands, merely enacts that no annual payment which falls to be paid out of profits, shall be deducted from profits in assessing for Income Tax.

The Respondent relied in argument upon Section 102, which directly charges with duty all annuities, yearly interest of money or other annual payments, whether such payments shall be payable within or out of Great Britain. That enactment is qualified by the proviso that no charge shall be made upon the person entitled to such payments in the case where the same shall be "payable out of profits or gains brought into charge by this Act." In that case the trader must pay upon the whole profits or gains brought into charge, but is authorized to deduct Income Tax in settling with the person entitled. These provisions do not appear to me to throw any light upon the question, because in order to ascertain what profits or gains are brought into charge by the Act, it is necessary to go back to the Rules of Case First of Section 100.

Section 40 of the Act of 1853 (16 and 17 Victoria, cap. 34), which was also referred to in the course of the argument, has really no bearing upon the present question. It empowers persons who are liable to the payment of any rent, or any yearly interest of money, or any annuity, or other annual payment, on making such payment to deduct and retain thereout the amount of the rate of Income duty payable at the time, but it has no application to annual payments payable out of profits or gains. Section 24 (3) of "The Customs and Inland Revenue Act, 1885", for the first time made it compulsory upon the persons liable in payment to retain Income Tax upon their making payment out of the capital which they employ in trade of any interest of money or annuities charged with the tax under Schedule D, and not payable or wholly payable out of profits brought into charge to such tax. Had that enactment been in force in the year 1885-6 the present controversy would never have arisen. The only point which remains for consideration is this—whether the annuities paid by the Appellants are payable, within the meaning of Rule 4, out of profits and gains? In order to bring annuities or other annual payments within the scope of the Rule, they must, in my opinion, either be directly charged upon profits, or be in themselves of such a character that they form a proper charge upon profits. Now I cannot understand in what sense the annuities in question can be said to be properly chargeable to profits. An annuity to the widow of a deceased partner, interest on capital advanced by a partner, or upon money borrowed for the purposes of the business, are truly payable out of profits earned, and therefore ought not to be deducted in estimating the income yielded by the business. On that ground I agree with the decision of the Court of Appeal in the *Alexandria Water Company v. Musgrave* (XI. Q.B.D. 174), although I am unable to concur in all the reasons which were assigned for it. But the business of the Appellants consists in employing their trading capital to pay annuities, as the counterpart of the consideration given by the annuitant, and the annuities are payable out of stock and not out of business profits.

On these grounds I concur in the judgment which has been moved by the Lord Chancellor.

Lord Herschell.—My Lords, the Appellant Society, as part of the business which it carries on, sells or grants annuities in consideration of a premium or lump sum paid down in the case of an immediate annuity, and of a similar payment or of periodical premiums in the case of a deferred or contingent annuity. In making up the balance sheet showing the amount of its profits and gains for three years the Society put on one side of the account the consideration money received for annuities granted, on the other side was put the sum paid in discharge of its annuity contracts. The Surveyor of Taxes contended that the latter sum ought not to have entered into the account for the purpose of ascertaining the profits and gains on which Income Tax was to be assessed under Schedule D of the Income Tax Acts.

This contention was based upon the Fourth Rule relating to assessment under Schedule D, which is in the following terms: "In estimating the amount of the profits and gains arising as aforesaid, no deduction shall be made on account of any annual interest or any annuity or other annual payment, payable out of such profits and gains." It was said that this expressly prohibited any deduction in respect of the annuities paid by the Society, that the Income Tax must be paid upon the basis that the entire sum received by the Society as the consideration for its undertaking to pay annuities was to be assessed as profits and gains, and that the Society would have a right to deduct Income Tax when paying the annuities to those who had purchased them. The contention of the Surveyor of Taxes has been sustained by the Divisional Court and the Court of Appeal. The learned judges in the Court of Appeal founded their judgment mainly upon a prior decision of that Court in the case of the *Alexandria Waterworks Company v. Musgrave*, which I shall have to consider presently.

It cannot, of course, be denied that as a matter of business profits are ascertained by setting against the income earned the cost of earning it, nor that, as a general rule, for the purpose of assessment to the Income Tax, profits are to be ascertained in the same way. "Money wholly and exclusively laid out or expended for the purpose of a trade, manufacture, adventure or concern" may, by the first of the rules applying to both the preceding cases, be taken into account in estimating the balance of profits and gains to be charged. It seems to me beyond question that the payments made by the Society to its annuitants are within these words. And those carrying on the business of selling annuities would be assessed on quite a different principle to those carrying on other businesses if their gross receipts were to be treated as profits without regard to the payment to which in consideration of those receipts they had bound themselves.

But it is said, and truly, that the Income Tax Acts have laid down certain rules to be applied in determining how the tax is to be assessed, and that even if the result should be to tax as profits and gains what cannot properly be so called, the requirements of the Acts must, nevertheless, be complied with. The controversy mainly turns upon the construction to be put upon the Fourth Rule which I

have already quoted, but the 102nd Section of the Act must be read in connection with it. That section, after enacting that Income Tax shall be charged upon all annuities, yearly interest of money, or other annual payment, provides that in every case where the same shall be payable out of profits or gains brought into charge by virtue of this Act no assessment shall be made upon the person entitled to such annuity, interest, or other annual payment, but the whole of such profits or gains shall be charged with duty on the person liable to such annual payment without distinguishing such annual payment, and the person so liable to make such annual payment out of the profits or gains charged with duty, "shall be entitled to deduct the Income Tax from the annual payment", and the person to whom payment is made is to allow such deduction.

It was admitted that reading this enactment in conjunction with the Fourth Rule the prohibition contained in that Rule only applied when the annuity was payable out of profits and gains brought into charge by virtue of the Act. What is the meaning of "profits and gains" as found in the concluding words of the Fourth Rule? Unless the context requires a different meaning, or the words appear to be used throughout the Act in another sense, I think they must be construed according to their ordinary signification. When we speak of the profits or gains of a trader we mean that which he has made by his trading. Whether there be such a thing as profit or gain can only be ascertained by setting against the receipts the expenditure or obligations to which they have given rise.

In the taxing provision of the Income Tax Acts, the tax is imposed in respect of the annual profits or gains arising or accruing to any person from any trade "to be charged for every 20 shillings of the annual amount of such profits or gains." Here it cannot be doubted, I think, the words "profits or gains" are used in the sense I have indicated. There is nothing in any part of the Act to show the contrary, indeed, when the Rules are considered, they confirm the view that this meaning is to be attributed to the words. Their general purpose is to prohibit certain matters being taken into account which ought not to be set against the receipts when profits are being ascertained. But the scheme of the Act obviously is to tax not receipts, but profits properly so called.

The 100th Section prescribes that the duties under Schedule D shall be assessed and charged under certain rules. The first of these is that the duty shall be computed on a sum not less than the full amount of the balance of the profits or gains of the trade, &c., upon a fair and just average of three years. The expression "balance of the profits and gains" is not a happy one, but the meaning obviously is the balance arrived at by setting against the receipts the expenditure necessary to earn them.

When we come to the Third Rule, and the first of the rules "applying to both the preceding cases", it must be admitted that the words "profits or gains" are not always used in their proper or ordinary sense. The Third Rule, for example, provides that in estimating the balance of profits and gains chargeable under Schedule D, "No sum shall be set against, or deducted from, or "allowed to be set against, or deducted from such 'profits or gains'

“on account of any sum expended for the repairs of trade premises, or for the supply, repairs, or alterations of trade implements, beyond the sum usually expended for such purposes on an average of three years.” Here it is obvious that the “profits or gains” against which the cost of repairs is not to be set, or from which it is not to be deducted, cannot be the profits or gains of the business properly so called. The subject matter necessitates the conclusion that language is being employed loosely and inaccurately. The Rule contemplates the making up of a balance sheet, and deals with what may be put on the debit side of it. And the First Rule “applying to both the preceding cases”, which commences with the same words as the Rule I have just referred to, evidently requires a similar construction. But in the Fourth Rule, which your Lordships have specially to construe, the language employed is very different. I can find nothing to show that the words “profits and gains” were intended to be understood otherwise than in their proper sense. The Rule prescribes that in estimating the amount of the profits and gains “arising as aforesaid”, no deduction shall be made, &c. This seems to refer back to the charging provision which I have quoted, and I have already pointed out the meaning which must be there attributed to “profits and gains.” And when read in connection with Section 102, the rule clearly relates only to annuities payable out of profits and gains “brought into charge” by virtue of the Act, and to cases in which the trader is liable to make the annual payment out of the profits or gains charged with duty. I have pointed out that in the charging provision it is the profits and gains properly so called, not the gross receipts, which are brought into charge. If these do not exceed the expenses, nothing is payable by way of Income Tax. In the Fourth Rule there is no mention of a balance, nor is it said, as in the other Rules, that no sum shall be set against the profits “on account of any sum expended” in the payment of annuities. I can see no reason why, if the intention was that contended for by the Crown, these words should not have been inserted in the Third Rule instead of a Fourth Rule with altered language being added. The expression in this Rule is altogether changed, the deduction prohibited is confined to annuities “payable out of” the profits and gains. This implies that the profits and gains out of which the annuity is payable are already ascertained. In the other Rule there is no reference to the fund out of which the moneys expended are payable, the only thing regarded is the fact of the expenditure and the purpose for which it has been incurred. It would be a strange use of language to speak of the annuities as “payable out of” the gross receipts of the trader merely because he is under liability to provide for them.

I think the Fourth Rule was primarily designed to meet such a case as that in which a trader had contracted to make an annual payment out of his profits, as, for example, when he had agreed to make such a payment to a former partner, or to a person who had made a loan on the terms of receiving such payment. But for the rule it might plausibly have been contended that in such a case a trader was only to return as his profits what remained after making such payment. It is unnecessary to define the limits within which the rule applies,

but I am unable to agree with the Court below that it is applicable to the present case. The annuities are not in my opinion payable out of the profits and gains of the Society. Until the payments which they necessitate have been taken into account, it cannot be ascertained whether there are any profits and gains or not.

I do not think this view conflicts with the decision in the case of the *Alexandria Waterworks v. Musgrave*. The payments of interest to the debenture holders were made out of the profits. These were ascertained by deducting from the moneys earned the expenses incurred in earning them, and of these expenses the payments to the debenture holders formed no part. A portion of the capital was raised by shares, and another portion by debentures. There was no more reason why interest on the debenture capital should be deducted from the profits than interest on the share capital. Supposing the whole capital had been raised by shares there would have been no pretence for making any such deduction, and the profits earned by the adventure could not be different according as the capital was raised wholly by shares, or partly by shares and partly by debentures. It is by no means clear that the case was not within the prohibition of the third rule.

Although it does not appear to me that the view of the present case which I have placed before your Lordships is inconsistent with the decision in the *Alexandria Waterworks Company v. Musgrave*, I think it is in conflict with the *Mersey Loan Company v. Wootton*, but the reasoning which has led me to my present conclusion applies equally to the facts of that case.

Lord Field.—My Lords, I have been asked by my noble and learned friend, Lord Morris, to express his entire concurrence in the judgment which has been moved, and I have no difficulty whatever in adding my own.

Questions Put.

That the judgment appealed from be reversed.

The Contents have it.

That the Respondent do pay to the Appellants their Costs, both here and below.

The Contents have it.

CORRESPONDENCE.

CONSUMPTIVE FAMILY HISTORY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The recent papers by Mr. Manly and Dr. Lyon on the extra risk arising from a consumptive family history (*J.I.A.*, xxx. 97. 120), and the discussions which followed, are among the most interesting which have occupied the attention of the Institute for some time, because the subject is one of such intense practical

importance. Anything which will aid in a more scientific and accurate classification of applicants is an onward step, and is of the greatest value to life assurance in general. I venture, therefore, to draw attention to a few points that arise in connection with these papers. We are all under too great an obligation to their authors to be at all super-critical, and yet I feel that it is not desirable to allow the matter to pass without comment.

1. The conclusion that "cases of consumption in the family, other than in a parent, may be disregarded where the applicant is perfectly sound", is one which cannot be accepted without strong confirmatory evidence. Even the statistics which Mr. Manly offers in its support are subject to criticism. The records include not only the cases which show an admittedly consumptive family history, but those which contain doubtful causes of death, which, in the opinion of Dr. Lyon, should really be classified as consumption. Now there is no doubt that Dr. Lyon did his work thoroughly and conscientiously, but it is quite possible that another equally competent and conscientious physician might make a different classification. So far as an outsider can judge, his inclusion of so many "childbirth" cases as being really consumption, leads to the belief that he has been unduly severe in his examination of the applications, and that the whole record has, perhaps, been swelled by the addition of a number of cases which, though doubtful, are not really consumptive at all. If this be so, the effect would probably be noticed, especially in the class containing the cases of supposed consumption among brothers or sisters, for these members of the family are more numerous than the parents, and probably more frequently separated from the applicants by long distances than the latter, and thus more likely to furnish cases with indefinite or unknown causes of death. The inclusion of any considerable number of cases which are, in spite of a doubt attaching to them, really non-consumptive, and therefore show a favourable mortality, would have the effect of greatly reducing the death rate from consumption in the whole class, and render any deductions therefrom valueless. I do not say that this has been done, but if individual judgment is allowed to enter at all into the classifications, a doubt must inevitably arise as to their reliability. There are surely enough cases to be found in which the record is beyond dispute, to make it unnecessary to include "doubtful" cases at all.

The Washington Life investigated 694 deaths which had a "doubtful" family record, owing to the presence of cases of "childbirth", "change of life", "exposure", "grief", "fever", "general debility", &c., and found that only 111 or 15.99 per-cent of these terminated in death by consumption, as against 16.02 per-cent of the total cases having no hereditary consumptive taint. We well may doubt, therefore, whether Dr. Lyon's apparent classification of all such cases as consumption is sound.

2. The conclusion that the record as regards brothers and sisters may be practically disregarded in applicants who are personally sound is, moreover, at variance with statistics derived from some other sources at least. The experience of the Mutual Life of New York contains a comparison between the family records of their 1,031

deaths from consumption, and an equal number of their deaths from accidents and zymotic diseases, with the following results:

Deaths	Total	SOME MEMBER OF FAMILY CONSUMPTIVE							No Member of Immediate Family Consumptive
		Father and Mother	Father only	Mother only	Father and Brother or Sister	Mother and Brother or Sister	Two or more Brothers or Sisters	One Brother or Sister	Total
Consumptives . . .	1,031	2	36	40	7	16	22	71	194
Non-Consumptives . .	1,031	0	18	20	2	12	5	45	102
									837
									929

These results were summarized as follows:

Deaths	PARENT CONSUMPTIVE			Brothers or Sisters only	Total with Family Taint
	Father	Mother	Total		
Consumptives . . .	45	56	101	93	194
Percentage	9.79	9.02	18.81
Non-Consumptives . .	20	32	52	50	102
Percentage	5.04	4.85	10.89

These figures, so far as they go, show that the history of brothers and sisters is of almost precisely the same value as that of the parents, since the ratios of the consumptive cases were just about double in each class. Important and reliable statistics of this kind should at least make us hesitate before coming to a different decision.

3. Some of the speakers apparently considered that since a consumptive tendency can only be inherited from the parents, the collateral record can be largely, if not entirely, disregarded where neither parent died of consumption. But, as Dr. Lyon pointed out, it is but rarely that the disease is inherited. What is inherited is a weakness, a lack of ability to resist as successfully as more robust persons, the exposure and infection which come in greater or less degree to all. The fact that the parent did not die of consumption is not necessarily proof of that parent's robustness. Is it not, then, a matter of the first importance to see how the brothers and sisters of the applicant, children of the same parents, have withstood the physical trials of life?

Yours truly,

Montreal,

T. B. MACAULAY.

16 Sept. 1892.

P.S.—Attention may also be drawn to the fact that the average age at entrance was not the same in the different groups. In tables

I and II, where father or mother alone died of consumption, it was 33-58 years, while in the three "collateral" tables it was 37-80 years. If there be any value then in the theory that "the greatest risk of hereditary taint being developed is in early manhood, and when that time is past and the applicant is of sound constitution, and in every respect, but for the family history, a first-class life, then the risk is small", it would naturally follow that the class of collaterals, being older, would show a more favourable mortality than the lineals, on the ground of age alone. The difference of four years is probably, however, only a minor factor in bringing about the results shown. But in this connection, if I might be allowed to throw out a hint, I would suggest that Mr. Manly would still further increase the obligation of the members to him if he could find time to divide his groups, say, at 40 at entrance (not by age at death), and tell us briefly how the actual mortality among those entering after age 40 compared with the expected. This would help in forming an opinion as to the theory in question. It would probably be found, however, that personal physique, as shown chiefly by the relation of weight to height has much more to do with neutralizing a consumptive taint in the family than mere age.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having been favoured with a perusal of Mr. Macaulay's letter, I should like to be allowed to make a few observations upon it at once.

1. In the first place, Mr. Macaulay, in quoting my conclusion with reference to "cases of consumption other than in a parent, &c.", has omitted my special qualification: "so far as these observations have any weight." The President, in the course of the discussion which followed the reading of my paper, expressed a fear that the conclusion might be accepted without the qualifying words, and apparently his fears were justified.

2. So far from "childbirth" cases having been classified as "consumption", as assumed by Mr. Macaulay, the greatest care was taken to separate them; and I do not see what other conclusion could be drawn from my "Tables of Observations XI and XII" (where the cases of consumption alone in collaterals are collated) than the one I have drawn.

3. I do not consider the experience of the "Washington Life" sufficiently long to give any weight to their deductions as to the causes of death. The experience of the "Mutual Life of New York", as given by Mr. Macaulay, is very interesting, *medically*, and is an additional proof, if such were needed, that children born of consumptive parents are more likely to contract that disease than others. I am also willing to admit that, amongst those who die of consumption, there may be found a greater number who had brothers or sisters die of that disease, than amongst a similar number who die from other causes.

This, however, does not prove that the *rate of mortality* amongst those who have had a brother or sister die of consumption is different from that of the average class of lives.

Perhaps my views, which are really the outcome of my investigations, will be rendered clearer by the following explanation.

First, then, I am inclined to believe, and in this I am supported by Dr. Lyon, that every person has a constitution which makes him more susceptible to contract one or more forms of disease and less susceptible to contract others; and that the child inherits a constitution with certain susceptibilities. Now, I hold that consumption is a particular case of a general theorem; and that if the family histories of those who die from typhoid fever, or cancer, or diabetes, or any other definite form of disease, were collated with reference to those diseases in the same way as has been done in Mr. Macaulay's statistics with reference to consumption, the results would be similar. The child born of a consumptive parent is severely handicapped, because he is not only born with a constitution susceptible of cultivating the germ, but he generally passes many years of his life in the midst of those germs.

Secondly, if my theory be correct, then I hold that it is unfair to select one form of disease in the parent and stamp the child as a peculiarly hazardous life, *unless it can be shown that the rate of mortality amongst such lives differs materially from that average rate upon which our premiums are based.*

I do not consider that the actuary is concerned with the *causes of death*, and that, therefore, statistics given in the form in Mr. Macaulay's letter are not only useless to him, but often very misleading. He is, however, concerned—and very seriously concerned—with the *rate of mortality*, and the premium to be charged for the special risk, if any, which is supposed to arise from a certain constitutional inheritance. That, I consider, can only be decided by an investigation such as I have undertaken in my "Attempt to measure the Extra Risk arising from a Consumptive Family History, &c."

Yours truly,

London,

23 December 1892.

H. W. MANLY.

THE INSTITUTE OF ACTUARIES.

EXAMINATION OF THE INSTITUTE, OCTOBER 1892.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).

Examiners—F. E. COLENSO, Esq., M.A.; G. F. HARDY, Esq.;
G. MARKS, Esq.; H. C. THISELTON, Esq.

First Paper.

1. Give a brief description of the various books usually employed in keeping a set of accounts by "double entry," pointing out those that are essential to the method and those that are merely auxiliary.

2. Apply the processes of contracted multiplication and division to obtain the following products and quotients to five places of decimals:

$$(a) \quad 37.9858761 \times .4872635.$$

$$(b) \quad .0072246 \times .56287.$$

$$(c) \quad 58.732196 \div 34.4827943.$$

$$(d) \quad 16 \div 1.4876532.$$

3. State and prove the rule for finding the Greatest Common Measure of two or more algebraical expressions, explaining what factors may be introduced or rejected in the process.

Prove that if a and b be any two integers greater than unity,

$$a^3b - ab^3$$

is always divisible by 3.

4. Solve the following equations:

$$(i) \quad \begin{cases} x(bc - xy) = y(xy - ac). \\ xy(ay + bx - xy) = abc(x + y - c). \end{cases}$$

$$(ii) \quad \begin{cases} z(x^2y)^{-1} = 1.5. \\ yz^2x^{-1} = 18. \\ xy^2z^3 = 108. \end{cases}$$

5. Having given the sum, the first term, and the common difference of a series in A.P.: show how to find the number of terms; and explain how you would interpret negative and fractional results.

Prove that if any term n of an arithmetical progression be the $\left(\frac{n+1}{2}\right)$ th of the series, the sum of the first m positive terms $= m^2$.

6. It is found that it takes 8 minutes to fill a certain cistern by means of one tap, 24 by means of another, and only $5\frac{1}{2}$ minutes when both taps are used. Show that this indicates that the cistern has a leak which would empty it in 66 minutes.

7. A bag contains 10 tickets, numbered from 1 to 10. A ticket is drawn and replaced four times. What is the chance that the sum of the numbers drawn is 33?

If in that case the drawer receives £1,000, what is the value of his expectation?

8. Show that the differences of the logarithms of consecutive integers are nearly constant when the integers are large.

Evaluate by means of 4-figure logarithms:

$$\{(12.34)^{5.67} \times (8.9)^{-2.345}\} \div \{(67.89)^{1.23} \times (45.67)^{-8.9}\}.$$

9. Find the sum of

$$\frac{1}{2.4.6} + \frac{1}{4.6.8} + \frac{1}{6.8.10} + \&c., \text{ ad inf.}$$

Prove that the sum of $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ ad inf. $= \infty$.

10. If $u_x = \frac{2x+7}{5x^2+26x+5}$, find $\Delta^2 u_x$, expressing your result in its simplest form.

11. Show that the equation

$$Ax + By + C = 0$$

represents a straight line; and find the length of the perpendicular falling upon it from the point (hk) .

12. The angles at the base of an isosceles triangle are equal to one another, and if the equal sides be produced the angles on the other side of the base are equal to one another.

Second Paper.

1. Multiply together $1\cdot3\dot{4}$ and $2\cdot5\dot{6}\dot{7}$, and extract the square root of the product correctly to seven significant figures.

2. Solve the following equations:

$$(i) \quad \frac{1}{x} - \frac{1}{1+x} + \frac{1}{2+x} - \frac{1}{3+x} - \frac{1}{4+x} + \frac{1}{5+x} - \frac{1}{6+x} + \frac{1}{7+x} = 0.$$

$$(ii) \quad \sqrt[5]{(1+x)^2} - \sqrt[5]{(1-x)^2} = \sqrt[5]{(1-x^2)}.$$

3. Compare the values of the arithmetic, geometric, and harmonic means between two quantities; and show that if a be the arithmetic mean between b and c , and b the geometric mean between a and c , c will be the harmonic mean between a and b .

4. One of two clocks, which both indicate the true time to-day at noon, gains a second an hour, and the other loses three seconds in two hours. When next will they both indicate the same time, and when next will they both indicate the true time?

5. Two vessels contain mixtures of wine and water; in one there is thrice as much wine as water, in the other five times as much water as wine; find how much must be drawn from each to fill a third vessel which holds seven gallons, in order that its contents may be half wine and half water.

6. What are oranges a gross, if, when their price is lowered five pence a score, 76 more are given for 19s.?

7. Find the number of combinations of n different things taken r together, without assuming the formula for the number of permutations.

How many different permutations, each consisting of three letters, can be formed from the letters of the expression

Statutory declarations?

8. Find the sum of the products of the co-efficients of the powers of x taken two together in the expansion of $(1+x)^n$.

9. Compare the chance of throwing 10 in a single throw with three dice, with the chance of throwing 11.

10. Write down the series for $\log_e (1-x+x^2)$ as far as x^6 , and show that

$$\log_e 10 = 20 \left\{ \frac{1}{9} + \frac{1}{3} \left(\frac{1}{9} \right)^3 + \frac{1}{5} \left(\frac{1}{9} \right)^5 + \dots \right\} \\ + 6 \left\{ \frac{3}{253} + \frac{1}{3} \left(\frac{3}{253} \right)^3 + \frac{1}{5} \left(\frac{3}{253} \right)^5 + \dots \right\}.$$

11. Find by the method of Finite Differences the sum of n terms of the series

$$1+6+15+28+\&c.,$$

and of the series

$$1^2+2^2+3^2+4^2+\&c. \dots$$

12. Given the co-ordinates of the angular points of a triangle, find the equations of the three lines joining the middle points of the sides to the opposite angles, and show that such lines pass through the same point. What are its co-ordinates?

13. If the square described upon one of the sides of a triangle be equal to the squares described upon the other two sides of it, the angle contained by these two sides is a right angle.

If the square on one side of a triangle be less than the squares on the other two sides, the angle contained by these sides is an acute angle; if greater, an obtuse angle.

The Foundation of the Institute of Actuaries.

MR. T. E. YOUNG'S valuable note upon the circumstances attending the foundation of the Institute, which appeared in the number of the *Journal* for October 1890 (*J.I.A.* xxviii, 436), will be fresh in the recollection of our readers. As an addendum to that note, the following extract from the *Athenæum* of 22 July 1848, written in reference to the proposed Institute of Actuaries, will be read with interest, as showing an independent opinion as to the principles upon which such an Institute should be based. The extract has been sent to us by Mr. G. H. Ryan, who found it

in the biography of John Francis, for 50 years publisher of the *Athenæum* (vol. i, p. 130), published a few years ago.*

“Looking at the science of which an actuary practises the application—and which, though at present it principally relates to *life* contingencies, yet must be held to include all *contingencies* to which calculation of probability can be applied—we should see every room to welcome the formation of a society which should take its place among those already established for the promotion of geology, astronomy, geography, statistics, &c. Such a society should be as republican in its constitution and as liberal in its principles as those which have preceded it. No man should have any rank except what he makes for himself out of the opinion of his comrades: no class of privileged members should exist.”

* “John Francis, Publisher of the *Athenæum*. A Literary Chronicle of Half-a-Century.” Compiled by John C. Francis. (London: Richard Bentley & Co.—1888.)

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NOTICE TO CORRESPONDENTS.

Communications for this *Journal* must be sent in at least one month prior to the day of publication, or their insertion will in all probability be deferred.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Enfranchisement of Leaseholds, and the Taxation of Ground Rents, Chief Rents and kindred Charges on Land in England and Wales.

I.—By THOMAS KYD, F.I.A., *of the Northern Assurance Company.*

II.—By ARTHUR WYNDHAM TARN, F.I.A., *of the Westminster and General Life Assurance Association.*

[To these Essays were awarded equal Prizes in the SAMUEL BROWN COMPETITION, the syllabus and conditions of which are given in *J.I.A.*, xxix, 479.]

[Read before the Institute, 30 January 1893.]

MR. KYD'S ESSAY.

I.—LEASEHOLD ENFRANCHISEMENT.

THE realm of England was claimed by the Conqueror and his successors on the throne. On the theory of possession by conquest, the feudal system of tenure was built and developed. The Sovereign granted land to his soldiers and favourites, or left it in possession of the ancient holders, at his own pleasure. In return, those who enjoyed the property undertook various obligations to the lord paramount, such as military services. The grants were at first made for the lifetime of the holders; but ere long the rights and duties were

Feudal Tenure.

extended to the succeeding heirs. Landowners holding thus of the king conveyed portions of their estates to their retainers, in return for services to be rendered. These services were usually either of a military or an agricultural character. With the advance of time, the progress of civilization, and the development of the arts of peace, the latter mode of compensation for the use of land gradually superseded the former; and after the Restoration the feudal burdens were abrogated by law. Since the abolition of these uncertain incidents of land tenure, the obligations subsisting among the holders of the several rights and interests pertaining to lands and tenements have been reducible to a money standard, with at least an approximation to justice and accuracy.

The same modern habits which discarded the uncertain burdens of knight service or tenure in chivalry have also, in much more recent days, generally discouraged the varying forms of entail, life tenure and copyholds, with their contingencies and anxieties; and substituted tenures either in perpetuity or else for a specified length of time. All landed interests now enjoyed by persons other than the representatives of the ancient holders have been created and acquired under definite contracts, carefully expressed and well understood by the parties concerned. The terms of conveyance are clearly and fully set forth in the lease or other instrument embodying the contract.

There are two classes of leaseholds—those held during the existence of a life or lives, and those granted for fixed periods. Both classes may be subdivided. A life lease may depend on the existence of one individual; it may terminate on the death of the last of several persons named in it; or it may endure during the existence of two or more lives to be named successively, the second during the lifetime of the first or at his death. Otherwise it may be for a term of years certain, and for the lifetime of the person holding it at the end of that fixed term, or of a person then to be nominated. Leases for terms-certain differ among themselves principally in the length of the period of lease, which may be for any time from one year or less to 999 years or more. Leases for lives and leases for years may alike be subjected to another classification, dependent upon the nature of the considerations in respect of which they are granted. Such considerations may consist of three or more portions. First, a payment at the commencement of the lease by way of fine or premium. Secondly, the annual ground rent

*Leases for Lives
and Terms.*

exigible during the subsistence of the lease. Thirdly, the expected increase in value of the property at the end of the lease, in consequence of buildings to be erected by the lessee, or operations undertaken by him. Other incidental remuneration may be stipulated for, such as a payment on the death of one of the persons named, or on the efflux of a stated period of time. In most cases it is the second and third of these classes of consideration that receive greatest attention on the negotiation of a lease. The shorter the lease, the more importance attaches to the reversionary expectations. When the period exceeds 100 years, the amount of the ground rent is the consideration that dwarfs all others in the eyes both of landowner and lessee.

Lands are generally leased either for urban or for agricultural uses. For the purposes of farming, a short lease, such as one for a period of 19 years, in most cases meets the views both of proprietor and of tenant. The question of legislative interference with such leases in England (here excluding Wales) has scarcely been mooted; and a measure for their compulsory enfranchisement is not within the horizon of practical politics. But land in towns and villages, to be used as sites for dwelling houses and trade buildings, must naturally be the subject of bargains of a more lasting character. Such contracts are embodied in building leases; and it is with reference to them that a movement has arisen for leasehold enfranchisement. A Leaseholders' Enfranchisement Bill was submitted to the House of Commons in 1883, and one or more measures dealing with the question have been announced every session since that time. Movements of a more official character have been made in similar directions, in Parliament and by the Crown, since the period of the first

Reform Bill. In 1852 and 1858 Acts were passed for the compulsory enfranchisement of copyholds.

And in 1839 and 1849 Royal Commissions had enquired into Church leases and episcopal revenues; and the Corporation

of Ecclesiastical Commissioners had been formed, with powers to enfranchise leases of Church property.

In 1884 a Royal Commission was nominated on the Housing of the Working Classes; and that Commission, in a Supplementary Report, adopted by a majority of its members, formally recommended "legislation favour-

able to the acquisition, on equitable terms, of the freehold interest on the part of the leaseholder." Following on this recommendation, a Select Committee of the House of Commons

Copyhold Enfranchisement.

Leases of Ecclesiastical Property.

Royal Commission on Working-Class Houses.

Select
Committees of
Commons.

was appointed in 1886 for the purpose of enquiring, among other things, into "the question of imposing a direct assessment on the owners of ground rents", and into "the expediency of giving to leaseholders facilities for the purchase of the fee-simple of their property." Select Committees with similar instructions were appointed in 1887 and 1888; and the latter Committee was continued until 1892. These Committees examined 106 witnesses from England and Wales, and 16 from Scotland, and had a great deal of Irish evidence before them also. They made various reports to Parliament, but their members did not agree with one another in their recommendations. The evidence laid before them may be assumed to give a full account of the facts relevant to the enquiry, and of the arguments on all sides. Extremely voluminous, it has, for the most part, been summarized and digested both by advocates and opponents of legislation. Of the views of the former class, we may take as a type the volume on *Leasehold Enfranchisement*, in the *Imperial Parliament* series, embodying, as it does, the summary of the evidence to the Royal Commission given by Mr. Howard Evans, in his *Doom of the Leasehold System*. The latter class will be best represented by three volumes issued by Messrs. Cassell & Co., entitled *Town Holdings*.

A notice in regard to the enfranchisement of copyholds and Church leases may precede consideration of urban leasehold enfranchisement in general. Copyhold estates originated time

out of mind; and the conditions on which they are held are regulated by ancient usage, according to the legal aphorism, "Custom is the life of copyholds." They may be held in fee or for a life or lives; and if the tenure depends on life, the estate at the tenant's death falls in a few cases into the hands of the lord of the manor; but more usually the tenant has the right to nominate another life in succession. Incident to the tenure are various exactions to which the lord is entitled, such as quit rents, heriots, fines and relief, some of them formerly arbitrary in amount, and payable at uncertain and inconvenient times. In specified contingencies the estate may revert to the lord of the manor. But, except in such cases, the lord has no other reversionary interest in the copyhold. Under the Acts 15 & 16 Victoria, cap. 51, and 21 & 22 Victoria, cap. 94, enfranchisement of copyholds may be effected compulsorily, in the option either of the tenant or of the lord. Failing agreement between the parties, the value of the lord's interest in the estate is to be

Copyholds.

ascertained by arbitration, and the enfranchisement to be effected by an award confirmed by the Commissioners. The law makes a distinction between the ease in which the tenant demands enfranchisement from an unwilling lord of the manor, and that in which the lord forces enfranchisement on an unwilling tenant. If the demand proceeds from the tenant, he must provide the cost of his emancipation in cash, or allow it to be charged on the estate by way of mortgage. If the lord compels the enfranchisement, the value must be commuted into a rent charge, varying with the price of grain, unless fixed in amount by agreement with the tenant. There is one class of copyholds excepted from the operation of the enfranchising Acts. It consists of those copyholds for lives which at death revert to the lord of the manor. The existence of the reversion in his favour is held to remove the copyholds out of the class in which enfranchisement can be equitably enforced. These copyholds with a reversion are, it may be noticed, the only ones which have much analogy to urban leaseholds.

The Church leases, on which two Royal Commissions reported in the early years of Her Majesty's reign, were contracts for lives, or for terms not exceeding 40 years. Church Leases. In many cases the lessors, who were the bishops or deans and chapters, lacked powers, not only to enfranchise, but even to let for a period long enough to encourage substantial building. To obviate objections incident to a tenure so temporary and uncertain, to remove temptations of swelling episcopal or capitular revenues by fines levied on premature renewals, and to exchange a uniform management for the differences of administrations in the various sees, Ecclesiastical Commissioners for England and Wales were incorporated with extensive powers, and the whole properties were vested in them. These Commissioners, however, have not enfranchised leases for terms of years, but they have granted enfranchisement of life leases.

The different classes and subdivisions of urban tenure sometimes exist side by side in the same locality. But, in general, each district of the country has its own favourite forms of holding. Varieties of Urban Tenure. A map of England and Wales, marked or coloured distinctively according to the different tenures of land, would give a clear impression of great variety. In general, it appears that freehold purchase is the prevailing tenure; dividing, with estates subject to a perpetual chief rent, the urban holdings in the great majority of the smaller towns of

England. And, if in the same category we include leases for periods of 500 years, such as prevail extensively in Sheffield, and of 999 years, such as exist in Huddersfield, in Bolton, Bury and Rochdale, and, indeed, Liverpool excepted, throughout Lancashire generally (nine-tenths of Manchester being held in this way), it appears that the superiors or original holders of urban land have parted with all practical interest in the reversion, except in a comparatively small number of cases. Prior to the Conveyancing Act of 1881 there was, indeed, a certain reversionary interest remaining to the lessor, in the case of a lease for any period, however prolonged, inasmuch as a breach by the lessee of one of the covenants, although technical and trivial, forfeited the lease, so that the granter could resume possession. This oppressive stipulation the Courts, however, can now set aside; while, on the other hand, their powers to enforce covenants by reasonable penalties have actually, in some quarters, made long leases preferred to freeholds; since by the covenants of the former tenure, one lessee can obtain protection from another, and so prevent injury to the amenity of a whole neighbourhood. Among the towns where building leases for terms of 99 years or shorter periods extensively prevail we may reckon Newcastle, Liverpool, Southport and St. Helens, Guisborough, Skipton, Huddersfield and Great Grimsby, Birmingham, Oxford and Cambridge, Croydon, Dover, Folkestone, Eastbourne, Brighton, Worthing and Bournemouth, many towns in South Wales, and, most important of all, London itself. A few leases for lives are found in various districts, as in Lancashire and Berkshire, but chiefly in towns of the south-west, as Bridgwater, Tavistock, Devonport, Redruth, St. Austell and Camborne.

Information on the subject of continental tenure was obtained by Government in 1884. It appears that the leasehold system is little known in most European countries. A conspicuous exception is France, where leases for terms of years prevail extensively, most of all in the larger towns. They are also found in Belgium to a small extent. In Denmark life leases exist in rural districts, although the ordinary tenure is freehold, except in Copenhagen. In Scotland, also, the urban holding is generally freehold, subject in most cases to a feu duty or chief rent in perpetuity. It will be observed that both at home and abroad, leasehold tenure seems rather to favour localities where land is costly. It is in such places that acquisition of the fee-simple is felt to be most onerous.

Foreign Tenure.

Scotch Tenure.

It should be remembered that building restrictions for the public benefit, frequently imposed here in contracts between freeholder and lessee, may be enforced on the continent by municipal regulations of a much stricter character than would be relished by the freeborn Briton.

We have found the term "enfranchisement" used in connection with the Ecclesiastical Commission half a century ago, and also in legislation affecting copyholds. In Church leases that term referred to transactions for which the consent of both lessor and lessee was necessary. In copyholds it expressed an operation which either party could enforce against the other. In both these cases, therefore, parties were on a footing of equality. In ecclesiastical leases neither could act alone. In copyholds either could do so. The advocates of leasehold enfranchisement, however, mean something quite different by the phrase. On their lips, and in the Bills which they have brought into Parliament, enfranchisement is to be granted only at the instance of the lessee, who may choose his own time for the purpose. The lessor has no corresponding option, but must simply wait the pleasure of his tenant, who, indeed, if he holds several independent leases, may compel the enfranchisement of any one or more of them, leaving the remaining ones unaffected. The consideration for the property thus appropriated is to be ascertained in modes prescribed by the several Bills submitted to Parliament. Colonel

Enfranchise-
ment Bills.

Hughes's Bill, backed by five Conservative legislators, is in this respect simplicity itself. It is restricted to cases where the ground rent is not more than one-half the rateable value. That ground rent is valued as an annuity for the currency of the lease discounted at 5 per-cent, and the reversion is assumed equal to an annuity of the rateable value of the property at the time of its seizure, deferred for the period of the current lease, and discounted at the same rate of interest. The sums so ascertained are payable to the freeholder, unless he exercises an option which the Act grants him of taking instead a perpetual rent charge. In that case the rent charge is to be calculated at 5 per-cent in respect of the ground rent, and at 4 per-cent in respect of the reversion, notwithstanding that both ground rent and reversion are capitalized at 5 per-cent, as part of the same enforced transaction. It is scarcely credible that the sponsors of the Bill understood the effect of this alteration in the rate of interest. In the case of a lease having 71 years to run it would cancel exactly one-half of the value of the reversion.

Their competence to deal with such questions may be further gauged by the consideration that the Bill values a life annuity as equal to an annuity-certain for the number of years shown by the expectation of the life, and also treats an annuity payable until the death of the last survivor of any number of lives, as equal in value to an annuity-certain for the period representing the expectation of the youngest life. Mr. Rowland's Bill, introduced in session 1890-91, may be taken as the other typical leasehold enfranchisement measure before the public, backed as it is by Mr. Broadhurst and Mr. Reid, the authors of earlier Bills on the subject, and by several other members of like political faith, as well as by a Conservative baronet representing a south-western borough where life leases prevail. This Bill provides that the purchase money shall be the value of the present interests and reversions as fixed by the County Courts. Provision is made for a rent charge instead of a payment in cash; but the option is here given to the lessee, with notice to the freeholder; and the terms of conversion are left to the discretion of the Judge. Compensation to the landowner for severance of the enfranchised property from the rest of his estate is tacitly excluded; although it is obvious that the enfranchisement of half an acre in the middle of a square mile of leaseholds might effectually take from him the power of dealing with his property as a whole. Messrs. Broadhurst and Reid treat the question of severance with a light heart in their volume on *Leasehold Enfranchisement*, and say the objection is of the most unsubstantial kind. They base their view on the length of time that in any case must elapse before the estate can be dealt with as a whole. They forget that 20 or 30 years, although great part of a human life, is but a short space in the history of a family estate, and that landed proprietors attach great importance to settlements extending over much longer periods. And it is well known that some excellent residential districts in London, such as Belgrave Square, Berkeley Square, Eaton Square and Grosvenor Square, owe their amenity to the fact that they comprise many leasehold houses all under the ultimate control of the owner of a single large estate. Evidence on the subject was given before the Select Committee, not only by Mr. Vigers, a Metropolitan surveyor, but by Mr. Johnson, Town Clerk of Nottingham; Colonel Sackville-West from Carnarvon; and Mr. Matthews, who gave particulars of a valuable public improvement effected by the Trustees of King Edward's School at Birmingham. If individual lease-

holders are empowered to enforce enfranchisement, such improvements will be possible only by Town Councils or other local authorities, exercising statutory powers at the cost of ratepayers.

Let us compare these proposals with the compulsory powers that Parliament has granted in the past. Such
Precedents. powers have frequently been given to the Crown, to municipalities and other public bodies, and especially to railway companies. They are conferred by specific enactment in individual cases, after enquiries and reports of Select Committees of both Houses of Parliament, directed among other things to ascertain that the powers are required in the public interest. And when granted they are subject to the provisions of the Lands Clauses Consolidation Acts, which entitle the unwilling seller not only to the value of the property taken away, but also to compensation for damage done by severing that property from the remaining possessions of the owner, or injury done to them in any other way. Besides these statutory rights, the denuded landlord has also a title, founded on invariable custom, to an additional allowance for compulsory deprivation. This may be taken as from 10 to 50 per-cent of the value otherwise ascertained. From the evidence of railway solicitors and surveyors before a Select Committee of the House of Lords, we learn that the average gross compensation used to amount to 60 years' purchase of the rental.

The Artizans' Dwellings Acts and the Housing of the Working Classes Acts furnish cases in which a landowner may be divested of his property on receiving its bare value from the local authority. But here it must be noted, in the first place, that the powers are conferred not on any private persons, but on a public body for public objects; and, secondly, that they cannot be put in force except where houses or areas have been neglected, and allowed to fall into an unhealthy condition. Further precedents are said to be found in the provisions of the Agricultural Holdings Act, 1883, and in the powers conferred upon the Irish Land Commission and the Scottish Crofters' Commission. In all these cases conditions of tenure have been modified; in Ireland and Scotland the Commissioners may reduce rent, and in Scotland they may also cancel arrears due by the tenant. But even powers so wide do not extend to denuding a landowner of his property against his will, nor has Parliament ever granted a general right of compulsory purchase.

Not only is the suggested legislation thus unprecedented in

character, but it is also gigantic in the amount of property which it would affect. It proposes to give the privilege of purchase by compulsion to every lessee whose land does not exceed a few acres in extent, and whose lease has to run more than a certain time, such as 20 years, or than a certain portion, such as one-fifth, of the original term. No one can determine precisely the amount of property within this category. But Lord Derby's agent in Lancashire has estimated at £134,000,000 the capital value of property within the 26 towns of that county affected by the enfranchisement proposals. A Birmingham authority gives the figure for that town at over £25,000,000. And the enormous preponderance of Metropolitan leaseholds assures us that we are within the mark in assuming that heritages yielding £20,000,000 a year, or one-half of the gross rental of London, would there come under the sweep of the proposed Bill. This property would be of a capital value of £400,000,000 or £500,000,000, and with the addition of provincial leaseholds the sum might exceed the giant figures of the National Debt.

These considerations, while they furnish no argument against the proposed legislation, create at least no presumption in its favour, and they demand of its advocates strong and cogent reasons. Let us then enquire what these reasons are. For them we naturally turn to the numerous Enfranchisement Bills which have been brought into the House of Commons. Since 1883, when the earliest of these measures appeared, we have had specimens tabled every session. In 1884 Lord Randolph Churchill proposed a Bill, incorporating section 63 of the Lands Clauses Consolidation Act, 1845, and thus granting compensation for severance and other injury; but the more recent measure, emanating from the same side of the House of Commons, supersedes this and every other consideration by tabular values applied to all property alike, and applied, as we have seen, with a total disregard of elementary principles of justice in matters involving questions of interest and mortality. Although a sevenfold explanation is prefixed to this Bill, nothing is advanced by way of justification of its provisions other than the usual and formal allegations of expediency found in its preamble. But Mr. Rowlands is more communicative. In a memorandum attached to his Bill he bases its claims on the Supplementary Report of the Royal Commission on the Housing of the Working Classes, and on the 1889 Report of the Town

Magnitude of
Interests.

Pleas for
Legislation.

Holdings Select Committee. Turning accordingly to these Reports, we soon discover that the latter one gives no approval to the Bill, or to any general compulsory scheme. The powers which it suggests in the case of registered co-operative and provident societies are not, as might be inferred from Mr. Rowlands' memorandum, applicable to leaseholds held by members of these societies. They apply solely to halls and other buildings held by the societies in their corporate capacity. The wider recommendation of the Report deals under careful restrictions, afterwards to come before us, with limited areas of industrial dwelling-house property. It declares that injustice would be done by any scheme of general application, and it ascribes the prevailing desire for enfranchisement to an idea that landowners ought not to be fully compensated for their existing rights.

We turn next to the Supplementary Report, signed by a majority (10 out of 17) of the Royal Commissioners on the Housing of the Working Classes. These 10 Commissioners allege that the system of building on leasehold land is a great cause of the many evils connected with overcrowding, insanitary buildings and excessive rents. This accusation they hold proved by several witnesses, three of whom they name; and, based on their expressed opinion that the prevailing system of urban leases conduces to bad building, to deterioration of property towards the end of the lease, and to want of interest on the part of the occupier in the house which he inhabits, they conclude that legislation, for the acquisition on equitable terms of the freehold interest by the leaseholder, would conduce to the improvement of the dwellings of the people. This deliverance is not concurred in by several of the Commissioners, such as the representative of the Church (Bishop W. W. How), nor by the Chairman himself (Sir Charles Dilke), whose approval was unlikely to be withheld on account of the radical character of its recommendation. Nevertheless, emanating as it does from the major portion of a body presumably well qualified to judge, and issued after hearing the evidence of numerous skilled witnesses, it is a deliverance demanding great respect and full consideration. Such consideration does not show that the opinion of the 10 Commissioners is deducible from the evidence on which it purports to be based. It is true that of the three witnesses whom they name two are adverse to the system of building leases. These are Lord William Compton and Mr. Vivian, of Camborne, Cornwall, men whose names are at least

Supplementary
Report of Royal
Commission on
Housing of
Working Classes

counterbalanced by the weight of authority of witnesses on the other side, such as Mr. Chamberlain and Sir Sydney Waterlow. Mr. Vivian's evidence bears chiefly on the life leases common in his district, four-fifths of the house property in West Cornwall being held on life tenures. He testifies to the overcrowded state of the Cornish mining villages. But it does not appear how the superseding of a tenure under which building sites are cheap, by a tenure under which they are dearer, would be likely to encourage building, and so avoid overcrowding. Building is checked by another cause, one which would operate strongly under freehold tenure. That cause is the rapidity with which the tin mines are wrought out. The exhaustion of a mine makes the miners' cottages worthless; and with such a contingency hanging over the builders there is a tendency to keep the supply of houses well within the demand for them. If building land could be had only by purchasing the freehold, a freehold of which the value would disappear as soon as the tin had been got above ground, dwelling houses might be expected to become scarcer still. Mr. Vivian farther adduces cases of cruel hardship under life leases; as, for instance, where a tenant had inserted the names of his children in his lease, and thus on their death found himself deprived at once of offspring and of home. But against the risk of loss of property in such a way life assurance gives complete protection. And this simple expedient is frequently adopted by Cornish leaseholders, in whose favour policies are often issued, without medical examination, on the lives of such public persons as peers' sons and royal princes, nominated by the holders of life leases. The only other witness who can be claimed by the 10 Commissioners is Lord William Compton. He testified to the deterioration of house property towards the close of a lease, and advocated, at all events as regards the better class of houses, facilities for the acquisition of the property by the ground landlord. This is quite a different proposal from that of the 10 Commissioners, or of the promoters of leasehold enfranchisement Bills. From the evidence on the other side before this Commission we shall content ourselves with quoting an answer of Sir Sydney Waterlow, Chairman of the Improved Industrial Dwellings Company. In reply to Lord Salisbury, he says, compared with freehold, property on lease "is more "profitable to a trading company, because money invested on "leasehold premises, that is to say in the building, ought to "yield 5 or 6 per-cent, but you cannot buy land to pay much

“ more than 3 or $3\frac{1}{4}$ per-cent; and if you have to invest a large sum at that percentage you have to earn more to pull it up to 5 per-cent; therefore we prefer leasehold property, always taking care to have an actuarial calculation of the sum to be put by yearly to redeem the cost at the end of the lease; and that is taken as the first charge out of the revenue.” This one sentence of Sir Sydney’s indicates, as clearly as a whole treatise could do, the nature of the leasehold contract, its inherent equity, and the means by which the lessee can protect himself in view of its ultimate termination. He must put by yearly the sum required to redeem the cost at the end of the lease. This operation, practicable for an industrial dwellings company, may be inconvenient for the individual holder of an isolated lease. But for the comfort of such a one there are societies which have published tables based on the calculations required by Sir Sydney, and which invite the lessee to accomplish the desired transaction through them.

Our farther examination of the remit made to the Royal Commission of 1884 and the work done by that body, does not add to the weight of the deliverance of the 10 Commissioners. The subject of enquiry was the housing of the working classes. Varieties of land tenure came before the Commission only incidentally. Nor does it appear that the Commissioners gave special consideration to the question of tenure. They directed their attention to specific localities inhabited by a working-class population; but while investigating such abuses as overcrowding, defective sanitation, excessive rents and unreasonable eviction, they made few enquiries as regards the tenure of the land on which the houses stood. Indeed, in many cases the evidence does not disclose the nature of this tenure. Information on the point has had to be gathered from other sources, and this has been done by the compiler of the first volume of *Town Holdings* already mentioned. It has thus been ascertained that most of the working-class houses censured by the Royal Commission in the report of their whole body, are built not on leasehold, but on freehold land. The Report condemns houses in six provincial towns, Alnwick, Bristol, Camborne, Doncaster, Liverpool and Newcastle, and in the Metropolitan districts of Bermondsey, Clerkenwell, St. Luke’s and Southwark. But of these six towns it is only the smallest one, the village of Camborne, that is leasehold; Liverpool is of mixed tenure, the other four are freehold towns. Of the Metropolitan areas, Clerkenwell is

leasehold, Bermondsey is mixed, the condemned houses at St. Luke's are freehold, as are also the majority of those at Southwark. The evidence laid before the Commission and the Report of the Commission itself, alike fail to support the recommendations of the 10 Commissioners, as far as that implies the inferiority of leasehold houses. It is true, however, that the lessee inclines to neglect the building during the closing years of the lease. This neglect is to the detriment of the ground landlord, who has his remedy in an enforcement of the covenant. And no ground landlord will desire protection from the risk of such an injury, by legislation which might deprive him of the whole property against his will. The temporary inconvenience which a lessee's neglect might cause an occupier, may be far more than counterbalanced by the public advantage arising from the operation, of which London can point to not a few examples, when on the simultaneous expiry of numerous leases, dilapidated houses are demolished, and whole neighbourhoods reconstructed. A third reason given by the 10 Commissioners for their recommendation is, that the building lease conduces to a want of interest on the part of the occupier in the house he inhabits. If we bear in mind the subject before the Commission, the housing of the working classes, we shall see how inapplicable this reason is. Not one member of the working class in a hundred either owns his house or occupies it under a building lease. Probably he does not know whether the reputed owner, to whom the rent is due, possesses the freehold of the property, or has merely a leasehold interest in it. A clear perception of this fact, so often ignored, cuts away the ground on which many public spirited men are disposed to favour leasehold enfranchisement. If the proposed legislation was for behoof of the actual occupiers, and of the working classes generally, it would present special claims to favourable consideration. But such classes would obtain no benefit; the advantages would be reserved for the middlemen, the builders, the speculators in house property.

There remains to be noticed one allegation of the 10 Commissioners put forth in the first sentence of the Report. It is that leasehold building is a great cause of excessive rents. Now the market rent of a house is primarily fixed by competition between those who have houses to let, and those who require them for occupation. The tenure of the land does not enter into the bargain. If two neighbouring tenements contain similar dwellings, they will fetch similar rents, although one of them may stand on

freehold and the other on leasehold land. The rent, however, ultimately depends on the cost of construction of the building. The builder or owner expects an adequate return for his outlay. If house rents are high, and the return consequently large, building is fostered, and houses become plentiful. If house rents are inadequate, building is discouraged, and a scarcity of houses redresses the grievance in course of time. Leasehold tenure, therefore, can cause excessive rents only by discouraging building or by making it more costly. But it is certain that leasehold tenure encourages Metropolitan building, and does not increase its cost; the ground rent, together with the sinking fund required to replace the capital, being usually less than the chief rent in perpetuity, or than interest on the capital sum required to purchase the freehold. This will clearly appear in the course of our enquiry. Building leases thus tend to keep down rents, and the opposing statement by the 10 Commissioners is contrary to the facts.

These opinions of the majority of the Royal Commissioners on the Housing of the Working Classes, weakly supported and in many respects erroneous as they are, gave force to agitation, encouraged a crop of amateur legislative efforts, and helped to secure the nomination of the successive Select Committees on Town Holdings already mentioned. An examination of the evidence tendered to these committees discloses the following additional pleas for leasehold enfranchisement:

Evidence before
Select Com-
mittees on
Town Holdings.

(1) Leasehold tenure is an unnatural and one-sided contract, into which a lessee enters only because he cannot help himself. The lessor is a monopolist. Land, the commodity of which he is possessed, is limited in quantity and incapable of increase; and the landowner can dictate his own terms. These terms are further restricted by family settlements, and by legislative interference with corporate action, so that the landowner, even if personally disposed to meet the reasonable wishes of an intending purchaser, finds himself debarred from doing so.

(2) The incidents of building leases impose arbitrary restrictions on the lessee, and enable the landowner to benefit by the skill and success of the lessee and occupier, by the improvements which they make on the property, and by the increment in value which the growth of population and prosperity adds to the land during the currency of the lease.

There is no material argument for leasehold enfranchisement

advanced before the Select Committees that may not be referred to one or other of these two heads, or to the allegations of the 10 Commissioners on the Housing of the Working Classes, already examined and found wanting.

It is quite true that land, considered as a species of property, differs from other possessions held by individuals.

Land.

In a prosperous and peaceful condition of society, with an increasing population, the rent of land surely tends to rise, just as the rate of interest surely tends to fall. This reasonable deduction from the consideration of the facts of nature and the circumstances of society, is confirmed by observations over a sufficient width of time and space. If we embrace the world in our view, and the world during whole centuries, we find that the man who owns the land has always an advantage over the man who seeks it. The former, not the latter, can afford to wait. But there are many circumstances which check, and even overbalance, this natural order of progression, as it applies to single countries and to single generations. Such circumstances we find in England at the present time. The tide of emigration and the development of foreign trade have brought the whole round world to our door, and have replaced the natural increase of the rent of English land by a most startling reduction. The President of the Surveyors' Institution has recently published statistics, obtained by extensive official enquiries in 11 central and south-eastern counties, showing a depreciation of land values of from 45 to 75 per-cent, the latter extraordinary reduction being approached in south-eastern Essex and actually reached in southern Oxfordshire. This undeserved decrement is found also to have extended to urban land, which in many towns has greatly receded in value, whether to purchase or to lease.

And the grievance which the community is said to have against landowners as holders of property restricted in amount as well as unearned by labour, is a grievance which leasehold enfranchisement would do nothing to abate. Indeed, legislation on the lines of enfranchisement, if extensively taken advantage of, would materially increase the numbers and the power of the landowning class. This is clearly seen by the most extreme section of the advocates of land nationalization, represented by the school of Mr. Henry George. Enfranchisement does not tend towards his goal, but leads away from it. For, in the words of one of his volumes, its "outcome is not to weaken landownership

“ but rather to strengthen it, by interesting a larger number in “ its maintenance.”

Enfranchisement would not benefit the artizan population. Few working men hold leases of their houses, either for life or 20 years, and therefore few would come under any of the Bills for leasehold enfranchisement. The vague expectation, influential over many philanthropic minds, that the proposals might to some extent redress the balance between landowners and the less favoured masses of society, is easily discovered to be without foundation.

Sir Henry James pointed out in the House of Commons, that if the allegations against leasehold tenure were true, the proper remedy was a measure to prohibit the granting of building leases in future. This would in due time bring to an end that dual ownership from which results so evil are said to flow; and it would terminate the wrong without introducing the knotty question of existing leases. Sir Henry perhaps surmised that the hopes of existing leaseholders were bound up with the agitation for enfranchisement. The inevitable effect of a measure on the lines of recent Bills would be to make more onerous the terms on which landowners would part with their property in future. No more building leases would be granted. No proprietor would enter with his eyes open upon a bargain so one-sided. Building sites would then be obtainable only by purchase of the freehold, by way either of a cash payment (with or without a mortgage to the landowner) or a chief rent in perpetuity. Such a restriction of tenure would have little effect in village districts and small towns, where sites are cheap. But in the Metropolis and other great centres of population it would discourage acquisition of building land, and would thus increase rentals just in the very localities where they are highest at present. It is in these localities that building leases mostly prevail; and the more prosperous the place the shorter is the term of the lease. Indeed, the prevalence of leaseholds in London is a good illustration of the fact that leasehold tenure suits Metropolitan conditions. It is incredible that the hundreds of Metropolitan landowners who are free to dispose of their property as they choose, and the tens of thousands of Metropolitan builders, and tenants, and dealers in houses, have not long ago discovered the kind of bargain that suits both parties best. Legislation in restraint of building leases, such as an Act for compulsory enfranchisement, would bring about results similar to those that were produced by antiquated

Antiquated
restrictive
Laws.

legislation in restraint of trade promoted by the ignorance of bygone centuries. Sumptuary laws, laws against forestallers, regraters and engrossers, laws fixing the price of bread, and the strength of ale, and the texture of cloth, all have had their day and have ceased to be. And a legislative discredit of urban leases, while it would be powerful in respect of existing contracts, would hamper proprietor and tenant alike in their future negotiations, and would, in course of time, disappear as those old laws have disappeared. It would be strong where strength is injustice, in breaking down the bargains of the past, and weak in the circumstances in which alone the laws should interpose, namely, in reference to the arrangements of the future.

In considering the other pleas for legislation suggested in the evidence before the Select Committees, we find that modern laws have swept away most of the disabilities which are declared to have been the origin of leasehold tenure, and have furnished landowners with easy methods of obtaining permission to deal freely with their property. The allegation of monopoly, whatever theoretic weight it may possess as against landowners generally, has least force of all in the case of building land, of which the available supply in numerous hands is more than ample in comparison with any demand that can arise for many generations to come. The restrictive covenants imposed in building

Covenants.

leases are of various kinds, and for various objects, but chiefly for the purpose of preserving the character and amenity of neighbourhoods, and protecting the lessees and occupiers in general from the eccentricities of any one of them. In so far as they serve this end, they make the position of the lessee a happier one than is that of the owner of a small freehold, surrounded by neighbours each of whom is a law to himself, within the very wide limits of municipal regulations, and covenants which no one has a title to enforce. In so far as they are oppressive or unreasonable, the Court, under the Conveyancing Act of 1881, is empowered to relieve the lessee. In so far as they prescribe a material element of the consideration between parties, they are on the same footing as the contract for payment of the rent itself, and are entitled to the same support of the law. In this category must be embraced covenants by which the lessee undertakes to pay rates and taxes, and on the termination of the lease to restore the land to its owner, with the buildings erected thereon. Under the latter stipulation, it will be observed that the freeholder enters not only on the improvements effected by the lessee, but also upon

the value of the goodwill of any business that may have grown upon his land. The loss by the tenant or occupier of this goodwill is one of the greatest hardships that arise under a building lease. Improvements on the building are made in the full knowledge of the rights of parties. But prosperity in trade depending so much on the personal qualities of the trader, he naturally regards all that flows from that prosperity as his very own. It must, however, not be overlooked that the loss of goodwill is no peculiarity of building leases, but that, in fact, it most frequently emerges in questions between building owners and occupiers, who hold their business premises on a very short, or even an annual tenure. The Select Committee's Report of 1889 states that, although the goodwill of a business carried on in his property gives a landlord considerable control over the terms of renewal, yet the cases in which it has been allowed to influence a renewal rent are few and exceptional. In any case it may be fairly presumed that the terms of the covenants, with all their contingencies, have been considered by the parties who engage to fulfil them, and have had due weight in determining the amount of the ground rent. This comes out clearly in the evidence laid before the Committee from localities where leasehold and freehold tenure exist side by side. There is a consensus of testimony that the shorter the lease the less onerous the terms. Indeed, the advantage to the holder of the short lease is usually far greater than the real difference in value of the different tenures. Two instances may be offered, one Metropolitan, the other provincial. Mr. Robert Castle testified before the Select Committee of 1887, that from one-third to one-fourth more ground rent would be charged for a 999 years' lease than for a lease of 99 years. Let us, then, on the assumption of annual payment of rent, and interest at rates of 3, 4 and 5 per-cent, consider a piece of land for which £1,000 a year can be obtained under the usual 99 years' building lease. The initial value of the whole of the payments of ground rent is as follows:

Leaseholds n.
Freeholds.

3 Per-cent	4 Per-cent	5 Per-cent
£31,517	£24,485	£19,810

On the alternative assumption of a 999 years' lease, at a rent one-fourth higher—being £1,250 a year—the value of the ground rent is:

3 Per cent	4 Per-cent	5 Per-cent
£41,667	£31,250	£25,000

The reversion in this case is of no value, a thousand years' term

being practically a perpetuity. If, then, the present value of the reversions to which the freeholder will succeed on expiry of the 99 years' lease may be taken as equivalent to the difference of these sums, or

3 Per-cent	4 Per-cent	5 Per-cent
£10,120	£6,765	£5,160

the two transactions will yield similar results to lessor and lessee. But in order that the reversion postponed 99 years should be of this present value it would have to yield, when it falls into possession, an annual rack rent of

3 Per-cent	4 Per-cent	5 Per-cent
£5,665	£13,141	£32,312

This statement shows that in London it is probably much cheaper to purchase an ordinary building lease than it is to acquire a lease for a thousand years, or a freehold subject to a chief rent. Our provincial illustration is from Southport, represented before the Select Committee of 1888 by witnesses both for and against the proposed legislation. Mr. Cockshott, the witness adverse to enfranchisement, states that the perpetual rent charge imposed on the Southport freeholds is from a little less than 3*d.* to 4*d.* a yard in the neighbourhood of 99 years' leaseholds given off at a penny a yard; but setting aside his evidence we find that the Town Clerk, a witness on the other side, admits that the Corporation's rate for freeholds is about double that imposed by the other proprietor for building leases. No increment in value, whether arising from the outlay of the lessee or from the progress of society, could ever reach the sum by which in this case the higher ground rent exceeds the lower. Indeed, the general allegations against the landowner of unendurable interference, deprivation, restriction and oppression, and the conveyance to himself of the fruits of the work and outlay of another, are substantially met by an observation of Sir John Ellis in the course of the enquiries of the Select Committee of 1887. "With the intellectual capacity of calculating the difference between a leasehold and a freehold, is there any difficulty or inconvenience about it? But if you have not the intelligence to understand the difference, then I suppose arises the difficulty which we hear so much about."

We have thus considered the origin and the varieties of land tenure throughout England and Wales, the proposals for legislative modification of the terms of long leases over small

areas of land, the recommendations of Royal Commissioners, and the proceedings of Select Committees of Parliament. We have found that allegations of the inequity and inexpediency of building leases have not been proved, and that no sufficient case has been made out for a general scheme of legislation permitting the terms of these contracts to be altered in the option of the lessee. And our examination of leasehold enfranchisement Bills brought into Parliament in recent sessions, has shown the inadequacy of the terms proposed for such a scheme, based as these terms are on tables which value rental and reversion alike too low, or on clauses which ignore material portions of the freeholders' rights.

But although thus adverse to the views and objects of the Leasehold Enfranchisement Association, it will be fitting that we should consider whether it is desirable to accede in a modified degree to any of the demands made in recent years. To assist our enquiry we return to the Report adopted in July 1889 by the Commons' Select Committee, and claimed by Mr. Rowlands, but claimed, as we have seen, with no sufficient warrant, as a plea for the Bill that bears his name. This Report, adopted after enquiries extending over more than three years, judiciously weighs the evidence of upwards of a hundred witnesses. In the opinions which it expresses, and the restricted character of its recommendations, we find but little warrant for those claimant demands. It pronounces that the allegations against leasehold tenure of inferior workmanship, a bad state of repair, unsanitary conditions and excessive rents, are, in general, not proved. The Committee, however, believe that where terms of leases are very short, or are dependent upon lives, enfranchisement would improve the quality of work; and they recommend the commutation of life leases into leases for terms of years. On the other hand, they pronounce the ground landlord's oversight to be beneficial in maintaining the standard of quality in buildings. They say that the leasehold system facilitates building, and increases the supply of houses. In their view enfranchisement would prolong the existence of old houses that might otherwise be rebuilt; it would discourage public improvements, now frequently effected by the owners of large estates when the leases fall in; it would interfere with the erection of cheap industrial dwellings, now sometimes promoted by such owners, who grant sites on liberal terms; and it would imperil the security now enjoyed by lessees under covenants for the general

Recommendations of Select Committee.

benefit. They add that compulsory enfranchisement would injure the revenues and estates of municipal corporations and public charities, and so deprive the community of important benefits for the advantage only of the few. They object to the one-sided and unprecedented nature of the proposed powers sought for the compulsory transfer of property merely from one individual to another, without any benefit to the people as a whole. Believing that the prevalent desire for enfranchisement is connected with a notion that the landlord's rights ought not to be fully recognized, they anticipate that the powers of an enfranchising Act would be used to but a small extent, yet they consider that the property would be injured in value none the less, from the uncertainty and insecurity that the measure would introduce. And they point out that the industrial classes could derive little benefit, as the principal result would be the conversion into a freeholder of the middleman under whom they occupy their dwellings. The Committee at the same time express an opinion that enfranchisement would often secure improvements and encourage the development of trades; and that it would abolish some irritating minor incidents of leasehold tenure. They believe that the power of acquiring a freehold would in many cases promote thrift among the industrial classes, and that an increase in the number of freeholders would in itself tend to benefit the community.

With these views the Committee embody certain recommendations in their report to Parliament. They advise that religious bodies be empowered to enfranchise their places of worship, educational bodies their schools, public authorities any house required for public utility, and registered co-operative and provident societies their halls and other buildings. Following out the first of these recommendations, Mr. Roberts brought a Bill into the House of Commons for the enfranchisement of leasehold places of worship. The Committee further suggest a measure permitting compulsory enfranchisement in a more general way. But they hedge about this suggestion with many qualifications and restrictions. They would apply it only to areas inhabited wholly or in great part by the working classes, including clerks and small traders; and there only if most of the occupants were leaseholders, and if after an enquiry under the direction of the Local Government Board, the local authority believed that thrift and industry would be promoted by enfranchisement of such an area as a whole, and that the operation could be undertaken without probable loss. Should all these contingencies seem to

Cases for Enfranchisement.

favour the experiment, the local authority might be authorized to acquire, compulsorily if need be, the reversions of the property so situated, with a view of disposing of the freeholds to the occupying tenants, so far as these latter chose to acquire them. The Committee admit the practical difficulties that would arise in carrying out so complex and withal so guarded a scheme; but they can discover no popular and royal road to the goal of so loud and persistent a political and social agitation.

This cautious deliverance is worthy of the reputation of the Select Committee, and consistent with the weight of testimony adduced before them. It seems fairly well established that any general measure for leasehold enfranchisement is likely to be either unjust or impracticable. It will be unjust if it is one-sided, permitting the leaseholder to prevent his landlord from retaining the estate; and not only refusing any corresponding power to the latter, but denying him the privilege of making any counter proposal. It will be unjust if it refuses to compensate a landlord for his loss by the partition of his estate. And it will be unjust if the lessee is not charged with the reasonable costs of the enfranchisement that he seeks. But if the measure is drawn so as to fulfil these necessary conditions of equity, the lessee will find the terms too onerous in the great majority of cases. There has been little if any demand by landowners for powers to buy up the leases affecting their freeholds; although, as we saw, one witness before the Royal Commission of 1884 suggested that such powers might sometimes be exercised for the public good. And we would not ask that the precedent of the Copyhold Acts, by which lord and tenant are on a footing of equality, should be applied to leasehold property. But confining to the tenant the privilege of making the first move, equity ought to entitle the landowner to reply by an offer to buy up the lease, rather than be compelled to surrender the freehold. Either mode of dealing with the property would meet the views of those who see the root of the evil in the duality of ownership. Indeed, the exercise of such an option is conceded, in really exceptional cases, in the fifth chapter of Messrs. Broadhurst and Reid's book. It is not too much to ask that the landlord, who is to be parted from his freehold, be allowed to judge whether or not his case is an exceptional one. And should the owner of the freehold so reply to the leaseholder, we would further extend to the latter permission in such a case to withdraw his notice and leave existing rights and obligations undisturbed. Two cases would remain, the first,

Equitable
Conditions.

where the lessee would proceed to acquire the fee, the freeholder not offering to purchase his lease; and the second, where the freeholder would buy out the lessee, the latter not withdrawing his notice on being apprised of the counter proposal of the freeholder. For the purpose of a valuation in either contingency, provisions similar to those of the Lands Clauses Consolidation Acts, or of the Copyhold Act of 1858, would be applicable. Both these Acts provide diverse modes of valuation, according to the greater or less value of the property or amount of the claim. The methods under the former Act include an award by two Justices; a submission to the arbitration of two men of skill, each party naming one, and they appointing an umpire before proceeding; and lastly, the verdict of a common or a special jury. Under the Copyhold Acts, the award of a valuer appointed by the Petty Sessions supersedes the direct action of the Justices themselves, and applies to cases where the annual rated value is not over twenty pounds, or where the rights to be valued consist only of heriots, rents and timber licenses at fixed rates. There is no provision for a jury trial; but an appeal is allowed from the valuers to the Copyhold and Inclosure Commissioners, and a further appeal on questions of law to the superior Law Courts. The Copyhold Act of 1852 directs the valuers to take into account all incidents of the tenure, all circumstances affecting the land, and all advantages to arise therefrom: thus guarding the equitable rights of parties, just as we have observed they are guarded by section 63 of the Lands Clauses Consolidation Act of 1845. These provisions of the Copyhold Acts supply a basis for the valuation of rights involved in leasehold purchase or enfranchisement. In either of the modes of merging the lease into the freehold, questions would arise giving scope to great differences of opinion. The freeholder to be bought out would have regard not only to his rental and his reversions, but to future prospects foregone, and to compensation for the severance; for the breaking up of the estate might do an injury far greater than the whole value of the leasehold taken from him. The tenant parting with his lease would look not merely for the present value of an annuity of the net revenue of the property, but also for compensation for such outlays as might not have had time to operate in adequately increasing that revenue, as well as for probable goodwill, and other reasonable expectations of increment. The difficulties involved in a fair solution of the question of goodwill might, however, be considerably lessened, if not altogether

removed. The freeholder would naturally, in all cases, acquire the lessee's interest subject to any existing short sub-leases to occupiers. In the absence of any such rights of occupation the Land Commissioners might be empowered to order the grant to the lessee or his nominee of an occupying lease for a period not exceeding 19 years. On acquisition of the building lease under such a burden, the freeholder would enter at once upon the rack rent, but the time would be postponed when he could deal with the occupation of the hereditament; and thus such disputable points as goodwill would be brought within a very narrow compass. Failing the appointment of a new tribunal (and there are objections to the increase of judicial institutions), the Land Commissioners for England might act as a controlling authority. Either party might appeal to the Commissioners from the valuers, whether on matters of fact or of law, and a further appeal would lie to the Courts on legal questions. The expenses would be borne by the party acquiring the property. This provision would fall somewhat hardly on the freeholder, as he is obliged to acquire the leasehold in order to escape deprivation of his fee. Its operation, however, would be one of the points to be considered by him in determining on his course of action. Where one person only was interested on each side, and where the titles were clear and simple, the consideration, whether for the lease or the freehold, might take the form of an annuity, terminable or in perpetuity, charged upon the land. In cases of limited or complex interests, the consideration would be payable to the Land Commissioners, who, after due notice, and after disposing of any objections, would divide the fund according to the respective interests of the parties. In any case it would fall to the Commissioners to confirm the valuers' award and grant a title to the property.

The value of heritable rights depends so much upon the situation of the property, the nature of the occupation, and other considerations, that any figures submitted in illustration must be of a very general character. Let us, however, take such a case as that of land let at a ground rent of £100, on a lease having 50 years to run, and covered with buildings yielding a rack rent of £600, but rated at £500, the last-named sum being assumed as the average net revenue derivable from the property. If the houses are well-built and in a good situation, we may capitalize them as a 4 per-cent investment, and the ground rent may be valued on a $3\frac{1}{2}$ per-cent basis. On these hypotheses, and assuming interest payable or convertible once a

year, the first step towards fixing the payments required in connection with the merging of the lease into the freehold is shown by the following calculations :

(1) <i>Enfranchisement :</i>	£
Sum payable by Lessee to Freeholder:	
Ground Rent—Annuity of £100 for 50 years, at 3½ per-cent	2,346
Reversion—Perpetuity of £500, deferred 50 years, at 4 per-cent	1,759
	<hr/>
	£4,105

(2) *Acquisition of Lease by Freeholder:*

Sum payable by Freeholder to Lessee:	
Rateable Value—Annuity of £500, for 50 years, at 4 per-cent	10,741
Less Ground Rent—Annuity of £100, for 50 years, at 3½ per-cent	2,346
	<hr/>
	£8,395

This case presents no features of difficulty, apart from questions of compensation for consequential damages, to the freeholder by severance of the estate, or to the lessee by loss of goodwill. Pleas on grounds like these must be considered on their merits in every case as it arises.

In the not unusual case of the enfranchising lessee holding by way of underlease on an improved ground rent, the intermediate or superior lessee would be paid out by the underlessee or the freeholder, whichever of the two acquired the consolidated interests. In the instances before us, if the buildings were charged over and above the original ground rent, with an additional ground rent of £200 payable for the whole period to an intermediate lessee holding no reversion, or only a nominal reversion of a few days, his annuity might be valued at £4,487, being at the rate of $3\frac{3}{4}$ per-cent. The enfranchising sub-lessee must in that case pay—

To the Intermediate Lessee	£
To the Freeholder:	4,487
Value of Ground Rent	£2,346
Value of Reversion	1,759
	<hr/>
	4,105
	<hr/>
	£8,592

While the freeholder, if acquiring the leases, must pay—

To the Intermediate Lessee	£	4,487
To the Sub-Lessee:		
Whole Value of the Property, being a		
Perpetuity of £500, at 4 per-cent . .	£12,500	
Less Value of his own Interest and that of		
the Intermediate Lessee, as above . . .	8,592	
	<hr/>	3,908
		<hr/>
		£8,395
		<hr/>

Under such an arrangement it might be impracticable to entertain any claim for indirect loss at the instance of the intermediate lessee, while at the same time he might find it highly inconvenient to have the capital value of his estate thrust upon him; and its re-investment might involve either a diminution of his income or the acceptance of a poorer security. An intermediate lessee is sometimes in the position of having the right to a reversion of a few years of the rack rental at the end of the sub-lease. This interest would be valued as a deferred temporary annuity. Mortgages might have been created on any or all of the estates, from the sub-lessee's to the freeholder's; and in that case the Commissioners would have to satisfy the mortgagees. The provisions of existing legislation, as found in the Land Clauses Consolidation Acts, would have to be applied in the case of persons absent, unknown or under disability.

Although we have submitted this short sketch of the mode in which the enfranchisement of leaseholds, or the consolidation of different estates in land, might be effected with the least inconvenience and disturbance, it is quite apparent from our

investigations that no sufficient case has been made out for legislative interference with existing contracts.

If Parliament should enact a measure for general enfranchisement, it will be to a certain extent influenced by that discontent which has grown formidable under the fostering care of lessees and middlemen who understand the question thoroughly, and of philanthropists and liberal politicians who are imperfectly acquainted with the details and probable effects of the movement.

II.—LAND TAXATION.

Concurrently with the cry for leasehold enfranchisement, a movement has been made for the subjection of ground rents and

other land values to local or Imperial taxation. This question also having been remitted to the Select Committees on Town Holdings, a great body of evidence in regard to it has been accumulated. Of the 122 British witnesses examined, 66 dealt with the subject, 50 of them being from England and Wales, and 16 from Scotland. The sittings of the Committee during the years 1891 and 1892 were devoted almost exclusively to this particular enquiry. The evidence seems fairly to represent all that can be said in support of the various opinions current; and there is no great preponderance of testimony on either side so far as the numbers show, 22 English and 7 Scotch witnesses favouring new or amended taxation, while 28 English and 9 Scotch witnesses testify generally in support of existing laws. The evidence on both sides is confined within narrow limits, the later witnesses ringing the changes on two or three familiar notes, and contributing at most only some new illustrations of points previously tested in the examinations and cross-examinations of the earlier witnesses.

Evidence
before Select
Committees.

(1). *Local Taxation.*

One of the earliest proposals under this head was made by the City of London Corporation in 1867, to the effect that ground rents should pay 6*d.* in the £, in addition to all other rates on the property. But the idea of any such exaction is now dismissed by almost every witness before the Committee. One insuperable difficulty lies in the arbitrary amounts of the ground rents. They are often fixed at figures much less than the worth of the ground, even at the time of their imposition; and the difference between the rent and the actual value increases as the years go by. The case of a freehold illustrates the extreme limit of this difference, as such a property would escape the suggested tax altogether. And freeholds apart, there are various reasons for the disparity between ground rents and ground values. The ground rents are fixed for a long period. No building lease, however valuable the site, would be entered on for much less than 50 years. The most common term is 99 years, while 999 years' terms, and perpetuities subject to chief rents, are also of frequent occurrence. The proposed 6*d.* rate would apply alike to ground rents fixed to-day and to those fixed 20 or 60 or 100 years bygone. But within the past century the value of urban building sites has greatly increased. Of two adjoining sites, one may have been

Rents v. Values.

leased 50 years ago, at a ground rent little more than nominal compared with the figures current to-day: the other, of no more real worth, may have fetched a ground rent a hundred times higher. Even confining our attention to leases entered upon about the same time, we discover important differences in the ground rents. This may arise from the payment of a premium or fine in some of the cases, or from the unequal apportionment, often convenient for the builder, of the ground rent of a large area, among the sites of the various houses built upon it. Thus the simplicity of a tax on ground rents at 6*d.* in the £ would be secured at the expense of equity.

Proposals for the taxation of ground rents and land values have come before the Select Committee in various forms, and have been urged on diverse pleas. The great majority of the witnesses in favour of the idea seek the relief of the occupier or building owner at the cost of the freeholder, upon whose ground rent is to be thrown a part of the taxation now charged in respect of the occupancy. But a few witnesses rather desire to impose new and additional burdens on the landowners for the benefit of the general community. Among these may be classed Mr. William Saunders, who desired that land alone, as distinct from buildings, should bear all taxes, both local and Imperial. This suggestion we reserve for after consideration, along with another proposal of the same witness, to the effect that land should be rated on 4 per-cent of its capital value, if that percentage brought out a figure exceeding its rental.

The question as between the ground landlord and the lessee or the occupier is comparatively a narrow one—belonging, indeed, to the same category as leasehold enfranchisement. The nation as a whole has no direct interest in the discussion whether freeholder or building owner shall bear the burden of municipal rates. The contracts of building leases bind the lessee to relieve the freeholder of all rates and taxes, property tax excepted. In

1870 a Select Committee, presided over by Mr. Goschen, enquired into the expediency of dividing between owners and occupiers the local rates imposed upon the latter alone, into the changes in the composition of local government bodies that ought to follow such a division, and into the proper classification of rates required for their equitable incidence upon occupiers and owners. The Committee, having heard much evidence, reported that the exclusive charge on occupiers of local rates, not only for current expenditure but for new objects and permanent works,

Diversity of
Proposals.

Mr. Goschen's
Committee.

was contrary to sound policy; but that in many cases the rates, although paid by the occupier, ultimately fell, either in part or wholly, upon the owner, who, nevertheless, had no voice in their expenditure; that owners as well as occupiers should have an immediate interest in the increase or decrease of local taxation, but that equitable arrangements must be made in regard to existing contracts. The arrangements suggested were the postponement for three years of the compulsory division of rates, and the subsequent increase of the rent by a sum equivalent to the owner's share of the rates, calculated on the average of these three years. We now, however, find the following arguments urged in support of a modification of existing contracts, and an imposition upon the freeholder of some part of the rates now paid by the tenant or occupant, without any such compensation as was recommended by Mr. Goschen's Committee:

**Pleas for
Modification
of Contracts.**

(1) There is no free contract between the owner of a large Metropolitan estate and the builder who seeks a lease. So said Mr. Knox to the Committee of 1888.

(2) Local rates have risen beyond anything that could have been contemplated by either party to a lease executed a generation ago. So said Mr. Rhodes in his examination in 1887.

(3) Rates are raised not merely for current expenses, but to pay off outlays incurred in improvements which will outlast existing leases, and benefit the freeholder when the reversion falls in. So Mr. Sargant, otherwise unfavourable to the proposal, admitted to the Committee of 1890. Mr. Moulton, a year later, put this position more broadly, claiming that the whole of the permanent element in communal expenditure went to increase the value of the land, ultimately excluding the buildings.

(4) Municipal expenditure directly benefits the freeholder in respect of the part of his estate unleased. Indirectly it secures his ground rents, which could not be got without municipal government of some kind. These considerations were laid before the 1888 Committee by Mr. Trevail, who also expressed an opinion that a tax on ground rents might be looked on as the freeholder's contribution to civilization generally.

(5) The capital value of imperishable land is much greater in proportion to the annual return derivable from it, than is the capital value of decaying buildings in proportion to the revenue from them. This is pleaded by Mr. Moulton in the first and second of his three examinations before the Committee of 1891.

Under one or other of these heads may be classed every important argument for the imposition of occupiers' rates on ground rents or ground values, submitted by any of the 22 English witnesses who gave evidence in favour of such a transfer, before the Select Committees in the years 1886 to 1891. In the session of 1891 Mr. Moulton was the only English witness who appeared on that side. He has given great attention to municipal rating, and has recommended a 5*s.* assessment on ground values, in his book on the subject, written at the request of the United Committee for the taxation of ground rents. Before tendering evidence to the Select Committee, he was able to ponder at leisure everything that had been advanced on either side during the investigations of four preceding sessions of Parliament. And he had the fullest opportunity of expounding and illustrating his views, having replied to upwards of one thousand questions put by the various members of Committee.

Scheme in detail. His scheme may be described as the model plan for imposing a local assessment on land as distinct from buildings. His first step is to make a valuation of all heritable property in the borough or other district subject to local government, distinguishing in every instance between the value of the land and the structural value of the erection built upon it. The assessments are to be levied on the basis of a valuation roll so made up, the rates on the land being fixed, either directly by Parliament or by powers conferred on the Town Council or other governing body, at a much higher figure than the rates on the buildings. The various persons beneficially interested in any property are to be rated according to the amount of their income from it, and at the higher rate or the lower, according as that income arises from the ground or from the structure. To explain the consequent incidence of the rating, we shall take Mr. Moulton's own illustration. A house of the rack rental of £250, stands on ground originally leased by the freeholder at £50, and sub-leased at £70. The compilers of the valuation roll decide that the present value of the ground is £100 a year, the difference of £150 by which the rack rent exceeds that figure, being deemed the annual value of the structure alone. The rate is supposed to be fixed at 6s. in the £ on ground value and 4s. on building rent. On these data the rates to be levied in the supposed case amount to £60, made up thus :

Building	£150 at 4s.	£30
Land	100 at 6s.	30
		<hr/> £60

This rating is collected from the occupier in full; and he is entitled to deduct the whole of it from the building owner, to whom he pays the rack rent. The building owner, on paying the ground rent due to his immediate superior, £70, deducts the rate at 6s. a pound on that sum, paying him accordingly £49. The net amount which he received from the occupying tenant having been £190, he has left in hand £141 as his own income from the property. That income consists of:

Building rent	.	£150 less rate at 4s. =	£30	.	.	£120
Ground value	.	30 less rate at 6s. =	9	.	.	21
			<u>£180</u>			<u>£141</u>

The land having been found to exceed in value the improved ground rent, the building owner is thus treated as a landowner to the extent of the excess. The lessee, on paying £50 to the freeholder, deducts £15, or the amount of the rate at 6s.; and the sum remaining in his hands is thus £14, being his £20 of improved ground rent under like deduction. And the freeholder finds his £50 diminished by an assessment at the same rate of 6s. in the £. The most important feature of the scheme is the

*Land v
Buildings.*

distinction between land and buildings. This distinction is unusual in British law, which looks on

a hereditament as a whole, whether it be waste land, land improved by agricultural outlays, or land improved by the erection of buildings thereon. The estates of the various persons interested may differ in quantity and quality—a man may hold the fee-simple, a life lease, or a lease for a term, his rights may be in possession or in reversion—but his estate, of whatever kind it be, is an estate in the land, with all its fixed adjuncts. Many difficulties were suggested as likely to arise in developing the new scheme of separate valuations of land and buildings, and it is clear that anomalies would arise in enforcing it. Mr. Moulton is by no means an extreme man; indeed, he does not go so far as some of the members of the Committee would willingly have led him. He would, for example, give some compensation for breach of contract in existing leases, probably on the lines of Mr. Goschen's Select Committee of 1870 on Local Taxation, by allowing the freeholder an equivalent for the rating current at the date of the lease. And he would not rate the value of the reversion, but would wait until it should be reduced into possession. This forbearance does not seem quite consistent with his adherence to Mr. Saunders's proposal for taxing vacant

building land on a percentage of the highest price which it is worth for any purpose. Such a basis of rating was recommended by the Royal Commission of 1884 on the Housing of the Working Classes. The Report of the Commission suggested a municipal rating of unoccupied land on 4 per-cent of its capital value. No evidence on the subject had, however, been laid before the Commission; and Mr. Ryde, formerly President of the Surveyors' Institution, a witness before the Committee in 1886 and 1891, showed the fallacy of such a method of rating. The capital value includes the market estimate of the present worth of all future increment; and if it be taken as the basis of rating, no increase of the rate ought ever to be afterwards imposed. Let us take, by way of illustration, an estate of vacant urban land, presently let at £2 an acre for temporary purposes. It might sell for £500 an acre; and Mr. Moulton proposes to rate it upon an annual value of 4 per-cent of this price, or £20. But such a price, if given, is given because the purchaser anticipates that in 10 years the land may fetch a rental of £40 an acre. He is remunerated for his outlay by the rental of £2 an acre for ten years, and an expected rent of £40 an acre thereafter. If the municipality discount future prospects by rating on £20 when the income is £2, it must continue to rate on £20 when the income rises to £40. Surely the existing mode of rating, on the rental for the time being, is at once easier, fairer, and more intelligible. By continuing in the old paths we escape many puzzling questions. No landowner will be so foolish as to let his land at a poor rent in order to escape paying rates on a higher one. It is just as likely that any person would decline an augmentation of his income on the ground that his income tax would rise also. And the rating of land, not on the income which it actually yields, but on the higher income which it might yield if applied to other uses, would bear some analogy to the action of a Surveyor of Inland Revenue who should decline to accept a shoemaker's return of £500 as his income from trade, on the plea that had the same capital and care been devoted to a public house, £1,000 a year might have been derived from it. Mr. Moulton concedes the necessity for special consideration of old ground rents; and goes so far as to suggest that chief rents in Manchester might be increased in amount before having rates imposed upon them. Such a proposal, in a direction precisely opposite to the rest of the scheme, introduces the most perplexing complications. And

yet Mr. Moulton sees that equity demands some such arrangement. The bitterest landlord-hater could not say that a very suitable object for fiscal exactions is to be found in the man who parted with his land 50 years ago in consideration of an annual rent not one-twentieth of its annual value to-day. Nor could it be explained why such a man is to have this small revenue diminished by 6s. in the £, while his neighbour, who sold land for a corresponding capital sum, is to escape altogether from rates on the income which he now derives from the investment of his capital, even if such investment was made by way of loan to the purchaser, on the security of the land itself. Mr. Moulton proposes to prohibit every contract by which one person undertakes to relieve another of the rates imposed on the latter, as such contracts tend to discourage municipal outlay by restricting the area of taxation. Very inconsistently with this position, he expresses disapproval of the system of ground rents, and prefers that those who want land should buy it, raising the purchase price by way of mortgage if they require to do so. He overlooks the fact that as mortgages are not rated, nor proposed to be rated under his scheme, such a financial operation would throw upon the lessee or occupier the whole of the rates, whether on the building or on the land. This fact, however, would soon be discovered by the landowner. And if it be true, as Mr. Knox alleges, that there is no free contract in negotiation with the owners of large estates, then on the introduction of the new style of rating, these powerful aristocrats may insist on selling the leases of their lands for capital sums, with ground rents nominal or non-existent, and with an understanding that they shall lend the purchase-money, or great part of it, by way of mortgage on the estate. Thus a more desperate remedy than the taxation of ground rent must be devised, if the disease of land monopoly is as desperate as some of the witnesses think it.

Let us now revert to the remaining four arguments above mentioned, gathered from the evidence of various witnesses; and consider whether they embrace any points not sufficiently examined in connection with Mr. Moulton's evidence. It is stated truly that local burdens have greatly increased of recent years. The growth of old rates, and the creation of new ones, have resulted in a figure which in many cases neither lessor nor lessee anticipated when the latter undertook to relieve the former of all such exactions. Now, if there be nothing on the other side to set against this rise of rates, the lessor has done

Increase of
Rates.

better, and the lessee has done worse than either party expected. This condition of things is by no means rare in commerce; yet traders do not invoke the aid of Parliament to undo their bargains. But now we have changed all that, and the law, declared to have in olden days favoured the strong at the cost of the weak, is invoked to do what is judged a smaller wrong, by helping the weak at the expense of the strong. We shall waive the question whether an urban leaseholder is not generally quite strong enough to protect his own interests, and also whether one-sided legislation for the masses is not as evil and dangerous as one-sided legislation for the classes. Let us rather enquire whether the leaseholder has not reaped an unlooked-for harvest to compensate him for his unexpected taxes. It is at once apparent that although rates have risen, yet there is one thing that has risen more rapidly, and that is rental. This comes out clearly in Mr. Clements' evidence before the Committee of 1890. His figures refer to London, which may be taken as showing in its fullest extent the recent movement in our progressive towns generally. The Metropolitan rates were £3,907,913 in 1873-4 and £6,244,476 in 1883-4, the increase being almost 60 per-cent in the ten years. But to judge of the comparative pressure of rates at different times by observation of their mere gross amounts, without reference to the rental on which they are levied, is to compare fractions with one another by examining the numerators and ignoring the denominators. Now the Metropolitan valuation was £24,756,711 in 1873 and £31,470,725 in 1883. The rental has increased more than four times as much as the rates; and these rates, although producing 60 per-cent more income to the local authorities, have risen only from 3*s.* 2*d.* to 3*s.* 7½*d.* in the £ of rated value of Metropolitan property. Of the increase in the valuation roll, more than one-half, or £5,820,967, is the rent of new buildings. But most of the balance is clear unforeseen profit, going into the pocket of the leaseholder from the rise of the market. Taking care to understate this profit rather than exaggerate its amount, Mr. Clements estimated it at £3,151,940, while the burden from rise of rates appears to be £1,147,147. Thus, the owners of Metropolitan building leases have gained nearly thrice as much by increment of rental as they have lost by increment of rates. Even if the rates have gone partly for improvements which will outlast the leases, these statistics surely forbid any attempt to re-open contracts in the interest of leaseholders.

But it falls now to be shown that in ordinary cases of new contracts of lease, whether under existing laws of rating or under any novel arrangement for the division of rates, no part of these exactions falls upon the building owner or lessee; not only does

the rate on the ground rent fall in reality on the freeholder, but it is he who bears the rate on the building also, except indeed in circumstances which may permit the occupier to be saddled with these taxes wholly or in part. Let it be supposed that a builder is in treaty for a piece of land on which to carry on his operations. He knows the house that he intends to build; he estimates the cost of its erection and the probable price or rack rent which it will fetch. He is then able to judge the amount of ground rent which he can afford to give to the landowner for a site. The probable rating is an essential element in his calculation. The rates either reduce the rent that an occupier will pay for the house or they do not. If they reduce the probable rent they reduce by the same amount the ground rent which he can afford to pay to the freeholder. If they do not reduce the rack rent, they fall not upon the building lessee but upon the occupier; unless indeed that occupier in his turn is able to impose the burden of them on his customers. In his *Manual of Political Economy*, Mr. Fawcett has shown (and has fortified his opinion by very simple and intelligible illustrations) that the actual burden of the rates depends in some measure on the question whether the supply of building sites is ample or restricted. As regards central holdings in towns where the building area is definite and limited, the major portion of the rates, whether imposed on the building rent or the ground rent, whether levied in the present way or in any modified form, ultimately falls on the owner of the land, because the ground rent obtainable is reduced by their amount. On the other hand in regard to suburban holdings, where the supply of building land is more than ample, the amount of local assessments exercises a less influence on the demand for sites, and the rates therefore may fall on occupiers to a considerable extent. But in neither case does the ultimate incidence of the taxation depend on the question from whom it is in the first place collected. Mr. Moulton in cross-examination admitted that his novel and elaborate scheme might probably in its final results differ from the present well tried and familiar system, only by the variations which it would introduce in the conditions of economic friction. If the object in view had been

Incidence of
Taxation

the imposition on ground rents or land values of taxes not at present levied in any way on the heritage, the object, if justifiable in other respects, might have been deemed important enough to warrant the adoption of a complicated valuation roll. This question will come up at a later stage of our enquiry. But, on Mr. Moulton's own admissions, it does seem but doubtful policy to bear and do so much for so small a gain. Detailed surveys of all urban property would begin the labours of the scheme, disputes and litigations might follow: on the tardy settlement of a valuation roll twice as long and complex as it is at present, the rates would be levied as heretofore from the occupiers. But each occupier would be entitled to relief from his building landlord, from whom he would deduct the whole amount of the assessment on payment of the rent. The building owner in his turn would collect from the superior lessee next in order the portion of the rate falling on him, and so on, in cases where there were several interests, the gradually diminishing claim would pass, until the freeholder was reached at last. To this labour must be added the delicate arrangements demanded in regard to existing contracts, in order to do substantial justice among the parties to them, while avoiding the intolerable complications of two concurrent and simultaneous modes of rating, applicable respectively to leases made before and after the change of the law. The great diversities on this point among witnesses favourable to the rating of ground rents, indicate that it taxes the wit of man to devise any such arrangements. Mr. Costelloe, for example, pleads that owners of improved ground rents with nominal reversions should have lenient consideration. But a discretionary power to tax-gatherers, such as this plea involves, is incompatible with the automatic operation of Mr. Moulton's plan, just indeed as is that witness's own proposal to increase ancient chief rents. These

Scotch Feus.

chief rents are analogous to the Scottish feu duties, which received so large a share of the Select Committee's attention during 1891. In the previous Session of Parliament, Mr. C. Harrison, who advocated rating very much on Mr. Moulton's lines, claimed Scottish law and practice on his side, inasmuch as rates in Scotland were divided between owners and occupiers. We may therefore devote a paragraph to the Scottish system of town holdings, especially in its bearing on local taxation. Most Scottish building sites are held in perpetuity, either free, or else subject to payment of chief rents

called fenduties, or ground annuals ; but whether held in that way, or on lease for any period exceeding 21 years, the possessor of the ground rent or chief rent is not regarded as the owner. He is the superior. In the valuation roll, the feuar or the lessee is entered as the owner. The local taxation is divided between the occupier and that owner. Last year in Edinburgh the rate on owners was about 1s. 1d. a pound of the rack rent, and that on occupiers about 2s. 6d. The position of the superior is in many respects similar to that of a mortgagee. Under old feu charters he has a claim to a casualty over and above his feu duty, that being a payment on descent or alienation, generally limited to an extra payment of the annual feu duty or ground rent, but unless so limited, equal to a year's net income from the building. No such casualties may however be claimed under charters granted since the Conveyancing (Scotland) Act, 1874 ; and that Act further provides for the redemption of casualties under ancient charters. The stipulation nowadays is generally for a double feu duty payable at intervals of 20 or 25 years. This duplication seems to be misunderstood by Messrs. Broadhurst and Reid in their volume on *Leasehold Enfranchisement*. In the second chapter of that work they speak of ground rent as being doubled every 25th year, apparently under the misconception that the increased sum once imposed, is exacted annually ever after. Seven witnesses from Scotland appeared before the Select Committee of 1891 on behalf of the rating of these feu duties, while nine Scotch witnesses spoke in favour of the existing law. One feature of the evidence for change was the hesitation with which witnesses recommended it, and the exceptions which they desired to introduce. Mr. Irons, solicitor in Edinburgh, would classify feu duties according to their age ; those constituted more than seven years ago should be taxed more lightly than those of recent years, while the full rating would be reserved for the feus to be granted in future. Mr. Cameron, of Greenock, would treat past contracts in a fair and reasonable way, dealing more severely with superiors in districts where the land was in few hands than in places where there was more effective competition. Mr. Gillies, a Town Councillor of Edinburgh, would tax feu duties which had remained in the hands of the landowners, at a higher rate than those which had been purchased by outside parties as investments ; and where there were different interests on account of sub-feus (corresponding to improved chief rents), he would ask the

Court to fix the respective liabilities of parties to taxation by a process of multiple-pointhing (somewhat similar to interpleader). It is easy to imagine the despair that would sink into the heart of the tax-gatherer if one half of those complex regulations were ever incorporated in an Act of Parliament. On the other hand, the Scottish witnesses for the maintenance of the existing laws give their evidence with simplicity and clearness. They appeal to the sacredness of contract. According to ordinary practice, the feuar or building-owner undertakes to pay all rates. This practice, however, is not universal. In some instances, on the contrary, the land-owner, in parting with his ground in consideration of the annual feu duty, covenants to pay all rates also. Two of the witnesses—Mr. Logan from Edinburgh and Mr. Murray from Glasgow—adduce an important case of the latter kind. Land was feued in the County of Caithness in 1803, 1807, and 1823, in consideration of feu duties of £169, the superior undertaking all public burdens. By and by the extension of the town of Wick, while it enormously increased the value of the property, rendered it liable at the same time to rates more than three times as great as the whole feu duty; consequently, the superior is in the position of having lost his land for ever, and being nearly £400 a year out of pocket besides. No proposal has been made for legislation in relief of such a hardship as this, which is in marked contrast to the leaseholders' grievance of advancing rates; for, as we have found, increase of rental more than compensates in his case. The witnesses further agree that, while the rating of existing feu duties would be unjust, the rating of future ones would be inconvenient—if, indeed, its enactment did not stop their creation altogether, and substitute sales combined with mortgages. One of the witnesses, however, conceded that the taxation of future feu duties might be good, in the way of stopping peoples' foolish talk. From other evidence it appeared that feu duties had been much in favour with churches, charities, public bodies, and trustees—yielding, as they do, a steady and secure income. The imposition of a fluctuating taxation would at once diminish their value by a capital sum more than corresponding to the reduction in the annual return; as the diminished revenue, being itself uncertain in amount, would fetch in the market a smaller number of years' purchase than has hitherto been obtained for the larger fixed and unvarying income. Mr. Martin, who appeared before the Committee both in 1889

and 1891, brought this circumstance out in his later evidence. "One of the very points," said he, "which has made ground rents valuable is the certainty of the income, and that would at once be destroyed, so that all owners would be very heavily fined in the capital value of their investment."

Mr. Moulton finds an analogy for the rating of ground rents in the fact that income tax is deducted from the dividends on preference stocks of railways and other public companies, as well as on the returns to ordinary shareholders. The illustration, however, is a singularly unfortunate one. Income tax is payable on revenue from whatever source derived. But joint-stock companies pay large sums in rates and taxes generally. These outlays are charged on the undertakings as a whole, and consequently they are borne by the ordinary shares or stock. Debentures, guaranteed and preference shares escape all exactions, so long as there is any margin of revenue enjoyed by the ordinary or deferred shares. Such an arrangement has grown with the growth of commercial enterprise. Investors of every class thus find something to suit them. Where security is of paramount importance debentures may be subscribed for. The return is small, but the risks of partnership are escaped. The various denominations of preferred shares yield higher returns with gradually increasing risk. Ordinary shares return a fluctuating income, but hold out to the sanguine a prospect of increased capital value. And deferred shares suit the tastes of the most speculative holders. A similar gradation is observable in the various estates in property, chief rents, leasehold ground rents—original and improved—building rack rents, and reversionary interests. Legislation which would divide a tax, such as the Railway Passengers' Duty, so as to throw a part of it on all the various railway stocks, ignoring the rights of priority among themselves, would supply a true analogy to ground-rent rating.

It has been shown that, disregarding a possible difficulty of shifting the incidence of rating from the shoulders on which the burden first falls, from sheer inertia and economic friction, the greater portion of the rates on property ultimately falls on the land. The question may then be reasonably asked in view of the complexities and difficulties that would arise from the division of rates among the various estates and interests in property, why not follow the precedent of 1869 in the matter of tithes in Wales,

Priorities among
Bonds and
Shares.

and in future contracts impose the entire rating on the ground landlord; little injustice could be done by the arrangement, for by raising his ground rent, he would probably be able to reconp himself for any part of the taxation that does not ultimately fall on the land even at the present day. An opinion in this direction has been expressed before the Committees by only one or two of the most socialistic of the witnesses. But it is really open to fewer objections than is the division of the rates. Two gigantic obstacles, however, present themselves. In the first place, if the freeholders become the sole ratepayers, they will be entitled to the chief voice in the expenditure of the funds directly contributed by themselves. This implies not only a revolution in municipal government, but a revolution backwards, a reversal of the democratic movements of the century, involving grave practical difficulties. And secondly, the rates on property are often so great in proportion to the value of the bare ground, that their mere possible fluctuation from year to year might amount to a very large part of that value. Thus, the landowner, who would naturally strive to keep himself safe, would refuse to lease his ground, or even to part with it for a rateable chief rent in perpetuity. He would insist on a sale for cash, allowing the price to remain unpaid, but secured by mortgage. Or if circumstances required a lease, a huge ground rent would be imposed. The Caithness case, quoted above, illustrates the danger from which he would seek protection. The landowner, called in that case to pay annual rates of about £550, on property yielding him only £169, will take very good care, should he again covenant to relieve a feuar or lessee of rates, to impose a five-fold ground rent. Is it answered that the circumstances are too exceptional to be used to point this moral? The exception is in the landowner taking the burden of the rates. It is not so exceptional to find these rates higher than the ground rent. A large number of Manchester chief rents, and a large number of old ground rents in the City of London are far below the rates now exigible from the properties to which they apply.

Our conclusions on the local rating of ground rents are thus of the same negative cast as those at which we arrived in the matter of leasehold enfranchisement. The rating of actual ground rents and similar revenues from land, without reference to its present value, we found to lead to very irregular results. The taxation of the present value of urban lands, irrespective of the rents charged upon them, would be

Imposition on
Land of all
Local Rates.

Objections to
Legislation.

inequitable in existing contracts, and inconvenient in future ones.*

The tendency of both movements is to benefit the lessee at the expense of the freeholder. The simultaneous adoption of

If adopted,
Enfranchise-
ment should
precede
Taxation.

both schemes would thus be a twofold injury to the latter. The like result would arise from taxation of ground rents, followed by leasehold enfranchisement.

The taxation would diminish the value of the freehold, and the enfranchising law would help the lessee to appropriate that freehold at its diminished value. Enfranchisement, followed after a due interval of time by ground rent taxation, would operate less inequitably, assuming that the enfranchising law had actually been enforced to a large extent in the meantime. The enfranchisement would probably injure the present freeholders, and it would replace them by the present lessees, changed into new freeholders, and so made liable to the burden that ground rent taxation would bring. But practically it would be found that the fear of a subsequent measure of ground taxation would greatly diminish the inclination of lessees to avail themselves of the power conferred on them by an enfranchising act. Therefore, if a leasehold enfranchisement act be passed, it might be expedient to limit the period within which its compulsory powers could be employed. If leaseholders were restricted to ten or fifteen years for this purpose, and if there was an understanding that legislation for the taxation of ground rents would be postponed until the expiry of the period, there would be some security against the infliction of a double wrong on existing freeholders.

(2). *National Taxation.*

Following the direction of parliamentary enquiries, we have confined our examination of the tax question almost wholly to urban subjects, and to municipal or other local rating. In this

* These conclusions are very fully borne out by the report from the Select Committee of the House of Commons on Town Holdings published in June 1892. The Committee agree that ground rents are already taxed, and that the imposition of a direct assessment upon them would lead to anomalies and inequalities. They declare the proposals for separate assessment of ground values and building values to be impracticable. They pronounce that no sufficient cause has been shown for interfering with completed contracts relating to payment of rates, and that such interference would inflict injury unfairly without any compensating benefit. But in regard to future contracts they recommend that local rates shall be equally divided between occupiers and owners, and that each owner shall be entitled to a proportionate deduction from the proprietor of the ground rate. This is the only change of the law that commends itself to the Committee. The main argument in its favour is, that it will bring the apparent incidence of rates nearer to their real incidence.

narrow field it is conceded that certain burdens fall on heritable property; and the debate has arisen in regard to the division of these burdens among the various persons who occupy and own the property. The question is not one likely to rouse general enthusiasm, especially in presence of the political economist who stands in the back-ground, invoking Mr. Ricardo, and seeking to mollify the combatants by an assurance that, no matter from whom they collect the rates, the ultimate burden of the exactions can fall only in one way, its incidence being determined by natural and unchangeable law.

But a much larger question has of late years come above the horizon of political speculation, if not within the range of practical statemanship. Municipal and other local rates are levied almost wholly from heritable property, in respect either of ownership or occupation. But the national exchequer is replenished from many sources, and a very small portion of the revenues of the United Kingdom comes directly from the land. Heritable property certainly contributes, under Schedules A and B, a large portion of the property and income tax. But this constitutes in no special sense a burden on the land, the tax being levied alike out of all kinds of income from whatever source. Indeed it has been said, apart from the question of a distinction between land and other possessions, that incomes derived from all kinds of realized property, whether rateable under Schedule A or Schedule C, ought to be more heavily taxed than incomes depending on trade profits or personal exertions, returned under Schedules D and E. Apart from income tax, and from a sum of £430,000 derived from crown lands, the only important direct contribution from the soil of the United Kingdom towards the national expenditure is £1,030,000 from the land tax. About half of the liability for the latter tax having, however, been discharged on payment by landowners of capital sums in the way of commutations, the whole revenue from land tax may be taken at £2,038,000, the amount received at the end of last century before the process of redemption began. This sum is certainly small in comparison with the gigantic figures of the Budget, which shows £89,489,000 of national income, £44,268,000 being drawn from customs and excise duties alone. Of recent years, formidable claims have been advanced, mainly in the interests of labour, for such fiscal re-arrangements as should throw upon ground rents and land values the whole burden of imperial as well as local taxation, to the entire relief of

taxpayers from every other source. The arguments for this social and fiscal revolution are based on the laws of nature, the theories of political economy, and the records of natural history. They may be summarized under the following heads.

(1) Land ought not to be private property. Like the atmosphere, it belongs to the whole race of man. Property in Land. This position has been upheld from time to time as a mere philosophic theory. It was favoured by Gregory the Great. Voltaire ridiculed Rousseau as a poor buffoon for his advocacy of the idea. It has been gravely expounded in sober treatises, as for example in an essay from the pen of Mr. William Ogilvie, himself a landed proprietor as well as a professor, published in London by J. Walter, a hundred and ten years ago, and lately re-issued by Mr. D. C. Macdonald to the present more radical generation. Mr. Ogilvie divides the price of land into three distinct portions. First, the original value, in a state of nature prior to cultivation. Secondly, the accessory value, imparted by the labours of the owner and his predecessors. Thirdly, the contingent value, or, to adopt the terms now in use, the probability of a future unearned increment. He maintains that landowners have a full and absolute right to the second only of these three parts of the value of their land, and that the first and third parts belong to the community at large. He claims that the expenses of the state ought to be met by a land tax until the whole amount of the original value of the soil be exhausted, and that active progressive industry should be freed from every public burden.

(2) Similar canons of taxation were promulgated at a still earlier period by John Locke in this country, and in France by the founders of the agricultural school of political economy. These physiocrats (to accept the modern name), of whom Quesnay and Turgot were chief, drew a sharp distinction between labour devoted to agriculture, and labour employed in manufactures. The latter form of work they denominated barren, not because it produces nothing, but because its result is no more than a mere equivalent for the capital expended and the labour and skill employed. But in agriculture, where human labour is directly assisted by the forces of nature, its result is sufficient to yield not only reasonable interest, wages and profits, but also to provide a margin of net product, corresponding to the rent of the land. They considered that the increase of this net product or land rent should be one of the chief ends of Government, and

The single Tax. that all national expenses should fall on that net product, and be levied by taxation on the value of the land. They maintained indeed that all taxes, howsoever imposed in the first instance, did ultimately fall on this net product of the land of the country.

English History. (3) English history is invoked in support of the pleas on behalf of the taxation of revenues from land, to the relief, wholly or in great part, of all landless taxpayers. In the middle ages the expenses of the English Government were defrayed chiefly by the rental of the land which the Crown had retained, and by the feudal burdens imposed on lands granted to the nobles. Tenure by knight service involved many incidents whose fruits upheld the charges of the Court. Wardship, fines, forfeitures on marriage, aids, reliefs, primer seisin, and possible escheat, all were sources of revenue to the Sovereign, as lord paramount, in ages when taxation in England was but little known. The taxes first granted were subsidies, or supplements to the royal revenues. At the Union of the Crowns of England and Scotland in 1603 the English Customs Revenue was but £127,000. Taxes on the fruits of the land were then in operation, a ninth, a tenth, or a fifteenth of the produce being claimed for the exchequer. It was just after the restoration of Charles the Second that land was freed from most of the heavy impositions under which it had till then been held of the Crown, tenure in chivalry being abolished by the Act 12 Car. II, cap. 24. The landholders, who were also the law makers, made up for consequent loss of national revenue by the imposition of excise duties, stamps, and taxes on bankers. The reign of William and Mary witnessed further developments of customs and excise Duties, as well as the enactment of a land tax assessed upon a valuation still in force. On the accession of the House of Hanover, land was still bearing more than one-third of the expenditure of the State. William and Mary's land tax was frequently dealt with by Chancellors of the Exchequer in the eighteenth century. The subsidies voted from landed property in the late Stuart reigns had failed to yield the revenue expected, and hence the land tax had been devised. It was associated with an equivalent tax on the profits of capital employed otherwise than in agriculture or loans to the Government. But being charged in definite portions on the various towns and counties, with permission for each district to collect it as the local powers might arrange, the tax on profits was to a large extent levied by way of

a house duty. Some of the changes of the land tax in the days of the Georges will fall to be noticed in connection with arguments for and against the re-imposition of such a tax at the present day.

(4) Pleas for the taxation of ground values are found also in a comparison of the fiscal arrangements of England with those of other countries. In spite of the inconveniently large revenue which the United States Government derives from customs, a tax of one-half per-cent on the capital value of land is levied in certain of the States, equivalent to about two shillings in the pound of rental. In India generally, with the exception of Lower Bengal, the Government absorbs the whole rent of the land. Confining attention to Europe, we find, to quote the words of Mr. Goschen in 1871, that as regards imperial taxation, land in this country is in a far better position than land in any other European state. In England it pays about $5\frac{1}{2}$ per-cent of imperial taxation. In Holland, land pays 9 per-cent, in Austria, $17\frac{1}{2}$ per-cent, in France, $18\frac{1}{2}$ per-cent, in Belgium, $20\frac{1}{2}$ per-cent, and in Hungary, $32\frac{1}{2}$ per-cent of the national taxes. The comparison is, however, defective, inasmuch as it takes no account of local rating. Local administration has been carried further in Great Britain than in any other European country, France being the only continental state nearly approaching us in the development of local government. But, of the gross revenues of £44,373,000 now enjoyed by local authorities in England and Wales, the rates yield no less than £28,275,000, or 63·7 per-cent, a great part of which ultimately falls on the ground rent or land value. In France on the other hand about one-fourth of the local taxation falls on real property, in Austria and Russia about one-half, and in Belgium and Holland about one-sixth. Taking national and local taxation together, there is probably no very great difference in the burdens on real property imposed in Britain compared with those in several of the continental states.

(5) Not only have national laws, economic speculation, mediæval history, and national example been invoked on behalf of ground rent taxation, but the confidence of prediction has been called upon to add its contribution to the height of the great argument. Mr. George, the most popular of its extreme advocates, assures us that the absorption by taxation of the whole value of the soil (other than what it has received from human labour), is the sure and speedy road to the

Foreign
Taxation.

Anticipations.

constant employment of all labourers, the abolition of all poverty, and the regeneration of society.

This subject naturally presents itself for our consideration under two heads:

(1) The nationalization of land by a tax approximating to its whole rent, whether urban or rural.

(2) The imperial taxation of ground rents and land values to a small extent, corresponding in some degree to the levies on other sources of national revenue.

These questions, although in one view they differ only in degree, demand separate examination.

Land differs in its origin and nature from all other kinds of property. "It is the original inheritance of the Landholder's Rights. "whole species." "When private property in land "is not expedient it is unjust." "The claim of the landowners "to the land is altogether subordinate to the general policy of "the state. The principle of property gives them no right to the "land, but only a right to compensation for whatever portion of "their interest in the land it may be the policy of the state to "deprive them of. To that, their claim is indefeasible. . . . This "is due on the general principles on which property rests. If the "land was bought with the produce of the labour and abstinence "of themselves or their ancestors, compensation is due to them on "that ground; even if otherwise, it is still due on the ground of "prescription." These quotations from Mill's *Principles of Political Economy* indicate at once the strength and the weakness of the cry for land nationalization. The right of the present proprietor is founded on contract, or else on immemorial usage. If the land has been in the same family from a period anterior to the great rebellion, it has probably been the subject of settlement and re-settlement generation after generation; and it would be highly unjust, at the cost of persons now owning the jointures, portions and trusts, the life interests, reversions and remainders, and even the mortgages and other debts, to enquire into the legislation by which feudal burdens were abolished two hundred and thirty years ago. If the property has been acquired in more recent times, unless it is land appropriated under a Commons Enclosure Act, or granted by Parliament for services to the country, it must have been acquired by purchase, either from previous private holders or from the Crown. In any such case it would be nefarious to confiscate the land without compensation, on the plea of an abstract distinction between land and other

property. An illustration may be taken from America, where these doctrines of a new and revised morality have been loudly preached. Land in the Western States is constantly being sold by the government to settlers, who indeed would not undertake the severe labour of bringing it under cultivation on any terms short of ownership. If in a few years the State were to adopt the tenets of the rent confiscation school, then it would be the very seller of the land that would seize it from the purchaser, and that without any return of the purchase price. At the Industrial Remuneration Conference of 1885, Mr. Shaw Lefevre pointed out that the various States of the American Union still held three hundred millions of acres of land unappropriated, where the new ideas could be applied without any injustice. But not one of the States has adopted any such principle, probably feeling that a refusal to grant permanent rights in land would drive away immigrants and check prosperity.

The main argument advanced for the seizure of the value of land in the name of the whole people, might indeed be carried a good deal further. The material substance of all human possessions of whatever kind is the free gift of nature, in the same sense as is the land in its primitive condition. All that man can do is to change the form or place of nature's bounty. We may appraise the original value of a cultivated field or of an urban building lot now levelled and drained and provided with roads, and may claim that value for the nation by a tax of twenty shillings in the pound; but on a similar principle we might enter the vaults of the Bank of England, and finding there twenty-five millions in gold, a raw material of nature having but little labour of man expended upon it except in the mere collection, we might estimate its original value, and carry that off to be employed for the benefit of the people. The problem of the liability of the bank in such case to its depositors, might be left in the same category as the question between a ruined landowner and his mortgagee. The gold thus seized would not be wanted at all events for the reduction of the national debt, and it might therefore be conveniently employed towards providing the ideal of Mr. Saunders, a reformer whose evidence before the Select Committee we had occasion to quote, and whose aim is expressed by his aphorism, "no taxes at all and a pension to everybody." The national debt is to be extinguished not by discharging it, but by a simple application of the doctrine that one generation cannot bind its

Communism.

Reputation. successors. Grants of land, whether made by the Conqueror to Norman nobles eight hundred years ago, or by grateful parliaments to victorious generals eighty years ago, are all to be resumed for the people. And in like manner the people are to "ask themselves why they should continue to pay "interest on the vast sums spent during former generations in "worse than useless wars", and to answer the question by making such "reforms" as that public debts are "abolished." (The quotations are from Mr. George's book on *Protection or Free Trade*.) Thus the one hundred and eighty thousand persons who have their money in the Government funds (from which about one third of them derive incomes of less than a hundred pounds a year) are to be involved in a common ruin with the many scores of thousands of British landowners, large and small. And all this havoc is to be effected by way of an introduction to the golden age of the world's history. Emergencies might however arise, even in those days of predicted prosperity, when the renovated Government would find its revenue from the land insufficient. No other taxes exist, and none would be borne. The "single tax", the famous *impôt unique* of the physiocrats, is the sole resource. Nor can a new loan be raised after repudiation of the old ones. It is re-assuring to learn that there is still one way in which the state necessities may be met. That mode is indicated in the following quotation from Mr. George's *Social Problems*, where he is regretting the creation of the debt of the United States in consequence of the civil war:

Spoil. "If when we called on men to die for their country, "we had not shrunk from taking, if necessary, nine "hundred and ninety-nine thousand dollars from every millionaire, "we need not have created any debt." Such are the financial methods of Mr. George's administration.

It is a mistake to attribute to private tenure of land the tendency of population to press on the means of subsistence.

Pressure of Population. This tendency is recognized by political economists generally; indeed it is seen in operation alike in Europe, Asia and America, amid civilizations of extremely diverse kinds. Nowhere is it more obvious than in our own Indian possessions, where the myriad inhabitants are poor alike in food and in permanent property, and unable to protect themselves against periodic famines. But in India the land is nationalized, and its rent is the chief dependence of the Government. The Indian Taxes on land and its produce yield three hundred and

twenty-five million rupees, or fifty per-cent of the whole State revenues, apart from such items as receipts from railways, post office, telegraphs, and irrigation, absorbed by corresponding outlays on these services themselves. The Indian Government, although conducted by an alien race, has never been seriously accused of gross partiality or corruption. In India then, according to the theories of the land nationalizers, the masses ought to be enjoying uninterrupted prosperity, and poverty should be unknown. The facts of Indian native life are of themselves enough to condemn the novel social panacea of landlord extinction. But we find its condemnation without going so far away. We have but to look around us. It is evident that no state appropriation of land, however general, and no devotion of its every acre to purposes of agriculture, excluding sport or luxury, will ever enable the soil of Great Britain to support more than a small fraction of the inhabitants, who have long ago exhausted the capabilities of their own country for the production of food.

The darker features of modern civilization are laid to the charge of the landowner. Not only so, but these evils themselves, sadly real though they may be, are magnified by the rhetoric of pessimistic exaggeration. It is said that as the rich have become richer, the poor have receded into more squalid and hopeless penury. Whatever foundation there may be for the allegation of increased destitution among the mere camp followers of the army of labour, it is by no means true that the industrial classes generally have gone back. Mr. Robert Giffen has deduced

Improved condition of working classes. quite opposite conclusions from more than one field of social statistics. He has shown for example that during the forty years ending with 1883, the aggregate income of the British working classes increased from two hundred and thirty-five million pounds to six hundred and twenty million pounds, or by one hundred and sixty per-cent, while the corresponding increase of the numbers sharing in that income was only about thirty per-cent, the individual working class earnings being thus doubled on an average. But between 1838 and 1883, the amount of British capital, exclusive of landed property, increased about one hundred and thirty-five per-cent, while the annual return from capital rose only about one hundred per-cent; interest and trade profits having declined. Pursuing another line of enquiry over a shorter period, based on statistics published by Mr. Leone Levi, it appears that in the seventeen

years ending 1884, the British working classes increased in number about eleven per-cent: but that, notwithstanding the general reduction of the hours of labour, the aggregate earnings rose about twenty-five per-cent: the average increase for each working man being from £38 to £42. 14s. or rather more than eleven per-cent. These facts indicate, at the least, that recent gloomy oracles concerning the position and fate of labour are not warranted by the facts of experience. Within the past half century the rich have become more numerous, but on the average not richer individually, while the poor are twice as well off as they were fifty years ago.

If it were certain that land nationalization would be for the general advantage, schemes might be devised by which, on easy, yet equitable, terms, the government could enter upon ground rent after the lapse of a period exceeding that of an ordinary human life, but short in the history of a nation. An estate yielding in ground rent, whether agricultural or urban, a revenue of £100, might be deemed of the value of £3,000. But an annuity of £5 a year amounts in 100 years to a sum exceeding £3,000, taking interest at three per-cent. Hence the full value of the reversion to the land of the country at the end of a century would be amply covered by the payment, during the century, of annuities of 1s. in the pound of the present rental. That rental, including land both in town and country throughout the British Isles, but excluding the value of buildings, may be taken at £100,000,000, so that £5,000,000 a year for a century would be sufficient to purchase the reversion. This price is less than would be the probable cost of a war with any European state. It may seem a large figure for so distant a heritage, but it is quite within the powers of our prosperous exchequer. And it is a far smaller price than we should have to pay, in extinguished national credit, with all its attendant evils, for a policy of rent confiscation, such as would make shipwreck at the same time of self-respect, conscience and honesty.

It may, however, be gravely doubted whether the absorption of a country's ground rent, rural and urban, by the State would not have the effect of diminishing the amount of that rent itself, of injuring the productive capacity of the country, and of ensuring a retrogression towards a less advanced condition of agriculture. It is said that production would increase, because land now held for luxury or display would be brought into use. Any addition from this cause

Practicable
Land
Nationalization.

Evil results.

might, however, be far more than counterbalanced in other ways. Adam Smith declares that in his day the Crown lands of Great Britain did not afford the fourth part of the rent which might have been drawn from them had they been the property of private persons; and he expressed the opinion that had they been more extensive they would probably have been still worse managed. "The revenue", continues he, "which in any civilized monarchy the Crown derives from the Crown lands, although it appears to cost nothing to the individuals, in reality costs more to the society than perhaps any other equal revenue which the Crown enjoys. It would in all cases be for the interest of the society to replace this revenue to the Crown by some other equal revenue, and to divide the lands among the people, which could not well be done better perhaps than by exposing them to public sale." But to impose a land tax approximately equivalent to the rent, would be to bring the entire country into the position of the Crown lands, with the imminent danger of a highly disastrous result. The stimulus of private enterprise is required for the development of any species of property. This fact the late Sir William Siemens recognized when he said, "If an invention lay in the gutter unowned, I would give it to a particular owner, that some one might have a particular interest to develop and push it." And the private ownership of land benefits the general community of Great Britain in an especial degree. It does this because British land is a possession so greatly esteemed, from the social and political distinction which it confers, that the landowner is willing to hold it, although receiving a return far less than would content commercial investors. The agriculturist thus secures his farm at a rent returning a very small percentage to the landlord. But if the landlord is taxed out of existence, with him will disappear the prestige of landed property, for which he has hitherto paid, in the manner indicated, a large price to society.

The advocates of the taxation of land values at 20s. in the pound claim to be the followers of the French economists of last century. But this pretension can hardly be sustained.

The physiocrats did, indeed, advocate the abolition of all other taxes, and the raising of the expenses of the Government wholly by an impost levied on the net produce of the land. But, so far from contemplating the ruin of the landowner by the approximation of this impost to the entire rental, they expected that his burdens would be lightened, and that the

Physiocratic
Theories.

revenue of his property would be augmented, not only by the amount of the *impôt unique*, but by the saving that direct taxation would bring. This was their inference from a belief that the soil was the source of all wealth, and that every person who had no land must be supported by an income derived directly or indirectly from the landowner. Hence they reasoned that, assuming the ratio between the demand for labour and its supply to be unaffected by the amount of taxation, then wages must be higher by the amount of the taxes and the costs of indirect collection, if the revenue is taken in the first place from the wage receiver, instead of being directly exacted from the primary source of all riches. The premises of the argument, however, are not unassailable, for the powers of nature work together with man, not only on the field but in the factory. And the practical conclusions are vitiated by many considerations, among which nowadays not the least are, the development of commerce and the freedom of intercourse between nations adhering to the most diverse systems of taxation. For example, the chief portion of the expenditure of the United Kingdom might be thrown upon agricultural rent. The consequent relief to commerce and labour might tend to a reduction of wages and to an increase in the demand for agricultural products; but the facilities which exist, alike for migration of labour and for international traffic in goods between nations, would prevent British agriculture, on which the burden of the fiscal change would fall, from reaping any large share in the compensations.

It may further be noticed that the value of land proposed to be claimed for the State by way of taxation is really, as far as concerns agricultural land in Great Britain, a merely hypothetical value. It is the original or unimproved value which is said to be no fit object of private tenure. The accessory value due to labour is generally conceded to belong to the labourer and his successors. In *Progress and Poverty*, Mr. George does, indeed, allege that the value of improvements is comparatively small, and that it may be merged in the greater value of the land in a state of nature. But in his more recent manifestoes he has refrained from repeating this glaring inaccuracy. The rent claimed for the community is the net rent, according to the definition of Ricardo: the value of the original and indestructible powers of the soil. But, as Mr. Macdonell shows, by many examples, in his *Survey of Political Economy*, the natural and inherent properties of the soil have in old

Original Value
of Land.

countries been exhausted long ago. Without a careful renewal of the properties of the soil, agriculture cannot long subsist, except in the most rudimentary form. And much more is needed in order that the earth may yield its increase. At the recent Industrial Remuneration Conference, many schools of thought on economic questions were represented. Mr. A. J. Balfour was followed by Mr. Frederic Harrison, and both speakers used the same language in testifying to the insignificance of the "prairie value" of British land. An ordinary farm is a highly elaborated instrument of production, as much artificial as a house or a factory. The land was at one time moor and fen, swamp and sandy wilderness, pathless jungle, wood and heath. In the absence of roads and bridges (many of them built by landlords or out of rates on land), without embankments, drainage or fencing, the valleys periodically inundated, the uplands bare and stony, the capital value of the land might hardly amount to one year's rent. On the basis of this insignificant value would require to be fixed the annual revenue claimed for the State, on the extreme theories of land nationalization.

Having thus examined proposals for the confiscation of ground rents and land values, we shall in conclusion consider the question of imposing any new or special imperial tax on this class of property. We have found that while local taxation is derived almost wholly from hereditaments, in respect either of ownership or occupation, Her Majesty's exchequer is replenished chiefly from other sources. Legislation has indeed dealt tenderly with land. Until 1853 freehold estates escaped the death duties; and although the anomaly is now redressed in great measure, yet a comparison of the present succession duty with the probate and legacy duties, shows that the favours granted to land, although diminished in this respect, have not been quite withdrawn. And the recent contributions by Parliament for the relief of local taxation, amounting twenty years ago to £1,250,000, and ten years ago to £2,860,000, but now increased to £6,870,000, indicate that the legislature is still disposed to lighten the burdens affecting the landed interest. We have seen that in Great Britain land bears an unusually small part of the public burdens, and that the land tax was so manipulated last century, and so neglected during the past ninety years, as to have become an insignificant item of the national income. Mr. Fawcett urges that this very neglect of the tax is a reason why it must not now be thought of as a possible source of

Claim for a
moderate
Imperial Land
Tax.

revenue to the exchequer. In his *Manual of Political Economy*, he states that the land tax in this country has long since been commuted for a fixed payment, and has consequently not increased with the enormous advances in the value of landed property. He points out that it would have been a boon to the tax-paying community if, when the land tax was imposed, its amount had been fixed not at a certain amount of money, but at a certain definite proportion of the value of the land. The revenue that would have been raised in that way, although constantly augmenting, would have been felt as a burden by no class, because the land tax, spontaneously provided by the appropriation of a national monopoly, would have naturally increased with the increase in the value of that monopoly. Nevertheless, Mr. Fawcett concludes that it would now be an unjust confiscation of property to increase the land tax: for the augmentation would be paid entirely from the rent of landowners, and would therefore be as indefensible as any other impost levied upon one special class. He might have strengthened his case by showing, as Mill shows in discussing the taxation of realized property generally, that the capitalized amount of the increase of such a tax would fall entirely on those who held land when it was imposed; as the selling price of their property would fall with the reduction in net revenue derivable from it.

Nevertheless, Mr. Fawcett's logic does not appear to be altogether conclusive. Between the imposition of the land tax in 1692 and its perpetuation in 1798, the rate was repeatedly altered by Parliament, and successive Chancellors of the Exchequer held themselves as free to deal with it as with any other source of revenue. Two examples of this procedure may be recalled, each of them having a historical interest of its own. A duty on salt had been devised in the same reign as that which saw the creation of the land tax, namely, that of William and Mary. The salt duty being found oppressive, an Act was passed in 1729 for its repeal at Christmas in the following year. But before that Christmas came round, the ministers of George the Second had concluded that their surplus might be better bestowed to lighten the land tax, which Sir Robert Walpole accordingly reduced to one shilling, re-imposing the salt duty before its abolition had taken effect. The land tax was soon raised again, the consideration of its amount being indeed an annual feature of the budget in those days. But in the early years of George the Third it was once more reduced by a shilling in the

Existing Land
Tax.

pound, and the events which followed this reduction made an epoch in British history. The date was 1767, and Charles Townshend was Chancellor of the Exchequer. In part compensation for the half-million foregone on the land tax, petty exactions were imposed on the imports of our American colonies. But the colonists would have none of the burdens which the mother country called on them to bear. And after a lamentable war, Britain recognized the independence of the United States of America. It was by the Act 38 Geo. III, cap. 60, that the

Provision for its
redemption.

land tax was made perpetual, with provision for its redemption or purchase, in consideration of consols sufficient to yield dividends exceeding its amount by one-tenth or one-fifth, according to the circumstances of the redemption. The debate in the Commons over this bill indicates that neither supporters nor opponents considered it to bar the imposition of a new and better-adjusted land tax. In advocating the measure, William Pitt contended that if it paved the way for a re-arrangement of the tax on a basis more equitable than the valuation made in William and Mary's reign a century before, that was a recommendation rather than a defect. But, added the great

Right of further
taxation
reserved.

commoner, "It leaves the question of a more equal repartition of the land tax precisely where it found it. Parliament now has the undoubted right to raise more than four shillings in the pound on the land; and what greater authority would it acquire were the present redeemed? The only thing necessary to be provided is that if ever a new land tax is imposed it shall not be imposed upon those who have redeemed, in any different proportion from that on those who have not redeemed. It would be necessary to provide that the amount of what may have been redeemed should be deducted from any new impost." The opponents of the bill maintained stoutly that it was no final settlement of the land tax. Sir John Sinclair said that the foundation of an additional tax was clearly laid in the bill, that such additional tax might at first be only one shilling or even sixpence in the pound, but that it would soon be found the readiest resource in all emergencies. Lord Sheffield understood that there was an intention of laying on a new land tax as soon as the old one was sold; and Mr. Pitt, so far from denying this, merely said that lands on which the tax might be redeemed would be liable for no other tax except such as should be imposed on them in common with all other property of the same description.

The history of the land tax, and the statements thus made in Parliament at the time of the last legislation on the subject, do not seem to warrant the high ground taken by Mr. Fawcett. Yet it may well be that the very magnitude of the claims set up against ground rents and land values, and the baseness of some of the pleas put forward for the spoliation of the land holder, incline the judgment rather in the direction of the views of that distinguished statesman and economist. But opinions of an opposite cast are expressed by authorities as famous as Mr. Fawcett, and supported by arguments not less cogent than his.

Adam Smith in his *Wealth of Nations* says "Both ground
Lands well adapted for Taxation. "rents and ordinary rent of land are a species of
 "revenue which the owner in many cases enjoys
 "without any care or attention of his own. Though a part of this
 "revenue should be taken from him in order to defray the
 "expenses of the State, no discouragement will therefore be
 "given to any sort of industry. Ground rents seem in this
 "respect a more proper subject of peculiar taxation than even
 "the ordinary rent of land. Ground rents, so far as they exceed
 "the ordinary rent of land, are altogether owing to the good
 "Government of the Sovereign, which by protecting the industry
 "either of the whole people, or of the inhabitants of some
 "particular place, enables them to pay so much more than
 "its real value for the ground which they build their houses
 "upon. Nothing can be more reasonable than that a fund which
 "owes its existence to the good Government of the State should
 "be taxed peculiarly."

We quoted John Stuart Mill's views, favouring the power of the state to appropriate land, but always subject to the indefeasible right of the holder to compensation. He condemns any new and general land tax on the ground that a peculiar tax on the income of any class, not balanced by taxes on other classes, is a violation of justice. At the same time he exempts from this censure a tax which, sparing existing rents, should content itself with the appropriation of future increase arising from the mere action of
The unearned Increment. natural causes. Indeed this proposal of Mill's, distinguishing the unearned increment of land from its previous value, is one of the features of his chapters on taxation; and it has become specially identified with his name. He points out that the ordinary progress of society tends of itself to augment incomes from land, giving both a greater amount and a greater proportion of the wealth of the community to land

holders, who grow richer without working, risking or economizing. He claims that the State is entitled to absorb this spontaneous increase, wholly or in part; but he concedes to a landholder, in the event of the enforcement of such a claim, the option of demanding for his property the market price of the day. Various objections have been made to Mill's proposal. It has been pointed out that land is not the only possession to which large unearned increments have accrued in recent times; and instances have been adduced of investments in the shares of commercial undertakings, which have enormously increased in value, without any effort on the part of the individual shareholder. While on the other hand, at an early stage of this enquiry, we found that much British land had greatly decreased in price within recent years. The adherents of Mill's system reply that traders undertake liabilities absent from the holding of land; and that, notwithstanding the present depression in real estate in this country, due to the competition of the whole world with the British Isles, fostered by the removal of almost all restraints whether physical or fiscal, yet it is demonstrable that, comparing age with age, land values must rise with the progress of society and the increase of the human race. Mill's plans, however, in the form in which he launched them, involve practical difficulties grave if not insuperable. The unearned increment will be found very unequally distributed. In one locality it will be but small even in prosperous times. In other more favoured situations it will be gigantic. Nowhere will it be greater than in the suburbs of private towns, where land formerly employed in agriculture becomes ripe for building. The adjustment of a tax on the mere increment would appear under such conditions very irksome, if not impracticable. And if the special character of landed property warrants any such exclusive exaction, substantial justice may be satisfied by the imposition of a small tax on the whole rent of the land, instead of the absorption of a large portion, or the whole, of the mere increment of its value. Acceptance of this position involves the admission of the justice and expediency of a moderate imperial taxation of ground rents and other revenues from land, including agricultural rent. The tax would be imposed on the actual value of the ground in its present condition. For while it was important to show the great difference between the original and the improved value of the ground when discussing a threatened confiscation of the rental by a taxation of 20s. in the pound, yet it would be quite unreasonable to attempt

Alternative to
Appropriation
of Increment.

to base any scheme of ordinary and moderate taxation on so hypothetical a basis. Such procedure would repeat in an exaggerated form the improprieties of the Georgian era, when the land tax was re-imposed year after year for generations, and finally declared perpetual, all on the basis of an imperfect valuation roll made up in the 17th century. The value of the buildings on the ground would, however, be excluded. The reasons which invite a special tax on land apply in no respect to buildings; notwithstanding that the ground and the erections are usually one subject in the eye of British law. Thus the taxation of ground values for imperial purposes would involve some such dealing with valuation rolls as was demanded by Mr. Webb and Mr. Moulton for the reform of local rating. We shall return to the figures already* used in illustration of Mr. Moulton's local assessment scheme, and shall suppose an imperial taxation of ground values at 6*d.* in the £. This would be a heavy exaction, for it would practically double the income tax on that description of property. The contributions to the new tax on ground values payable by the persons respectively interested would be as follows:—

Illustration.

	£	s.	d.
Freeholder, in respect of his ground rent of £50	1	5	0
Intermediate lessee " " £20	0	10	0
Building Owner, in respect of the increased value of the ground £30	0	15	0
In all, 6 <i>d.</i> in the £ on the full ground value of £100	2	10	0

We found that such a sub-division of values was likely to cause more difficulties than any consequent re-arrangement of local taxes would compensate, seeing that the ultimate incidence of these local burdens does not depend on the question from whom they are levied in the first place. But it is a matter demanding much deeper consideration, whether a valuation of land apart from buildings would be expedient, in order to collect from the land, taxation now falling on trade or labour or on the general community.

Aided by the researches of Mr. Giffen, assistant secretary of the Board of Trade, we showed that labour had profited more than capital from the recent progress of society; and we thus undermined a corner of the foundation on which arguments have been built for the extinction of landlords by the confiscatory taxation of ground rents and land values. On the general

subject of land taxes, that painstaking economist points out that there could not be a wider divergence of ideas than is discovered between the proposals of the Land Tenure Reform Association on the one side, and the agitation against local rates on the other. The latter cry comes from the landowners, formerly if not now the most powerful party in the State. They have discovered that, although ground rents are exempted by contract from local taxation, yet the burden of local expenditure does ultimately affect the land. Their aim, therefore, is to throw local rates on the imperial exchequer; and in this effort, as we have already seen, they have been surprisingly successful, Parliament now granting more than five times as much in relief of local taxation as it did twenty years ago. This procedure benefits chiefly the very classes that have profited most by the increase in the value of urban land. That increase is much greater than the rise in rates; indeed it is only the amount by which the growth in value exceeds the advance in rates that becomes visible and measurable, seeing that the higher price of ground is fixed after giving effect to the stipulation that the freeholder is exempt from all rating. But were the poor rates and the police rates to be thrown on the consolidated fund, as is desired by those who press for local relief most eagerly, the concession would represent a gift to landowners of more than eleven millions a year, equivalent to a capital sum of from three to four hundred millions of pounds. The degree of success achieved in this direction, in spite of the power of democracy, indicates how great is still the strength of the ancient landed interests, and how little the ordinary citizen understands the effects of the Government grants that are passed so quietly in aid of local rates.

The Land Tenure Reform Association on the other hand, like the United Committee for the Taxation of Ground Rents, would fain secure for the State a share of the unearned increase of land values, according to Mill's idea. Nothing can be said against the principle of the association, which respects the existing rights of individuals, but asserts the paramount right of the State. The imposition of special charges on land, such as a royalty or reserved rent-charge, resembling many other burdens by which Government limits the rights of ownership, would be an innocent restriction of exclusive privilege. Especially would such a reservation bear lightly on land in a progressive community. Mill's own scheme, however, would be of most difficult application, for

Grants in aid
of Local Rates.

reasons already indicated; and the presentation of his alternative of surrender to the State on equitable terms, would tend to weaken confidence in the continuity of private tenure. A national increase of special taxes on land levied according to ordinary methods, would, in Mr. Giffen's opinion, reserve for the nation in a more practicable way a portion of the augmenting value of that description of property. In his *History of Political Economy*, Mr. Ingram adopts the view that a rise of rent benefits landlords alone, and that their interests are permanently opposed to those of all other classes. This doctrine savours of the unenlightened idea that the interests of buyer and seller are always opposed. Such opposition of interests indeed refers only to the incidents of a single transaction. In the general scheme of commerce, buyers and sellers have a common concern. And in the universal intercourse of society, landlord and tenant, producer and consumer, have but one interest. None the less, however, has rent obligations of its own. It is the revenue spontaneously offered by nature or accident. This is the definition of Mr. Nassau Senior, who thus distinguishes it from wages, the reward of labour, and profit, the reward of abstinence. Agricultural rent is the gift of nature. Urban ground rent is the outcome of social progress. And the possessors of rent, whether of the former kind or of the latter, owe special duty and consideration to the rest of the community.

MR. TARN'S ESSAY.

I.—THE ORIGIN AND DEVELOPMENT OF LEASEHOLD TENURE.

THE custom of letting land for agricultural or building purposes is a very ancient one, the earliest records of such a custom being those which have been preserved in the Institutes of Justinian. From these records we discover that under the Roman Empire there were in use four distinct forms of leasehold tenure. The most important tenure was that known as *Emphyteusis*—a Greek word meaning *planting*—which was introduced in the 2nd century, and arose out of the practice of making perpetual leases of provincial lands belonging to the State and acquired by the rights of war. This tenure was afterwards adopted by private

Leasehold
Tenure under
the Roman
Empire.

proprietors, who, through inability to devote sufficient time or trouble to the cultivation of their property, preferred to let it to a suitable tenant called the Emphyteuta, who agreed to pay an annual rent for the use of the land. Of a similar nature to Emphyteusis was the tenure called Superficies, by which a landed proprietor conceded to another person an area of ground for the erection of buildings without parting with the ownership of the soil. A third class was that known as Precarium, which, being equivalent to a tenancy at will, could be determined by the grantor at his pleasure. Lastly, there was a general system of short leaseholds, the usual term being five years, at the end of which the lease might either be renewed or be considered as one from year to year. In all leases the tenant was bound to pay the burdens attached to the holding of the land and to deliver the receipts to the owner ⁽¹⁾.

Although it would appear that none of the above systems of land tenure were introduced into this country during the occupation of the Romans in the first four centuries of the Christian era, yet it will be seen that every system which, in the course of the subsequent 14 centuries has come into vogue, has been based upon, and is of a similar character to, one or other of the Roman systems.

In the middle of the 5th century the Romans withdrew from their occupation of Britain, and were succeeded after a brief period by the Anglo-Saxons, who invaded the country and established a rule lasting for over 600 years. They had been accustomed to a tenure of land known as the Mark System—a system which had prevailed amongst the Teutonic races for a long period of time. Under this system the landowner held his tenancy not of any superior, but in token of his status as a freeman and head of a household; and on his death the headship of the homestead was conferred on a man chosen by an assembly of markmen. With the Anglo-Saxons the possession of landed property was the one qualification by means of which a Thane could exercise political functions, and the notion that all civil rights arose from the ownership of land was inherent in their minds. In fact, the distinction between real and personal

Introduction
of the Mark
System into
Britain.

⁽¹⁾ For fuller details as to tenures under the Roman Empire, reference may be made to the standard works of Hunter, Sheldon Amos, and Lord Mackenzie; as well as to Gibbon. *Decline and Fall*, vol. v, ch. xlv; and Seeböhm, *The English Village Community*, ch. viii.

property was most strongly marked among our Saxon forefathers ⁽¹⁾.

During the later period of their rule there existed the following species of land: Folk-land, which was held in immemorial usage for the benefit of the nation: Erf-land, which, being private property, passed to the heir on the death of the owner; Book-land, which was invariably the subject of grant either to the Thanes for services in war, or to the religious houses, which, at that time, had considerable influence in the State; Common-land, which consisted of meadows held by communities for the benefit of the members ⁽²⁾. A fifth form of tenure was that possessed by the burgesses in the large towns, which, in the time of Edward the Confessor, had grown considerably in size and importance; this tenure was governed by local customs, especially those relating to the descent of land and the devising of it by will, one such custom, namely, Borough English, being still extant in different parts of the country ⁽³⁾.

It is to the Saxon period of our history that we must turn for the origin of leasehold tenure in England. In the 10th century it seems to have been customary for the holders both of Folk-land and Book-land to lease their estates to free cultivators in such quantities and on such terms as the holders pleased. These leases were usually granted for the survivor of three lives, the lives being the tenant's own and those of two other persons nominated by him as his successors, and on the failure of these lives the land reverted to the lord. Leases thus granted conferred a freehold interest with its concomitant privileges, and in their creation were attended with great ceremony. The land, of which the lease was the subject, was known as *lœn-land* ⁽⁴⁾, from which is derived our word *loan*. Under the religious houses this was a favourite form of tenure, since a fine was exacted for the renewal of the lease upon the failure of each life, and these fines became a considerable source of revenue to the bishops and abbots, who seem to have appropriated them for their personal advantage ⁽⁵⁾. In the Domesday Book of St. Paul's a record exists showing that it was the practice of the

Origin of
Leases in
England.

⁽¹⁾ Hoskyns, *Essay on the Land Laws of England*; Kemble, *The Saxons in England*, vol. i, ch. ii; Brodrick, *English Land and English Landlords*, p. 3.

⁽²⁾ Pollock, *The Land Laws*, p. 20 *et seq.*; also Note B.

⁽³⁾ An interesting account of this custom is given in Elton, *Origins of English History*, ch. viii.

⁽⁴⁾ Kemble, *The Saxons in England*, vol. i, App. E.

⁽⁵⁾ Thrupp, *The Anglo-Saxon Home*, p. 215 *et seq.*

Dean and Chapter to let their farms for the term of one or more years, and to require the tenant to supply the provisions for the use of that body with or without money payments in addition ⁽¹⁾.

Having glanced at the various modes in which the land of this country was held in the early days of our history, we will now proceed to consider the effect of the Norman Conquest upon tenures of land.

Folk-land, which at that time consisted of the greater part of the country, was confiscated by the king for his personal use, being registered in Domesday as Terra Regis. Erf-land became merged into copyholds. Book-land changed its character entirely, and that portion of it which was not seized by the Norman conqueror, was retained almost exclusively by the religious houses. Common-land was allowed to remain untouched, and traces of it may still be found in the village greens and Lammas-lands of the present day ⁽²⁾.

These changes were brought about by the introduction and gradual development of the complicated system of real property law, known as the feudal system ⁽³⁾. The main feature of this system was the grant of land by the king to his lords, in return for which they were required to supply armed contingents of men for military service, together with fixed sums of money, whenever called upon to do so. Although, however, this system was an essential feature of land tenure under the Normans, yet it is now generally conceded that its leading characteristics were not unknown in former days. "Feudalism", says the late Mr. Freeman, "consists of two main elements: the feudal relation implies the union of two other relations. There is the personal relation of lord and vassal, bound together by mutual will and mutual fidelity, the one owing service, the other owing protection. . . . In our old constitution we find the elements of feudalism; but they were not as yet worked into a systematic shape; they had not as yet become the materials of an elaborate jurisprudence. Homage was there; for the relation of every man to his lord was a relation of homage. Military tenure was there; for much land

⁽¹⁾ Digby, *History of the Law of Real Property*, ch. i, sect. ii, § 3; Milman, *Annals of St. Paul's Cathedral*, ch. vii, p. 136.

⁽²⁾ Pollock, *The Land Laws*, p. 34; Digby, *History of the Law of Real Property*, ch. i, sect. ii, § i.

⁽³⁾ For an account of the causes which led to feudal tenures, see Palgrave, *Rise and Progress of the English Commonwealth*, vol. i, p. 495.

“ was held by military tenure. Heriots, too, and other feudal incidents existed. But these feudal elements had not yet been wrought together into any harmonious feudal system ” (1).

In the early days of this system, the estate granted to any lord was limited to his life, but it soon became customary on the death of the lord for the land to pass to his issue, who thus inherited what was called an estate tail, this term being derived from the French word *tailleur*, to cut, since by the Statute *De Donis* (13 Edw. I, c. 1) the inheritance was cut down and confined to the tenant and the heirs of his body. To evade the operation of this Statute with the object of enabling the lord to alienate his land, and thus defeat the expectation of the heir, various devices were attempted, and in course of time this object was attained by means of a fictitious action at law called a common recovery, by which the estate could be barred, and converted into a fee-simple, this being the largest estate in real property, and the tenant of which possessed full powers of alienation by deed, as well as, by means of “uses”, disposition by will (2).

Such in brief was the feudal system, the practical effect of which was to subject the great majority of the English race to a servitude lasting for two centuries. It entirely abrogated the freedom, characteristic of the Saxon rule, which recognized the authority of the Sovereign only when sustained by the advice and sanction of the Witan. Instead of the unfettered right of alienation, of disposal by will, and of transmission by inheritance, there was substituted a network of tyrannical and oppressive laws, the effects of which are felt down to the present day (3). “The Norman Conquest”, says a modern writer, “has not been exceeded in violence, cruelty, or in consequent desolation and misery by the most terrible catastrophes recorded in European history” (4).

In the large towns, however, we find that the merchants and burgesses seem to have been but little affected by the Conquest, since they were only required to render a

Its effect on
Tenures.

Contrast
between
Norman and
Saxon
systems.

Urban
Tenures.

(1) *History of the Norman Conquest*, vol. i, ch. iii, p. 97. Compare Stubbs *Constitutional History*, vol. i, p. 261; Taswell-Langmead, *English Constitutional History*, ch. ii, p. 53; Round, Art. on “Knight Service” in the *English Historical Review* for July and October 1891.

(2) Williams, *Law of Real Property*, part i, ch. iii; Pollock, *The Land Laws*, ch. iii.

(3) Hoskyns, *Essay on the Land Laws of England*.

(4) Thrupp, *The Anglo-Saxon Home*, p. 18.

light and easy tribute and to perform moderate or nominal services, in return for which they were protected against compulsory taxation, and were accorded the privilege of self-government.

Two other forms of tenure deserve a passing notice, namely, base tenure and religious tenure. The former included copyholds as well as small plots of land given to the labouring classes in exchange for their services. Copyholds originated in the custom of sub-dividing the estates which had been granted by the king. The lord on receiving his grant of land parcelled out amongst his Norman followers estates of inheritance, reserving the right to demand their services in time of war. The remaining portion he retained for his own use, compelling the "villeins" whom he found attached to the land, to perform the manual labour necessary for its cultivation. In order, however, to secure their adherence, he permitted them to occupy portions of the land upon the condition of the performance of certain services, retaining at the same time the absolute ownership of themselves and their goods. The estates thus created were said to be held at the will of the lord, and subject to the custom of the manor of which they formed part, the tenant's title being recorded in the Court rolls ⁽¹⁾.

Copyholds were originally divided into five classes, of which the two highest were the tenures of ancient demesne and of customary freehold: these tenures were subject to nominal quit-rents and to fines, and from one to two years' rent on death or alienation. The third class was composed of copyholds of inheritance, which descended from father to son, and in addition to the payment of quit-rents and fines, were characterized by the custom of seizing the tenant's best beast, or sometimes the best chattel of any kind as a heriot on the death of the tenant, the custom being of Teutonic origin. Next came copyholds for lives, usually held till the death of the survivor of three lives. Finally there existed copyholds for years, these being generally renewable every seventh year in a lease for 21 years, but no right of renewal was possessed by the tenant ⁽²⁾.

Religious tenure was of two kinds—the tenure of Frank-almoign, and the tenure by Divine Service. The former was held

⁽¹⁾ Williams, *Law of Real Property*, part iii; Pollock, *The Land Laws*, Note C.

⁽²⁾ Scratchley, *Treatise on Copyholds, &c.*, sect. ii; Kemble, *The Saxons in England*, vol. i, p. 178.

subject to the prayers of the Church, and the latter to Masses being regularly said on behalf of the lord or his predecessors. Frankalmoign still survives in the tenures of ecclesiastical corporations, although its incidents have long since fallen into desuetude, and it is the nearest practical approach to absolute ownership on the part of the tenant ⁽¹⁾.

It is under this last class of tenure that we must look for the development of the leasehold system in its earliest stages. For a long period the only mode of granting leases was for the duration of one or more lives, and no record of leases granted for terms of years exists till the middle of the 15th century, when a reference is made to such a method in the Statute of Marlborough, which was enacted towards the close of the reign of Henry III. This Statute contains the following clause: "Also fermors during their terms shall not make waste, sale, nor exile of house, woods, and men, nor of anything belonging to the tenements that they have to ferm without special licence had by writing of covenant making mention that they may do it."

It does not appear, however, that there was any large number of leaseholds in existence at that time. In the majority of cases it is probable that although the land was cultivated by the tenant, the stock employed for the purpose of tilling it, as well as a fixed proportion of the produce, remained the property of the lord; and in the few instances in which the tenant held a lease, his condition differed widely from that of a lessee of modern times, the transaction being a matter of necessity on the part of the tenant, who was forced to farm the land upon such terms as the lord chose to fix. Leases for terms of years were always regarded as inferior to those for lives, the idea being that no man was free whose tenure could be disturbed during his lifetime. By the Statute of Gloucester (6 Edw. I.), all claims of the tenant for additions made to his property became forfeited upon the expiration of his lease, and he was even forbidden to erect any building on the land where there was none before, or to convert one kind of edifice into another, even if it improved the value of the estate ⁽²⁾.

During the Middle Ages leases were frequently granted on very peculiar tenures, such as a particular service to be rendered

(1) Ellis, *General Introduction to Domesday*, vol. i. p. 258; Williams, *Law Real Property*, part i, ch. v.

(2) Hoskyns, *Essay on the Land Laws of England*.

to the king, the baiting of a mad bull on a certain day, or the delivery of provisions for the use of the royal table. Rents payable in kind were not uncommon, and we find instances of a white bull, two white hares, the service of a farrier or shoeing smith, a peppercorn, and a pound or half a pound of cummin, as the mode of payment stipulated for in the leases of those days ⁽¹⁾. It was not till the middle of the 14th century that money payments were substituted for labour dues or personal service, and a fixed rent became payable by the tenant for the use of the land. This change was owing in a great measure to the scarcity of labour at that period on account of the depopulation of the country by the epidemic known as the Black Death, which made it unprofitable to keep farms on hand; and leases, which had hitherto been chiefly granted by monastic houses as a profitable means of cultivating their lands, began to be universally adopted. The lessees, however, were considered as merely the bailiffs or servants of the lord, who were to receive and account for the profits: they were not allowed to have a freehold estate, but their interest vested after their deaths in their representatives ⁽²⁾.

The gradual introduction of long agricultural leases in the latter part of the Middle Ages was principally due to the increase of personal extravagance among a great proportion of the upper classes, who, in order to gratify their luxurious tastes and habits, were obliged to exact the utmost rent out of their lands. On the other hand, the inability of individual landowners under the old system of strict entail to grant leases which should effectually bind their successors, drove the more enterprising farmers to seek tenancies under the monastic houses and other corporate bodies, and it was mainly under the Improvement Leases granted by these corporations that the earliest experiments in drainage were undertaken ⁽³⁾.

Throughout the 15th century the increasing wealth of the nation brought about many changes in the character of land tenure. Villanage had practically ceased, and in its place had sprung up a superior class of petty freeholders, who became the occupiers of farms ⁽⁴⁾.

Early forms of
Rent.

Monastic
Leases.

Changes
during the
15th Century.

⁽¹⁾ A description of curious forms of tenure in different parts of the country given in Blount's *Tenures of Land*.

⁽²⁾ Rogers, *History of Agriculture and Prices*, vol. i, p. 265.

⁽³⁾ Pollock, *The Land Laws*, p. 71.

⁽⁴⁾ Froude, *History of England*, vol. i, p. 13.

Following upon the state of anarchy which had prevailed during the Wars of the Roses, the peaceful reigns and firm government of the early Tudors greatly enhanced the security of this class of property, and a general rise in prices, though not apparently in rents—which were practically fixed by custom—naturally took place ⁽¹⁾.

The reign of Henry VIII marks an important epoch in the development of leasehold tenure. In 1535 was enacted the famous Statute of Uses ⁽²⁾, by which land could be conveyed to a purchaser to his use whilst he paid the rent, and in default of such payment, then to the use of the vendor, who was thus able to reclaim his land until the rent was satisfied and the covenants duly performed, after which the purchaser was again entitled to the land. One result of this Statute was the creation of the system of freehold Chief Rents ⁽³⁾, a form of tenure which was held under a perpetual rent charge, and which, at the present day, prevails to a great extent in Lancashire. It also contained provisions which had the effect of putting an end to the devising of landed property. These provisions, however, being found burdensome, an Act was passed in 1540 which empowered all persons having an estate of inheritance, either in fee-simple or in fee-tail, and whether in their own right or in the right of their wives, to devise their land by will, as well as to lease as against reversioners for a term of 21 years, or for the survivor of three lives ⁽⁴⁾. The Statute of Enrolments (27 Henry VIII, c. 16) enacted that every bargain and sale should be by deed, indented and enrolled within six months: a provision which, however, was successfully evaded by making such bargain or sale for a term of years, and at the same time granting a release. This mode of conveyance was called a lease and release, and was partly a statutory and partly a common law conveyance, its effect being to vest the legal estate in the purchaser ⁽⁵⁾.

But the great event of the reign was the suppression and dissolution of the monasteries and abbeys, the first act taking

⁽¹⁾ Green, *Short History of the English People*, vol. ii, p. 19; Rogers, *History of Agriculture and Prices*, vol. iv, p. 3.

⁽²⁾ According to Sir F. Pollock and Mr. Wren Hoskyns, this statute, which was passed to restore the ancient simplicity and notoriety of titles to land, became the chief instrument of secrecy and complication.

⁽³⁾ Evidence of Mr. C. Harrison before the Town Holdings Committee (1887), 3,226.

⁽⁴⁾ Scratchley, *Treatise on Copyholds, &c.*, sect. iii, art. 23; Grey, *Church Leases*, p. 10.

⁽⁵⁾ Digby, *History of the Law of Real Property*, ch. vii, § 3; Shearwood, *Law of Real Property*, part ii, ch. ii.

place in 1536, and the second 10 years later ⁽¹⁾. These bodies had, in the course of many centuries, acquired land to the extent of fully one-third of the kingdom, and possessed revenues yielding over £100,000 a year, which, in addition to a large amount of personal property, were diverted to the Crown. Large grants of land from this source were made to the king's favourites, some of whom became founders of great families, such as the Russells, the Manners, and the Cavendishes, whose extensive estates in different parts of the country, were, for the most part, originally monastic property. Prior to the change in ownership, estates for terms of years had been placed in the same rank with those of freeholds, both in regard to security and to stability of enjoyment; under the new régime, however, the privileges thus conferred, as well as the rights enjoyed by the tenants under the express terms of their leases, were deliberately ignored, rents being raised to an exorbitant pitch, and payment exacted under pain of eviction.

Effect of the
Dissolution
of the
Monasteries
on Land
Tenures.

The 16th century was remarkable for the enormous increase in agricultural rents, which took place during its latter half, it being no uncommon occurrence for farms, which at the beginning of the century were let at £4 a year, to yield at its close a rental of £40, £50, and even £100 ⁽²⁾. In the following lines the agriculturist poet Tusser complains of the increased burdens on the land:

Rise of Rents
in the 16th
Century.

“Great fines so neere did pare me,
Great rents so much did skare me.
Though country health long staid me,
Yet lease expiring fraid me” ⁽³⁾.

In the middle of the next century there was developed a new tenure, which has continued down to the present time. This was the system of strict settlements, which has been the instrument by which land has accumulated into few hands ⁽⁴⁾. It arose under the following circumstances. After the defeat of the Royalist party by the Parliamentary forces, the landowners who had supported the king found themselves in great danger of ruin, and, in order to avoid the risk of forfeiture of their estates, they engaged

Origin of
Strict
Settlements.

⁽¹⁾ Full details are given in Gasquet, *Henry VIII and the English Monasteries*. See also Hook's *Church Dictionary*, Art. "Monasteries."

⁽²⁾ Froude, *History of England*, ch. i, p. 26.

⁽³⁾ Quoted in Cullum's *History and Antiquities of Hawsted*, p. 246.

⁽⁴⁾ Brodrick, *Essay on Primogeniture*; Rogers, Art. "Land Tenure" in *Dictionary of English History*.

two skilful lawyers—Sir Orlando Bridgman and Sir Geoffry Palmer—to draw up deeds of settlement, tying up their property for long periods of time. Upon the Restoration, when these two lawyers became officers of the Crown, an Act was passed confirming the validity of these settlements.

The final abolition of tenancies in chivalry, with their attendant incidents of knight service, which, since the Wars of the Roses, had been little more than empty forms, was effected by the Long Parliament in 1646. This Act was confirmed in 1660 by the Statute of Tenures, the Crown being compensated for the loss of its hereditary revenues from the feudal incidents by the grant of £1,000,000 per annum, which was raised by means of an Excise Tax ⁽¹⁾.

It was about this period that the character of leasehold tenure began to undergo an important change. The long period of commercial prosperity, which had marked the reigns of Elizabeth and her immediate successor, resulted in a great number of people from the country districts coming to settle in the large towns, and especially in London, to provide for whom houses were erected and new streets formed. A strong prejudice,

however, existed in the minds of the statesmen of those days against the extension of the Capital, and building beyond certain limits was prohibited under severe penalties: but this prohibition proving futile in its operation, it gradually fell into abeyance, and the building continued with renewed vigour ⁽²⁾. “As the population of the “Metropolis increased”, says Mr. Charles Harrison, “it was “impossible to obtain freehold sites for building. There was no “competition amongst landowners. The Church and other “corporate bodies could not sell the freehold; and hence they “and the landowners adopted the system then in use in “mortgage transactions. They offered to the lessee a short term “of years, such as they would offer to a mortgagee lending them “money on land as security for a temporary loan, taking care, “however, to preserve the freehold in themselves, and as they “were a body of monopolists, dictating such terms as they “thought fit as the price of allowing any house to be erected” ⁽³⁾.

(1) Digby, *History of the Law of Real Property*, ch. ix. One effect of this Act was to render all freehold lands capable of being devised by will.

(2) Evidence of Mr. J. R. Bourne before Town Holdings Committee (1887). 11,312.

(3) Paper on “Leasehold Enfranchisement”, read before the British Association in 1888, p. 21.

The earliest date of which any record exists as to the grant of such leases is the year 1620, when houses were erected on the grass land belonging to the Bedford estate, special permission being obtained from the king. The term of these leases was limited to 31 years, a period composed of 21 years and a bonus of 10 years ⁽¹⁾.

In 1667 an Act was passed providing for the immediate re-building of London after the Great Fire. It was stipulated that the new houses were to be built either of stone or brick, "reasonable satisfaction" being given for the ground. "And
Earliest known Illustration. "forasmuch as the houses now remaining and to be
Re-building of London after the Great Fire. "re-built will receive more or less advantage in the
 "value of the rents by the liberation of the air and
 "free recourse for trade, and other conveniences, by such regulation or enlargement"—to quote the words of the Act—it was provided that in case of dispute as to the assessment, a jury should be empanelled to assess the amount to be paid, by the owners or other interested persons. The money thus raised by the Corporation was to be applied "wholly towards the payment
 "and satisfaction of such houses and grounds as shall be converted
 "into streets, passages, markets and other public places." Any disputes as to the title of the property were to be settled by the Courts of King's Bench, Common Pleas, or the Exchequer. It was further provided that a tax upon coal brought within the boundaries of the City should be levied.

The work of re-construction was entrusted to Sir Christopher Wren and Mr. R. Hooke, the former undertaking the task of re-building St. Paul's Cathedral and many of the City churches, and the latter supervising the laying out of new streets and the erection of the houses. At this time the whole of London north of the Thames consisted of only 668 acres, and the southern side of 611 acres, making altogether a total of barely two square miles ⁽²⁾.

Evelyn, writing in 1684 ⁽³⁾, speaks of London as having nearly doubled within his own recollection, his statement being confirmed a few years later by Petty ⁽⁴⁾, who asserted that the value of houses in London was twice as great then as it was 40 years earlier. It seems, nevertheless, to have been a matter

⁽¹⁾ See Mr. Bourne's evidence (1887), 11,313.

⁽²⁾ Wheatley, Art. "London" in *Encyclopædia Britannica*.

⁽³⁾ *Diary*, 12 June 1684.

⁽⁴⁾ *Essays on Mankind*:—"Of the Growth of the City of London."

of frequent complaint at this time that London houses were built of bad bricks, and in appearance were more like edifices of dirt and mud than of brick. Muralt ⁽¹⁾ also tells us that in his day houses seldom lasted more than 40 or 50 years, and sometimes dropped before the end of that term. According to Davenant ⁽²⁾ the general rental of the country during the course of the 17th century rose from £6,000,000 at its commencement to £14,000,000 at its close, the purchasing value of land having likewise increased during the same period from £72,000,000 to £252,000,000.

As illustrating the value of house property in the reign of Queen Anne, the following extract from Mr. J. Ashton's well-known work ⁽³⁾ is of interest. "Houses were not always let by agreement, but the leases were sold; and it is by means of such advertisements that we are able to get at the rents, which seem to have been very low, even reckoning the difference of value in money. Certainly they had none of our modern appliances and conveniences, which add so considerably to the cost of the buildings, nor do they seem to have been saddled with exorbitant ground rents. 'To be sold, a lease of 33 years to come in five houses standing together on the north side of Pall Mall, whereon £25 per ann. rent is reserved. The houses are let at £200 a year.' 'A gentleman has occasion for a lightsome, fashionable house in some genteel part of the town, or very nigh the town, and if accomodated with coachhouse and stabling it will be better lik'd: of about £30, £40, or £50 a year rent.'"

The earliest known Act of Parliament authorizing the grant of building leases in the Metropolis was passed in 1702, and related to the parish of St. Martin's-in-the-Fields, the term being limited to 61 years. It may be mentioned that in the same year the Civil List Act provided that no grant of manors, leases or tenancies belonging to the Crown, should be made for a longer period than 31 years or three lives. Between this date and the year 1750, there are records of 21 private Acts having been passed in connection with settlements, the leasing powers in which were inadequate, and required to be

⁽¹⁾ *Letters on the English*, p. 76.

⁽²⁾ *Essay upon Ways and Means*.

⁽³⁾ *Social Life in the Reign of Queen Anne*, vol. i, p. 61. See also Report of Town Holdings Committee (1890), App. 16.

extended ⁽¹⁾. It was not, however, till the year 1783 that the term of 99 years was authorized by Statute, this being the term for which powers of leasing were authorized in the particular settlement which was the subject of legislation. From this time to the present, the term of 99 years has become the customary one to insert in building leases in the Metropolis, and it has also found favour in many provincial towns.

Many explanations have been given as to the origin of this term, the most plausible being the following :

In former days lessees and mortgagees in possession for terms of 100 or 1,000 years, frequently demised the whole or a great part of the property at a certain rent, retaining the reversion of the last year of the original term. This stipulation was made by under-lessees in order to prevent their becoming bound to the performance of the tenancy covenants contained in the original lease, and it was formerly necessary to the recovery of the reserved rent by distress that the reversionary interest should remain in

the person to whom the rent was payable. Reversions of three days or even one day were thus usually reserved on the grant of derivative terms, but out of terms of 100 or 1,000 years the last year was retained. The practice gradually grew into a recognized custom, although the reason for its continuance had long since disappeared ⁽²⁾. The idea has in some few instances been carried to a ridiculous extent in the grant of leases for 9,999 years.

“The London 99 years’ system”, says Mr. Harrison ⁽³⁾, “is the outcome of an application to houses of the conveyancing system of short terms of years formerly used for mortgage transactions, and for securing jointures to widows and such like purposes, where the object was to confer as little interest as

possible on the person in whose favour the lease or term of years was created. It owes its origin, in a great measure, to the legal restraints on free alienation by sale or long lease attached to the bulk of the land surrounding the old City of London—into which the present Metropolis was forced to expand—and which belonged then, as now, to the Church. It is also to some extent attributable to the circumstances that those restraints against leasing or selling which were imposed by Parliament

Origin of
Terms of 99
and 999 Years
in Leases.

Theory of
Mr. Charles
Harrison as
to Building
Leases.

⁽¹⁾ A List of these Acts is given in the Report of the Town Holdings Committee (1887), App. 5.

⁽²⁾ *Notes and Queries*, 1st Series, vol. x, p. 294.

⁽³⁾ Paper on “Leasehold Enfranchisement”, p. 29.

“owing to the improvidence throughout the country generally
 “of the bishops, prebends, and life-occupant clergy, and for
 “their protection, produced exceptional evils in London, and
 “prevented leases being granted for a longer term than 40 years.
 “When the houses were worn out, they could not be pulled
 “down and rebuilt, or improved, on the ground that so doing
 “would be ‘waste’ as against the person next entitled. For
 “above 200 years these statutable restraints remained irremovable
 “except by special Acts of Parliament, which were granted only
 “to exceptionally favoured persons, and as each bishop’s or
 “prebend’s properties were required by the ever-increasing popu-
 “lation of the Metropolis, as sites on which it was to be housed.”

Although the greater part of London has been built on the 99 years’ system, in the City the term is now usually 80 years upon either a renewal being granted or a rebuilding authorized. During the 50 years succeeding 1839 there have been built 500,852 new houses, and 1,833 miles of new streets and squares have been formed. This gives an average of 10,017 new houses and about 36½ miles of new streets, &c., per annum. In a recent address by a former President of the Surveyors’ Institution ⁽¹⁾, it is estimated that the average number of houses built in London since 1856 has been about 17,000 per annum, the largest number in any one year being 29,271 in 1881, since which date building has declined considerably.

Outside the boundaries of the City, and excepting the eastern and southern districts, which are principally owned by small freeholders, the greater part of London is owned by a small number of ground landlords, who have adopted the 99 years’ system as a means of developing their property. The largest estates are those belonging to the Crown, the Dukes of Westminster, Bedford, Norfolk and Portland, the Marquises of Camden and Northampton, Earl Cadogan, Viscount Portman, Lord Southampton, Lord Amherst of Hackney and the Ecclesiastical Commissioners ⁽²⁾.

Passing from the Metropolis to the provinces, and commencing in the North of England, we will proceed to examine the different systems in use in the principal towns ⁽³⁾.

(1) *Transactions of the Surveyors’ Institution*, vol. xviii, p. 6.

(2) On this last estate the tenants having unexpired terms of 40 years have been recently granted the option of converting their holdings into terms of 999 years.

(3) See App. 2 of the Report (1887) of the Town Holdings Committee, where a detailed account of tenures of 261 English Boroughs is furnished.

Newcastle, Gateshead, Berwick, South Shields, Durham, Hartlepool, Darlington and Carlisle are built almost entirely on the freehold system, the landowners selling the sites to the builders for a sum down. In Jarrow, however, the 99 years' system prevails to some extent. Both in North Shields and Sunderland land is let for building subject to a chief rent, which is fixed at about 4 per-cent on the estimated capital value of the land, and redeemable at 25 years' purchase. The low price of land in these parts greatly facilitates the purchase of the land for building.

Freeholds also appear to be the rule in the greater part of Yorkshire, the only important towns under a different system being Sheffield and Huddersfield. The former has been developed entirely on the leasehold system, the term being 99 years on the Arundel Estate and from 500 to 900 years on other estates. In Huddersfield, nearly the whole of which belongs to Sir John Ramsden, leases are now granted for 999 years. This term has, however, only been in operation since 1867, prior to which the terms had been 60 years, renewable on payment of a fine, and subsequently 99 years. This town is a fair illustration of the advantages of the short leasehold system under a good landlord.

In the adjoining county of Lancashire various systems are to be found. Manchester has been built almost entirely either under leases for 999 years or on the chief rent system. Liverpool, on the other hand, is subject to the short leasehold system, the terms varying from 75 to 99 years. A large portion of this city belongs to the Corporation, who have no power to lease for more than 99 years; but their leases are invariably renewed on expiration, fines being exacted instead of increased rents.

Both in the Eastern and Midland Counties the tenure of urban land is, with few exceptions, freehold. In Great Grimsby, where leasehold tenure prevails, the option of enfranchisement is contained in the leases, but is rarely taken advantage of. The large towns of Birmingham and Nottingham have been built almost entirely on the leasehold system, the Corporations of these boroughs owning a considerable area, and granting leases for 75 years in Birmingham, and for 99 years in Nottingham. In Oxford freeholds are rare, the general rule being 99 years' leases, both by the Corporation and the colleges. Some leases, however, exist on the perpetual

Tenure in the
North of
England.

In Yorkshire.

In Lancashire.

In the Eastern
and Midland
Counties.

renewal system, the term being 10 years, subject to a fine of from $2\frac{1}{2}$ to 3 years' rent at the end of every 11 years.

South of the Thames we find a great variety of tenures. In the small towns freehold tenure prevails, but at such rising places as Croydon, Tunbridge Wells, Folkestone, Hastings, Brighton, Eastbourne and Bournemouth, the 99 years' system is almost universal. In the two last-named towns the option of enfranchisement is frequently granted in the leases. In the South of England. At Brighton, since the year 1820, all leases have invariably contained this option, which must be, and generally is, exercised within 7 or 10 years, on payment of 25 years' purchase of the ground rent. The covenants, which would ordinarily have been contained in the lease, are inserted in the conveyance. In the Isle of Wight three distinct systems have been successively tried—99 years certain, 99 years determinable on the death of the survivor of three lives, and leases for 999 or 1,000 years. This last tenure, which is the present one, also prevails in Southampton and Winchester.

In the West of England various kinds of tenure are in vogue. Freeholds predominate in the large towns of Devonshire and Cornwall, though short leases are the rule in Tavistock and Truro. In some places the tenure is that of 99 years determinable on the death of the survivor of three lives, but such leases have become very unpopular owing to their uncertainty and the difficulty of meeting the necessary fines, and they are gradually dying out, being replaced by terms of 99 years certain. In the West. In Bath and Bristol the customary tenure is that of freehold chief rents, but in the latter city the Corporation grant leases for 75 years, and the Bristol Industrial Dwellings Company, Limited, let their land for terms of 10,000 years. Further north freeholds may be said to be universal, though on one estate in Malvern leases of lives are the custom.

Passing on to the Principality of Wales, we find that the modern system of tenure is of three kinds—freehold, leasehold for years, and leasehold for lives—the large towns of Cardiff, Swansea, Merthyr Tydfil and Llandudno being all In Wales. under the short leasehold system, the usual term being 99 years. In the quarry districts leases for lives were formerly common, the term being 21 or 30 years determinable on the failure of one or more lives; but recently terms of 60 and 99 years have been substituted.

Having now briefly passed in review all the various systems of tenure which exist in England and Wales, we will
 Summary. proceed to summarize them and to point out the features peculiar to each system.

1. *Freehold (absolute purchase)*.—This system is only prevalent in those parts of the country where either land is cheap or the tenure is regulated by local custom. It has the advantage of placing the tenant in the position of absolute
 Freehold (absolute purchase). owner of his property, and of being under no restrictions as to the use which he may choose to make of it.

It is, however, seldom the case that purchasers possess sufficient funds without having recourse to a mortgage on the property; and it thus happens that the interest which they have to pay, together with the expenses incident to such a proceeding, frequently become a heavy burden. If the tenant erects a building on the land, he has to borrow again at a still higher rate of interest; and in the case of a purchaser who is able to pay for the land out of his capital, the disadvantage exists that under this system his money is locked up without any immediate prospect of return.

2. *Freehold (Chief Rent)*.—Under this system the purchaser of the fee-simple, instead of sinking his capital in the purchase money, agrees to pay the vendor a perpetual annuity in the form of a rent charge secured on the property. One of the stipulations of the sale is that the purchaser shall build houses upon the land of the aggregate annual value of at least double the amount of the rent charge. In default of payment of the annuity, the owner of the chief rent can collect the rents from the
 Freehold (Chief Rent). purchaser's tenants and thus recoup himself; and if the chief rent remain in arrear for the time named in the conveyance (generally two years), he can take back the land he sold, together with all buildings thereon, and keep them as his own property. The chief rent is a rent which is received from the land, and the owner of the chief rent is a distinct person from the owner of the property, which can only be mortgaged subject to the chief rent. This system gives a distinct advantage to the builder, as by it he obtains the freehold without having to find a large sum to pay down for it, and he has thus more capital to devote to his business. It is, however, liable to become extremely complicated through the creation by the purchaser of sub-chief rents, which together exceed the chief rent payable to

the landlord. In such a case every house is liable to the payment of the whole of this excess to the original landlord, as well as of the second chief rent to the purchaser; and the existence of these complications often involves trouble and expense in determining the title to houses built on this system. Another objection to this system is, that as money fluctuates in value in the course of long periods of time, so it may happen that the amount of the chief rent fixed at one period exceeds or approaches closely to the amount of the rack rents, which have been fixed at another period, thus throwing a considerable loss on the purchaser of the freehold ⁽¹⁾.

3. *Long Leasehold*.—The terms for which long leases are usually granted under this system range from 200 to 1,000 years, the most common term being 999 years. In practice there is very little difference between this tenure and the preceding one, except that the former, being personal property ⁽²⁾, devolves in case of intestacy on the representatives of the tenant instead of upon the heir, and this circumstance gives greater facilities for determining the title to the property. A reversion attaches to this tenure at the termination of the lease, but it is practically of no value owing to the great length of time which must elapse before it can be realized.

4. *Short Leasehold*.—This system is used exclusively in the creation of building leases, the term generally varying from 40 to 99 years. The nature of the contract entered into on the grant of a building lease is, that the tenant shall, within a specified time, erect a certain number of houses, the rack rents of which shall not be less than the amount agreed upon, and that, during the term of the lease, he shall pay a fixed ground rent to the owner, as well as all rates and taxes which may become chargeable in respect of the property. Covenants are inserted in the lease which bind the tenant to keep the premises in repair, to pay the insurance premiums as they fall due, to observe certain conditions as to tenancy or the carrying on of particular trades, and on the expiration of the term, to give up the land with all buildings upon it, free from dilapidations, to the lessor or his successors. On some estates the option of enfranchisement may be exercised during the first few years of

⁽¹⁾ 1,000 *Answers to 1,000 Questions*, vol. i, p. 156: *Town Holdings* (Cassell), vol. i, p. 22.

⁽²⁾ For reason of Leaseholds being Personal Property see Digby, *History of the Law of Real Property*, ch. v, § 1.

the lease, the price being regulated by the ground rent ⁽¹⁾. Under this system the houses or other buildings are erected under the supervision of the landowner's surveyor, and the lease is not granted until he is satisfied that the terms of the agreement have been carried out. It is usual, in practice, to apportion comparatively high ground rents to the houses first built in order to secure the total rent as early as possible, the subsequent houses being subject to a ground rent of decreasing amount, which, in the case of the last house, is merely nominal; and it thus frequently happens that two houses of equal value are subject to ground rents widely differing in amount ⁽²⁾.

5. *Leases for Lives*.—The usual term of leases of this class is that of the survivor of three lives who are named by the tenant and inserted in the lease with the option of renewal on the failure of each life by payment of a fine. The tenure is freehold, and hence greater than that of a leasehold even for 1,000 years. It is subject to a low rent called a conventional rent, but to this payment should be added the premium for the insurance of the lives in order to provide the fines as they become due. The lives chosen are generally young and healthy, being frequently members of the Royal Family, who are easily traced through life. The system has been condemned as being productive of bad building and dilapidated houses ⁽³⁾, and is gradually becoming obsolete, renewals being now seldom granted.

6. *Beneficial Leases*.—These were formerly granted by Ecclesiastical Corporations, both in conjunction with lives and terms of years, and also for terms only. The tenure was for terms of 21, 30 or 40 years, or for three lives, renewable every 7, 10 or 14 years, or on the failure of each life.

They originated in the days when rents were fixed, and payable partly in money and partly in wheat and malt. When changes took place in the relative value of money and corn to the disadvantage of the corporations, they adopted the method of levying fines on renewal in order to recoup themselves for the loss, and the custom has prevailed ever since ⁽⁴⁾. In the mining property formerly belonging to the See of Durham, the terms of

⁽¹⁾ This option is common in Eastbourne, Brighton, Worthing, Christchurch, Walsall, Westward Ho! and Carlisle. A draft of a lease granted on the Devonshire Estate at Eastbourne is given by Woodfall, *Law of Landlord and Tenant*, p. 948.

⁽²⁾ Underhill, *Leasehold Enfranchisement*, p. 39.

⁽³⁾ See Mr. Nott's evidence before Town Holdings Committee (1888), 14,149-51.

⁽⁴⁾ Rogers, *History of Agriculture and Prices*, vol. i, p. 25, and vol. v, p. 805.

the leases were generally 21 years or three lives, renewable by payment, either of a fine, or of an annual rent contingent to some extent upon the quantity of coal worked during the year from the mines forming the subject of the lease. Ecclesiastical leases contained few, if any, covenants or restrictions, and seldom any reservations but those of timber, mines and quarries. Upon the appointment of the Ecclesiastical Commissioners in 1835 this body at once refused to renew these leases, which were allowed to run out their terms. Subsequently, leaseholders were empowered to acquire the freehold of their holdings on most advantageous terms.

7. *Occupation Leases.*—These are for the most part granted for terms of 3, 7, or 21 years. Houses let for a term of three years are, as a rule, held under an agreement for a lease, this term being exempted from the operation of the Statute of Frauds, which required all leases to be granted by deed. Shops and mansions are usually let for terms of 21 years, a clause being generally inserted giving the tenant the option of terminating his lease at the end of 7 or 14 years. Comparatively few occupation leases are held direct from the freeholder or even the original leaseholder, it being not unfrequently the case that three or four intermediaries stand between the owner of the fee-simple and the occupier. It sometimes happens that at the expiration of a building lease the freeholder, on coming into possession of the house, offers the occupying tenant a repairing lease for a term varying from 14 to 40 years, on condition that the tenant will undertake to make a stipulated outlay upon repairs and improvements, a reduction of rent being granted in consideration of the due performance of the contract ⁽¹⁾.

8. *Agricultural Leases.*—These are altogether different in character from any of the preceding, and have been made the subject of special legislation. They are never granted for a longer term than 21 years, and are usually held from year to year.

With few exceptions, it is customary to renew them to a good tenant without raising the rent, and in Lincolnshire, as well as in a few other districts, a custom of tenant right prevails by which the outgoing tenant can by virtue of established usage claim from the landlord or his successor, repayment in certain definite proportions of the cost of improvements of a

(1) Mr. Bourne's evidence (1887) 11,223-39.

specified kind, which he may have made during the last years of his tenancy, and the benefits of which his removal prevents him from enjoying. A similar right has recently been conferred on tenants throughout the country, under the provisions of the Agricultural Holdings Act, 1883 ⁽¹⁾.

9. *Copyholds*.—This species of property still retains most of its original features, which have been already briefly described. Modern copyholds may be considered as belonging to one of two classes—(1) Those in which the tenure is at the will of the lord, and of a burdensome character by reason of the extraordinary claims in the form of heriots and fines which are incident to the tenure; and (2) Those in which the tenure is on the contrary light, the quit-rents, fines, suits and services being all nominal and of a fixed amount. In this latter class the tenants **Copyholds.** can without license build, and obtain building materials for the purpose within their respective tenements; and further, they are able to demise for any number of years, and in some parts of the country may sell minerals to the exclusion of the lord. As an illustration of the small amount of quit-rent payable under this tenure, it may be mentioned that in one instance the copyholder receives from his sub-tenants a rental of about £6,800 per annum, while the whole of the quit-rent that he pays to the lord amounts to not more than £4. 14s. 5½d. ⁽²⁾ In November 1891, the Council of the Central and Associated Chambers of Agriculture unanimously resolved that copyhold tenure, having ceased to serve any useful purpose, was injurious to industry, and should be extinguished upon equitable terms.

During the present century, and particularly within the last 50 years, legislation has effected much towards facilitating the transfer of land, and has also enlarged the powers of ecclesiastical and other corporate bodies, as well as of the holders of land under settlement; and in the various measures that have been passed within recent years many useful and necessary reforms have been introduced. The subject is, however, an extremely complex and difficult one, and, hence, progress on sound and equitable principles must necessarily be gradual and cautious.

Among the legislative reforms of the present century may be

⁽¹⁾ See Rogers, Art. on "The History of Rent" in *Contemporary Review*, April 1880, in which he traces the progress of these tenancies from the reign of James I.

⁽²⁾ Woodcock, *Note on the Copyhold of the Manor of Clitheroe* in Report of Town Holdings Committee (1887), App. 2.

mentioned the Municipal Corporation Acts, 1835 and 1882, the Ecclesiastical Leases Acts, 1812 and 1858, the Lands Clauses Consolidation Act, 1845, the Settled Estates Acts, 1856 and 1877, Lord St. Leonards' Act, 1859, the Conveyancing Act, 1881, the Settled Land Act, 1882, the Agricultural Holdings Act, 1883, and the Housing of the Working Classes Act, 1890. Under these Acts the powers of corporations and tenants under settled estates, for the purpose of granting leases, have been considerably extended; relief from forfeiture of a lease, owing to some breach of a covenant by the lessee, has been provided for; tenants under agricultural leases have been accorded compensation for unexhausted improvements; and local authorities have acquired increased powers to purchase sites and to grant leases for the erection of artisans' dwellings and model lodging houses. The various Acts relating to copyholds, and providing for their compulsory enfranchisement at the instance of either the lord or the tenant, have greatly facilitated the extinction of this obsolete form of tenure.

In addition to these and many other Acts, the subject of leasehold tenure has engaged the attention of several Royal Commissions. Royal Commissions, such as those on Church Leases in 1839, on Episcopal Capitular Revenues in 1849, on Agriculture in 1880, and on the Housing of the Working Classes in 1884. It was also the main subject of enquiry by the Select Committee of the House of Commons on Town Holdings, appointed in 1886, whose report was published in 1889.

II.—THE ENFRANCHISEMENT OF LEASEHOLDS.

During the last ten years an agitation of a political nature has arisen, having for its main object the acquisition by urban leaseholders possessing a substantial interest in their holdings, of statutory powers for the purchase of the fee-simple upon terms which shall be fair and equitable to both parties. With this object in view various schemes have been put forward, and the subject has been repeatedly discussed both in the House of Commons and at public meetings held in different parts of the country.

In 1883 a society called the Leaseholds Enfranchisement Association was formed for the purpose of obtaining for leaseholders: (1) Compulsory power either of purchasing the freehold of their property by payment of a lump sum down, or of

converting the ground rent into a perpetual rent charge; and (2) Compensation for permanent improvements effected by tenants during their occupancy ⁽¹⁾. This society, which has been presided over successively by Mr. Henry Broadhurst and Mr. H. L. W. Lawson, M.P., has published and circulated many pamphlets pointing out the defects which, it is asserted, are inherent to the short leasehold system, and alleging, on the strength of evidence obtained on behalf of the society, that this system is radically and irretrievably unsound, and that the only means by which any substantial benefit can accrue to the inhabitants of urban districts, is the universal adoption by Act of Parliament of the freehold purchase system.

To effect such a change in the law as is desired by the society, three Bills have been promoted by the council and introduced into the House of Commons. The first ^{Proposed Legislation.} Bill, which has been an annual one ever since the society was formed, seeks to obtain the enfranchisement of leaseholds, in cases where at least 20 years of the lease remain unexpired; the second relates to compensation for improvements effected during the tenancy; and the third has for its object the acquisition of the freehold of sites occupied by Nonconformist chapels.

Up to the present time, however, all the efforts towards legislation in this direction have been unsuccessful owing, as might be expected, to a strong opposition in the House of Commons. The opponents of the movement may be divided into four distinct classes: (1) The ground landlords, who naturally object to being compelled to part with their property even in exchange for compensation. (2) The bulk of the Conservative party, who see no necessity for such a sweeping change in the law. (3) A considerable portion of the professional and mercantile classes, who regard any measure which interferes with the principle of free contract between man and man as economically unsound. (4) The advocates of Land Nationalization and of other socialistic reforms, who contend that the proposed legislation will fail to achieve the objects for which its supporters are striving, and further, that the idea of individual ownership in land is in itself an evil, which the multiplication of freeholders will only tend to perpetuate ⁽²⁾.

⁽¹⁾ For an extended list of the objects of this Society see Report of Town Holdings Committee (1887), App. 1.

⁽²⁾ See *Fabian Tract*, No. 22, "The Truth about Leasehold Enfranchisement."

The following arguments have been adduced for and against the principle of leasehold enfranchisement :

For the principle :

- Arguments in favour of Leasehold Enfranchisement.
1. The leasehold system has not been developed by the action of free economic forces, but is the outcome of legal restraints, which have, for many centuries, been in operation over land held by corporations or under settlement.
 2. Under this system the ground landlord reaps the entire benefit of the "unearned increment" which has accrued through the efforts of the community, and to which he has not contributed his share.
 3. It discourages thrift in the working classes, who have no inducement to save sufficient money to purchase their houses.
 4. It is the direct cause of jerry building, and of dilapidated houses towards the end of a lease, the lessee having no permanent interest in the property.
 5. Under the existing system the tenant derives very little benefit from any improvements he may make during the last few years of his term, on the expiration of which he is obliged to pay a higher rent for renewal, in consequence of the enhanced value of the premises resulting from such improvements.
 6. The restraining covenants contained in leases are frequently vexatious and burdensome, especially those relating to dilapidations on the expiration of the lease.

Against the principle :

- Arguments against Leasehold Enfranchisement.
1. The supervision exercised by the freeholder on granting a lease, and the covenants under which a building is erected and kept in repair, have proved highly beneficial to the property, and in comparing houses built on the freehold and leasehold systems the former is at a disadvantage in this respect.
 2. Under the leasehold system estates and towns have been developed to an extent that would have been quite impracticable if the builder had been obliged to lock up his capital in the purchase of the freehold.

3. The real benefits of enfranchisement would be obtained, not by the occupier, but by the middleman, who is to blame for all the evils incident to the system, since in very rare instances does the occupier happen to be the actual leaseholder.
4. Compulsory enfranchisement would set aside existing contracts, and would thus inflict great hardship on the owners of ground rents and reversions, which are held, to a considerable extent, as investments by hospitals, colleges and insurance companies.
5. The evils attributed to the leasehold system are exaggerated and exist only in isolated cases, and the disadvantages arising from depreciation can be readily provided against by the creation of a sinking fund at the commencement of the lease.
6. The amount of compensation payable to the owner of the reversion would, in most cases, be extremely difficult to assess, especially if the ground rent formed the basis of the calculation, and disputes would constantly arise causing annoyance and litigation.

Before examining these arguments in detail, it will be convenient to give a brief summary of the various attempts at legislation which have been made since the subject first engaged the attention of Parliament.

The Bill promoted by the Leaseholds Enfranchisement Association was first introduced into the House of Commons in 1883. Its principal provisions are that, in the case of leases having an unexpired term of 20 years (including sub-leases) and all leases for lives, the lessee shall be entitled to purchase the reversion on terms to be mutually agreed upon ⁽¹⁾. In the event of disputes arising, the price—which may be by way of rent charge—is to be fixed by the County Court judge of the district in which the property is situated. The Bill contains clauses relating to settled land and to leases held by trustees, as well as statutory forms of conveyance and of notice.

In 1884 a Bill on similar lines was introduced by Lord Randolph Churchill. The unexpired term was extended to 21 years, and the machinery to be employed for settling disputes was to be the County Court when the annual value of the land did not exceed £50, and

⁽¹⁾ As introduced in February 1893 this Bill includes *all* leases not held at a rack rental.

Lord Randolph
Churchill's
Bill.

arbitration under the provisions of the Lands Clauses Consolidation Act, 1845, when the annual value exceeded £50. The amount of compensation was to be taken as the sum which the property would fetch if sold at auction, with an addition of 10 per-cent less the value of the interest of the leaseholder therein. Allowance for improvements by the leaseholder was to be made only when the consent of the reversioner had been obtained. This Bill, which was restricted to England and Wales, was withdrawn without discussion, and has not since been introduced.

In the following session a Bill was brought into the House of Commons by Mr. Daniel Grant, with the object of enfranchising leasehold building land. It contained provisions for the purchase of the fee-simple in future leases by the lessee upon his giving six months' notice of his intention to do so, property under settlement being included. The purchase money was to be an amount equal to 25 years of the annual ground rent reserved on the lease. This Bill was also withdrawn.

Mr. Grant's
Bill.

The next attempt towards enfranchisement was made in 1887 by Colonel Hughes, a Conservative member. His Bill applied to leases held at a ground rent of not more than half the net annual rateable value of the property, and having at least 20 years to run, as well as to all leases for lives. Tables were appended showing the amount of compensation to be paid to the ground landlord, such tables being calculated throughout at 5 per-cent interest, and, in the case of leases for lives, the rate of mortality being that of the Government Tables (male and female). Table 1 indicated the number of years' purchase to be paid for a ground rent of £1 per annum under a lease having an unexpired term ranging from 20 to 100 years. Table 2 gave the present value of a perpetual annuity of £1, payable after the expiration of 20 years and upwards. Table 3 contained the present value of a reversion of £1, due at the end of 21 years and upwards. In Table 4 the expectation of life from age 0 to 49 (male and female) was shown.

Colonel
Hughes's
Bill.

The lucidity of the process to be followed may be judged from the instructions appended to Table 4: "To ascertain the value of
" a freehold interest in a lease for a life or lives, ascertain the
" expectation of life by the above table for the life (or the youngest
" life if more than one) of the person or persons on whose life or
" lives the lease is determinable, and assume the year's expectation

“to be a term of years;—thus, 49 years of age is for one male
 “equal to 20 years unexpired term,—and ascertain the
 “present value of the ground rent for years by
 “Table 1, then add the value of the reversion by the
 “second table. In the case of a lease for years
 “determinable on a life or lives, the value of the ground rent is
 “to be ascertained by Tables 1 or 4, calculated by that table
 “which gives the fewest number of years unexpired or expected
 “duration, and the value of the reversion is to be ascertained by
 “Table 2, and when the two amounts are added, that is the
 “value of the freeholder’s interest.”

Provision was made in the Bill for the purchase money to be paid by way of rent charge, “equal to five pounds per-cent per
 “annum upon the value of the ground rent purchased
 “from the freeholder, and four pounds per-cent per
 “annum upon the value of the reversion.” In case
 of dispute the amount of compensation was to be settled in the same manner as in Lord Randolph Churchill’s Bill, the appended tables being taken as the basis of the calculations.

An annual Bill, called the Occupying Tenants’ Enfranchisement Bill, has also been before Parliament at the instance of another Conservative member, Mr. Bartley. It provides that every occupying tenant holding a lease of his premises, with certain exceptions, and having an unexpired term of 20 years, shall be entitled to purchase the freehold of his house, shop, chapel, &c., at a price to be fixed by the County Court, or, in the event of difficulties arising as to title, by way of a rent charge
 equal to four per-cent on the purchase money. Any
 freeholder so selling by compulsion may require the
 Municipal Corporation or County Council or other local authority to purchase the adjacent property belonging to him, at a price to be settled either by a professional valuer or by a jury; and the property purchased in this manner shall be offered for sale to the occupiers, who may thus purchase their dwellings at such prices as in the aggregate shall cover the entire cost of the transaction. The purchase money may be paid either in a lump sum, or by a sinking fund spread over 15 years at four per-cent interest, or by a perpetual rent charge as before. Occupiers not purchasing will remain tenants of the local authority. Certain restrictive covenants in the original lease are to be annulled on purchase, but the remaining and more important covenants are to be enforced by the local authority.

Instructions
 as to the
 Tables
 appended
 to the Bill.

Provisions for
 Purchase by
 way of Rent
 Charge and for
 Compensation.

Mr. Bartley’s
 Bill.

In 1888 Sir Edmund Lechmere brought in a Bill to provide for the conversion of leases for lives into leases for years, by means of the machinery of the County Court. The term of years to be substituted in the lease was to be estimated in the following manner: "The actuary to the Treasury shall calculate the probable future duration of such lease from the date of notice, having regard to the tables showing the expectancy of life for the time being in use in Her Majesty's National Debt Office, and shall thereupon certify to the Registrar in writing the probable future duration of such lease."

Mr. Lawson's Town Holdings Bill contained various provisions for carrying out the recommendations of the Select Committee as to compensation for tenants' improvements. Disputes were to be settled by a referee appointed under the Arbitration Act, 1889, or by the Local Government Board. The procedure adopted was taken from the Agricultural Holdings Act, 1883.

Mr. John Ellis's Places of Worship Sites Bill was before the House of Commons in 1890. Its object was to acquire sites for places of worship upon application being made to the owner of the land required for the proposed site, by not less than 20 householders. In the event of the owner failing to grant the site within six months, a memorial, together with the sum of £100, was to be lodged with the County Court. The Court was then, after taking into consideration all the circumstances of the case, to determine whether the site was necessary, and whether the acquisition would not injure the adjoining property.

The Places of Worship Enfranchisement Bill, which was introduced by Mr. S. T. Evans into the House of Commons during the sessions of 1890 and 1891, provided that trustees of a religious body should have power to purchase the freehold of the chapel, school, or minister's house, together with the buildings and lands attached thereto, of which they held the lease or tenancy. The amount of the purchase money, in case of disagreement, was to be settled by the County Court. After a sharp debate in the House of Commons on 22 April 1891, the Bill was read a second time, but was subsequently withdrawn.

In all these Bills there is much to criticize, and especially is this the case with the provisions contained in the Bills of Colonel

Hughes and Sir Edmund Lechmere, which relate to the conversion of leases for lives, in the one case into freeholds, and in the other into leases for years. In the first place, the adoption of the expectation of life as a basis of conversion cannot be regarded as actuarially sound. As is well known by all students of life contingencies, the number of complete years in the average duration of life after a specified age is not the equivalent for the term of a life of that age, and in the event of more than one life being involved, as in the great majority of such leases would be the case, the analogy fails completely, and the tables are useless. Nor is the Government Experience Table a suitable one for general use, unless the lives happen to be of an advanced age—a highly improbable circumstance, having regard to the practice of selecting very young lives for such purposes.

A better mode of estimating the value of the unexpired terms of such leases would be to make the Institute of Actuaries' *H^M* Table the basis of calculation, and to assume a rate of interest throughout not greater than $3\frac{1}{2}$ per-cent. Proper annuity-values could then be deduced for any number of lives by a mathematical process, such as Lubbock's formula. Having obtained these values, it would then be easy to determine the equivalent term of years, which should be substituted in the leases. It would probably, however, be a matter of some difficulty to ascertain to what extent the influence of selection operated, when the option was exercised by either party. It might very well happen that if the lives were in bad health, the lessee would be anxious for conversion, while the fact of their being sound and healthy would no doubt be an inducement for the lessor to exercise his right to the disadvantage of the tenant, and thus the grievance would continue, though under a different form.

It will be observed that none of these Bills are altogether satisfactory, either in their principle or in their working, and they would probably, if passed, be of little use in remedying the evils against which they are directed—such as bad building, the insecurity of the occupying tenant, or unsanitary arrangements. It does not necessarily follow that because the ownership of his house changes hands the occupier will take more interest in his property, or that by facilities being given to obtain freehold ground for building the builder will be induced to spend more of his capital in the materials used in the construction of the houses which he purposes erecting. It has been proved over and over again that many of the worst houses

Criticism of
Bills dealing
with Leases
for Lives.

Doubtful
remedy
against bad
building.

have been built on freehold ground, such as, for example, in Bedfordbury, Seven Dials, and large portions of Soho, Bermondsey, and Southwark, while, on the other hand, the splendid mansions built by Cubitt, Seth Smith and Freaque in the West End of London, were all erected under the 99 years' system.

It is often asserted by those who advocate compulsory enfranchisement, that it was recommended in the Report of the Royal Commission on the Housing of the Working Classes issued in 1884. Such a statement, however, is misleading, as the recommendation formed part, not of the main Report, but of a supplementary Report signed by only 10 out of the 17 members of the Commission. This supplementary Report ran as follows:

"The system of building on leasehold land is a great cause of the many evils connected with overcrowding, unsanitary buildings, and excessive rents. This appears to be conclusively proved by the evidence of Lord William Compton, Mr. Boodle, the agent of the Marquis of Northampton, Mr. Vivian of Camborne, and by the incidental evidence of other witnesses. The evidence of the two former witnesses contains a strong condemnation of the whole system of building on leasehold tenure.

"Those of your Majesty's Commissioners whose signatures are appended to this supplementary Report are of opinion that the prevailing system of building leases is conducive to bad building, to deterioration of property towards the close of the lease, and to a want of interest on the part of the occupier in the house he inhabits; and that legislation favourable to the acquisition on equitable terms of the freehold interest on the part of the leaseholder would conduce greatly to the improvement of the dwellings of the people of this country."

In view, however, of the fact that the subject of leasehold tenure did not form part of the enquiry by the Commission; that the evidence taken on the subject was very meagre and of limited scope; that Mr. Boodle has on more than one occasion emphatically denied that he was unfavourable to the leasehold system¹; and that Mr. Vivian's evidence related mainly to leases for lives, it may not unreasonably be inferred that the above supplementary Report is not based upon the evidence taken before the Commission, but merely expresses the political opinions held by a portion of the Royal Commissioners.

Supplementary
Report of the
Royal Com-
mission on the
Housing of the
Working
Classes.

Its practical
value.

(1) See *Standard*, 11 May 1885, and Mr. Boodle's evidence before the Town Holdings Committee (1887, 6, 216-29).

In a further supplementary Report, which is entirely ignored by the promoters of the movement, Lord Salisbury makes the following remarks :

“ I understand that it is proposed by a portion of the Commission to report in favour of a measure for enabling the holder of a long lease to force the freeholder to sell his freehold to him at an arbitration price. This measure, which is objectionable and wholly novel in principle, appears to me to have little to do with the housing of the poor. It has not been shown in evidence that any considerable number of the working classes, especially in overcrowded localities, hold their tenements on long leases. As far as this class of house is concerned, such a measure would have no other effect than to put the house farmer in the position now occupied by the ground landlord. On the other hand, such a proposal would entirely arrest the grant of building leases, and the supply of new houses for the working classes in suburban districts would be very materially checked.”

Report of Lord Salisbury.

It may thus be concluded that the Report of the Royal Commission has but little value in eliciting the truth as to the advantages of the proposed scheme for enfranchisement.

Of a very different character is the Report of the Select Committee of the House of Commons on Town Holdings, which is the result of minute and independent enquiry into the various kinds of tenure of house property in the principal towns of the United Kingdom. This Committee was appointed in 1886 “to inquire into

Appointment of a Select Committee on Town Holdings.

“ the terms of occupation and the compensation for improvements possessed by the occupiers of town houses and holdings in Great Britain and Ireland; and to inquire into the expediency of giving to leaseholders facilities for the purchase of the fee-simple of their property; and also into the question of imposing a direct assessment on the owners of ground rents, and on the owners of increased values imparted to land by building operations or other improvements.”

The Select Committee, after taking the evidence of more than 100 witnesses during three sessions, made a very long and exhaustive Report on the first two subjects of enquiry.

The conclusions which the Committee arrived at with regard to leasehold enfranchisement appear in the following extracts from their Report :

“ We are not of opinion that leasehold enfranchisement would, on the whole, have much practical effect in improving the quality of houses built; we think that under some circumstances, as where terms of leases are very short, or (as under the life system) of

uncertain duration, it would have this effect: on the other hand, we are of opinion that the oversight of the ground landlord on well-managed estates undoubtedly has, in this respect, a beneficial operation. Upon a general view, our conclusion is that the quality of building depends upon other circumstances than those arising from tenure. Nor are we satisfied that the bad state of repair of much house property in large towns, or the unsanitary condition of many of the dwellings of the working classes, is to any large extent to be attributed to the leasehold system. As regards excessive rents, we are not inclined to think that leasehold enfranchisement would have any appreciable effect in reducing them, and we think that the leasehold system rather facilitates the building of houses, and so increases the supply.

We are, however, of opinion that the conversion of leaseholds into freeholds would often promote improvements, and encourage the development of trades and businesses by giving lessees the opportunity of securing the full benefit of their outlay in improvements, and the full value of any goodwill they may have created. It would also get rid of some minor incidents of leasehold tenure, which are felt to be of an irritating character, and would tend to remove the feeling of grievance among lessees, which, to whatever extent it may be well-founded or not, exists amongst them. We believe that the power of acquiring the fee-simple of their holdings would, in many cases, promote habits of thrift and providence among the industrial classes, and we think that in places where leaseholds are now prevalent, the effect of such an Act, so far as it tended to increase the number of freehold houses in the market, would stimulate among the industrial classes the desire to acquire houses of their own. It would also tend to bring about the advantages to the community connected with an increase in the general number of owners of freehold property.

Although the advantages we have referred to appear to us as likely to arise to some extent from leasehold enfranchisement, yet the evidence laid before us does not enable us to come to the conclusion that the power of enfranchisement will be generally taken advantage of by lessees, especially if it is coupled with provisions, as must in fairness be the case, to secure the reversioner both the value of his property, and compensation for any injury he may sustain by its being taken from him, and the expenses which he may necessarily incur. We are clearly of opinion that any measure of enfranchisement must proceed upon the basis of giving full compensation for all interests and rights, which the reversioner at present possesses by law, and of which he would be deprived."

Among the reasons against the proposed measure which must be taken into account, are the objections, that it will, in many cases, prolong the existence of old houses which ought to be pulled down, and which, under the leasehold system, would probably have been rebuilt on the dropping in of the leases: that it will discourage or prevent many improvements of a more or less public character, which are, undoubtedly, frequently

Opinion of the Committee as to the effect of Enfranchisement on Building.

Minor advantages of Enfranchisement.

As to Compensation.

Probable effects of Enfranchisement—

effected by the owners of large leasehold estates when their property falls into hand, and which would probably not have taken place at the expense of the ratepayers; that it would interfere with the encouragement sometimes given by such owners to the creation of industrial dwellings, by granting sites for such buildings on liberal terms; and that it would imperil the security at present enjoyed by the holders of leases on a well-managed estate, through the power and interest of the ground landlord in enforcing covenants which are for the general benefit.

On Improve-
ments, &c.

On the value
of Municipal
property.

“Another objection, to which we think much importance must be attached, is the effect that enfranchisement may have in depriving municipal corporations and public charities of the prospective improvement in the value of their estates, and in some cases in seriously diminishing their present income. It is possible that, in some of these cases, important benefits may be lost to the community for the advantage of a comparatively limited number of individuals.

On Ground
Rents and
Reversions
generally.

“An objection of more general application arises from the very extended scope of the proposed scheme of enfranchisement, which subjects the great bulk of ground landlords, and the owners of ground rents and reversions in leasehold property throughout the country, to the liability of having their property compulsorily taken from them at any time during a long period, and where, as in so many cases, there are a number of lessees upon one property, of being deprived of it piecemeal, as may suit the convenience of the several lessees. We consider this power to be unprecedented and one-sided. The operations of the measure can only benefit the community in an indirect way, as each transaction under it would be a compulsory transfer of property from one individual to another, and however fully the right to pecuniary compensation may be safeguarded, a feeling would widely prevail amongst those affected, that their rights had been unjustly interfered with and needlessly disturbed. The persons chiefly affected by the proposed measure would not be only

On small
Owners.

the large landowners, but the much more numerous class of persons, often of very moderate means, who are in the habit of investing in ground rents and reversions in house property. The operation of the Act on any such property comprising, as is so often the case, several houses, if carried out piecemeal, would be to subject the owner to repeated trouble, annoyance and expense, in the settlement of each case, and the successive re-investment of each portion of the purchase money. The statements of experienced witnesses leave no doubt on our minds that the measure will seriously lessen the desirability, and thus diminish the value of the class of property affected by it. In the

On
Mortgagors.

case of mortgaged property it might act very hardly on the owners by inducing the mortgagees to call in their money, and it is clear that the liability which a mortgagee would be under to receive back the investment in small and uncertain payments, will seriously interfere with the power of raising money upon such property on reasonable terms. The injury we thus

anticipate to the property affected by a general enfranchisement measure will, we think, arise from the uncertainty and insecurity which it will introduce, even though in the result the powers of the Act might not be very largely made use of."

"We are doubtful whether an Act of this sort would be generally taken advantage of by lessees, and we doubt whether it is calculated to confer much benefit on the industrial classes, in whose interest it has been chiefly advocated, inasmuch as the majority of the working-class occupiers would not be within its provisions. As regards these classes, we think the principal effect of such an Act would be to convert the middleman, under whom they hold, into a freeholder."

On the Work-
ing Classes.

The Committee then proceed to make the following recommendations as to the direction which efficient legislation should take:

1. That local authorities should be empowered to facilitate enfranchisement, subject to the approval of the Local Government Board, in districts where the majority of the occupiers are leaseholders and are desirous of purchasing the freehold of their houses.
2. That leases for the duration of lives should be converted into fixed terms of years.
3. That religious bodies should be enabled to purchase the fee of their places of worship and schools.
4. That similar powers should be granted to public educational bodies, co-operative and provident societies and local authorities, so far as concerns their schools, halls and public buildings.
5. That the powers of tenants for life of estates under settlement should be extended, so that they might be able to grant leases for long terms and to give the option to lessees to purchase the freehold.
6. That tenants should be entitled to compensation for sanitary improvements, made by them during their tenancy, and also for material improvements made with the consent of the landlord ⁽¹⁾.

Recommendations by the
Committee.

Three of these recommendations have been, as we have already noticed, under the consideration of Parliament. It may be doubted, however, whether the best mode of carrying them out is by Bills introduced by private members. A Government measure which contained provisions embodying the main features

(1) Report of Town Holdings Committee (1889), p. 37 *et seq.*

of the Report of the Town Holdings Committee would be probably more effective in removing existing grievances, and, if drawn up on a broad and workable basis, would meet with but little serious opposition.

We are now in a position to examine the arguments upon which the principle of leasehold enfranchisement is grounded. Some of these have already been disposed of in the foregoing pages. We have seen, for instance, that the leasehold system is far from being an artificial form of tenure, but that, on the contrary, it has been in use in one shape or another for many centuries, and that it is admirably adapted for rapidly developing estates, which would otherwise have yielded only a small return. Nor can the system, in any sense, be held to discourage thrift in the working classes, since there appears to be no general desire on the part of these classes to purchase their own dwellings, for the reason that it is essential for them to live within an easy distance of their work, and consequently to be obliged to occupy a particular house all their lives would place them at a great disadvantage with their employers, and destroy their independence to a considerable extent. Besides, with the doubtful exception of Mr. Bartley's proposal, no scheme has yet been submitted which could assist them in the slightest degree, since the only result of the change would be that the freehold of their tenements would pass from one landlord to another.

Examination
of objections
to the
Leasehold
System.

Then it is said that the present system is the direct cause of jerry building, from the fact that the builder's interest in the property ceases at the end of 99 years. No proof has yet been given that this is the case, or that builders are influenced at all by the length of their lease. It is no doubt true that, in the suburbs of large towns, whatever the system may be under which building is carried on, attempts are frequently made to run up houses in as cheap a manner as can be safely done, without being called to account by the district surveyor or the freeholder; but this may be remedied, not by altering the tenure, but by enforcing more stringent regulations, both as to the quality of the materials and thickness of the walls in the buildings which are thus erected, and to the obligation of the landlord to keep the houses in good repair. If district surveyors would insist upon certain rules being strictly observed before granting certificates, we should soon see an end of jerry building, and of the harm which it undoubtedly causes. The way in which houses are erected by

the speculative builder has been the subject of recent comment by the Inspector-General in Bankruptcy, who remarks:

“The speculative builder seems to consider it his business to erect buildings without due regard to the laws of supply and demand. The only limit upon his operations is that which is imposed by his power of mortgaging. Frequently starting with little or no capital, he has no difficulty in obtaining sites of land on lease; the lessor, knowing that the moment the requisite materials are brought on the ground, the value of his land is certain to be improved in any case, is perfectly indifferent to the lessee’s means. Having thus obtained the lease, the builder proceeds to order the materials. Timber, bricks, cement, &c., are obtained on long credit, the usual period being about six months. As the buildings progress there is no difficulty in obtaining advances on mortgage to the extent of about two-thirds or three-fourths of their actual cost. These advances are obtained long before the bills for materials become due, so that the builder is no longer hampered by want of ready money. Sometimes a building may be disposed of for cash, but as a general rule it is by mortgage and not by sale that the requisite funds are supplied. . . . But the day of reckoning inevitably comes: a temporary pressure in the timber trade, or the closing of some avenue of supply by failure or otherwise, is the frequent precursor of collapse” (1).

It is clear from the above description that whatever might be the prevalent tenure of land for building purposes, the speculative builder would find some way to obtain the ground and cover it with buildings composed of the cheapest materials, and, further, that it would require legislation of a character very different from the proposed enfranchisement of leaseholds to put an end to jerry building and its concomitant evils. The true remedy lies not so much in Acts of Parliament as in a healthy public opinion on the subject, and so long as people will live in houses built in this fashion, so long will the system continue to flourish.

With regard to the assertion that lessees who make permanent improvements to their holdings do not obtain the full benefit of such improvements, it may be replied, that no sensible man would incur the expense of improvements unless he felt certain that he would be eventually recouped during his tenure. He knows perfectly well that his lease expires on a certain day, and that after that time he will have to pay a rack rent on renewal, and he calculates his expenditure accordingly. This objection was repeatedly raised in the evidence before the Town Holdings Committee, and on each occasion was fully answered.

(1) Eighth Report by the Board of Trade (Bankruptcy), p. 6.

Lastly, we may notice the argument that the leasehold system possesses the drawback of placing the lessee in the position of being obliged, under the terms of this contract, to surrender his holding at the expiration of his lease, and, in addition, to pay a large sum for dilapidations. This is, of course, perfectly true, but then it is precisely what he agreed to do, and the lease was granted to him on more favourable terms in consideration of the due performance by him of this part of the contract. The following illustration will be sufficient to show that the lessee is not eventually out of pocket by taking a lease, instead of purchasing the freehold ⁽¹⁾:

Profits of
Lessee.

Take 10 per-cent on the outlay (say £10,000) as the net rental	£1,000
Deduct—	
(a) Ground rent which may be assumed to be one-third of the rental value (say)	£266
(b) Four per-cent interest on borrowed capital	400
	<hr/>
	666
Net annual profit	<hr/> £334 <hr/>

He can also, by means of a single payment of about £400 or of an annual payment of about £15, effect a leasehold assurance in a first-class office for £10,000, payable at the expiration of 99 years from its commencement, thus recouping himself for the loss of his property at the end of the term. The addition to the rent incurred by taking out a policy of this description is so trifling, that it is difficult to discover the grounds upon which the objection to the leasehold system in this respect is founded ⁽²⁾.

It would appear, then, that the only real grievance which leaseholders can be said to possess is the sentimental one of feeling that the property which they hold is not absolutely their own, but is subject to the covenants contained in the lease. This feeling, however, only operates in the rare case of an occupier being the actual leaseholder, and it seems unreasonable, that so sweeping a change in the law should be made for the benefit of such an inconsiderable proportion of the community, especially

⁽¹⁾ *Transactions of the Surveyors' Institution*, vol. xviii, p. 22. These figures, though hypothetical, may be fairly applied to the profits arising from the construction of blocks of City offices.

⁽²⁾ A complete list of tables is given in Mr. Tyssen's little work on "Leasehold Assurance."

General
conclusions as
to Leaseholds.

when, by the means of the natural laws of supply and demand, such a grievance can be easily removed. In the North of England, and especially in the large towns of Yorkshire, public opinion demands freehold houses, and, as a consequence, freehold houses are built. It therefore follows that what is done in Yorkshire can also be effected in London and elsewhere, without having recourse to special legislation. It is sometimes replied that the power of the large landowners in London is so great that the leaseholders are practically compelled to accept their terms. However this may be the case in the interior of the Metropolis it is by no means so in the suburbs, where notices of land to be sold may constantly be seen, and even with regard to land in the more populous parts of London it is not unusual for freeholds to be sold by auction.

We will now pass on to the consideration of the various methods of assessing compensation, adopted in the Bills that have been presented to the House of Commons. It is probably assumed by the authors of those Bills that, in the great majority of cases, the terms of compensation would be mutually agreed upon. If,

Provisions in
Bills as to
Compensation.

however, it should happen that no agreement can be arranged as to the terms, a settlement must be decided upon by a Court of Law. Three modes of settlement are provided for by the different Bills, the favourite one being by means of the County Court. In two Bills the course to be adopted is that provided by the Lands Clauses Consolidation Act, when the annual value of the property exceeds £50. The third mode is under the provisions of the Arbitration Act.

The provisions of the Lands Clauses Consolidation Act, which were incorporated in Lord Randolph Churchill's and Colonel Hughes's Bills, are Sections 25 and 119. Section 25 authorizes the parties interested to select an arbitrator to make the award; and in the event of their not concurring in the selection, then each party may appoint an arbitrator, whose decision shall be final. Section 119 applies only to disputes settled in the County Court, and refers to the apportionment of rents of superior leases where the notice is given by an under-lessee. It runs as follows:

"If any lands shall be comprised in a lease for a term of years unexpired, part only of which lands shall be required for the purposes of the special Act, the rent payable in respect of the lands comprised in such lease shall be apportioned between the lands so required and

Section 119 of
the Lands
Clauses Act.

the residue of such lands; and such apportionment may be settled by agreement between the lessor and lessee of such lands on the one part, and the promoters of the undertaking on the other part, and if such apportionment be not so settled by agreement between the parties, such apportionment shall be settled by two Justices; and after such apportionment the lessee of such lands shall as to all future accruing rent be liable only to so much of the rent as shall be so apportioned in respect of the lands not required for the purposes of the special Act; and as to the lands not so required and as against the lessee, the lessor shall have all the same rights and remedies for the recovery of such portion of rent as previously to such apportionment he had for the recovery of the whole rent reserved by such lease; and all the covenants, conditions and agreements of such lease, except as to the amount of rent to be paid, shall remain in force with regard to that part of the land which shall not be required for the purposes of the special Act, in the same manner as they would have done in case such part only of the land had been included in the lease."

The Arbitra-
tion Act.

The Arbitration Act, 1889, the provisions of which were incorporated into Mr. Lawson's Town Holdings Bill, relates to the appointment of an arbitrator on each side, who may, on disagreement, select an umpire whose decision shall be final.

The Machinery
of the County
Court.

A few words as to the machinery of the County Court will not be out of place. These Courts were first established in 1846 and superseded the old Courts of Request, which had existed in England and Wales for the recovery of small debts, since the time of Henry VII. To County Court Judges the power of settling disputes as to compensation has from time to time been conferred by Parliament, as, for example, in the Succession Duty Act, when the amount in dispute is not less than £50; in the Metropolitan Building Act, 1855; in the Building Societies Act, 1874; and in the Agricultural Holdings Acts, 1875 and 1883. Various provisions are to be found in these and other Acts with respect to the settlement of such disputes by County Courts.

Hence, the machinery of these Courts having been proved to furnish a convenient, rapid and inexpensive process by which the amount of compensation in small cases may be assessed, it has

As to the
competency of
County Court
Judges to
assess
damages.

been adopted in the schemes for estimating the amount that should be paid by the leaseholder for the purchase of the fee-simple of his property. Whether such Courts are the most competent to adjudicate upon the complex questions that must inevitably arise in connection with conflicting interests in landed property, is a

matter which has occasioned much difference of opinion. It may be submitted that, as a class, County Court Judges are far from being men of exceptional ability or acuteness, and in but rare cases have they such a deep knowledge of the intricacies of real property, as would be absolutely necessary if the proposed powers were entrusted to them, since the only requisite qualification for the appointment is that of being a barrister—not necessarily practising—of seven years' standing ⁽¹⁾. It may also be noted that in some of the more important Acts, which have reference to the compensation of landowners, such as the Lands Clauses Consolidation Act, the Ecclesiastical Leasing Act, the Artizans' Dwellings Acts, and the Housing of the Working Classes Act, provision has been made for disputes to be settled

The appointment of professional assessors. by competent surveyors, selected either by the parties themselves, or by the Court, or by the Local Government Board. Such a provision is omitted from the Bills promoted by the Leaseholds Enfranchisement Association, probably on account of the expense entailed by such a mode of settlement. Having regard, however, to the fact that it is not proposed to place any limit on the amount of compensation to be awarded by the Courts, it may be questioned whether the omission is altogether satisfactory ⁽²⁾.

We may now sum up the conclusions which may fairly be drawn from the consideration of the arguments for and against leasehold enfranchisement. These conclusions may be briefly stated as being:

- Concluding Observations on the subject of Enfranchisement.
1. That leasehold enfranchisement is desirable only on the lines indicated by the Town Holdings Committee.
 2. That the proposals hitherto before Parliament are, to a great extent, unsound in principle and unworkable in practice.
 3. That any legislation on the subject should be by means of a Government Bill, and should be adapted to the particular circumstances which exist in different parts of the kingdom.
 4. That the awards of compensation should be made by competent and independent surveyors, appointed

⁽¹⁾ See County Courts Act, 1888, sect. viii.

⁽²⁾ The creation of a tribunal composed of non-practising surveyors is suggested by Mr. Sidney Woolf, Q.C., in a paper read before the Surveyors' Institution, see vol. xxii of *Transactions*.

either by the Local Government Board or by the Judge of the County Court of the district.

5. That in estimating the value of leases for lives, or of the fines accruing upon the failure of such lives, a valuation should be made upon approved scientific principles.

III.—THE TAXATION OF GROUND RENTS, CHIEF RENTS, AND GROUND VALUES.

It is, as we have already seen, an essential condition to the grant of a lease, whatever the length of its term, that an annual rent should be paid to the freeholder for the use of the land. The amount of this rent is a variable quantity, being dependent not only on the prospective value of the land, but also upon the circumstances surrounding the transaction between the owner and the speculator who undertakes to develop the estate. The latter covenants to cover the land with houses according to a specified plan and within a fixed period of time. As the buildings progress he obtains at short intervals, either from his solicitor or from the freeholder himself, advances of money upon the mortgage of the buildings, with which to pay wages and professional charges, the materials being obtained on credit. From the moment that the builder takes possession to the time when he has completed his contract, no less than four distinct kinds of rent are created, namely: peppercorn rents, original ground rents, improved ground rents and rack rents ⁽¹⁾.

Peppercorn rents are often charged in lieu of ground rents, in consideration of the lessee compounding for the greater portion of the ground rent in a lump sum. They generally exist for the first few years of the lease, after which a much higher ground rent than would have been paid in the first instance is agreed upon. In Birmingham there are two modes of creating this class of rents. One is by charging only a peppercorn rent for the first year, then so much for the next year, and an additional amount for every succeeding year on a fixed scale, till the whole block of houses stipulated for is finished. The other way is for the builder to take an agreement from the

How Ground
Rents arise.

Peppercorn
Rents.

⁽¹⁾ Kerr, *The Consulting Architect*, sect. viii.

ground landlord instead of a lease, by which he undertakes to cover the land within a certain period at a total ground rent of so much, the ground landlord agreeing to grant leases to his nominees when required. Then the grantee will secure the whole of the agreed ground rent on a portion of the block, taking the remainder at a peppercorn. In the event of either the grantee or lessees making all the necessary street improvements, a very much smaller ground rent is charged than if the lessor had made them ⁽¹⁾. These rents are also frequently reserved in leases of houses which are the last built on the block, the object being to secure the total rent to the ground landlord as early as possible by apportioning comparatively high rents to the houses first built, the amounts gradually diminishing till the whole block is completed. On the Portman estate, in Marylebone, there are a very large number of peppercorn rents, which have existed during the whole continuance of the lease ⁽²⁾. A recent illustration of the creation of such a rent may be taken from the action of the Ecclesiastical Commissioners, who, in 1888, let a piece of ground in Red Cross Street, Southwark, to the Kyrle Society for the purpose of a garden and playground, the lease being for 999 years at a rent of one farthing a year.

Original ground rents may be considered as representing the estimated annual value of the ground at the time that the lease was granted, and so long as the lease continues they remain unaltered in amount. Many considerations determine what this amount shall be, such as the number and class of houses to be erected on the site, and whether the road, sewers, &c., are to be made by the lessor or the lessee, it being invariably stipulated in the lease that all rates and taxes, of whatever nature, property tax alone excepted, shall be paid by the lessee. As has been already observed, ground rents are frequently distributed unequally amongst the houses in a block, and hence, the amount of ground rent attached to a particular house affords no criterion as to the value of the lessee's interest.

Improved ground rents consist of annual payments for a term of years secured upon buildings as a charge second to the original ground rents. The difference between the amounts of the original and the improved ground rent represents, when

⁽¹⁾ Evidence of Mr. Mathews before the Town Holdings Committee (1888) 1,279-83.

⁽²⁾ Evidence of Mr. Hunt, 13,085.

capitalized, the sum which the middleman has earned by developing the land to such an extent as to be ready for building operations. Improved ground rents may, however, be created in another way, such as by the lessee receiving an advance from the lessor and repaying it by means of a sinking fund added to the ground rent. In the former case it is the usual practice for the middleman, whose interest in the land ceases after its development, to sell his improved ground rent either to the ground landlord or to the lessee, who has undertaken to erect the houses, and thus it not unfrequently happens that the amount of rent mentioned in the lease is not the same as the amount actually paid by the lessee. Leasehold improved ground rents possess a reversion of only one day to the rack rents, and hence, they are practically terminable annuities pure and simple ⁽¹⁾.

Rack rents are the rents payable by tenants for the use of houses occupied by them, and represent their full annual market value. They form the basis upon which all local rates, as well as property tax and the inhabited house duty, are levied. The amount of the rack rent, which generally varies from twice to six times the ground rent, is determined entirely by the natural laws of supply and demand, and rises or falls in accordance with those laws.

This brief description of the four classes of rent, which are usually attached to urban property in this country, and which it is proposed to assess for the purpose of re-adjusting the burdens of local taxation as between the owner and the occupier, will enable us to elucidate much that has been said and written on the subject since it has come within the range of practical politics.

Before, however, we consider this portion of our subject, a short account of the history of the taxation of landed property since the time of the Commonwealth will be of interest.

Previously to that time, we hear of occasional taxes being imposed on land for purposes of war, such as the Danegeld of Anglo-Saxon times, levied to bribe the Danes from invasion; the tax called "scutage", which formed an essential incident to knight service under the feudal system; the Saladin tithe, charged at the close of the 12th century upon rent and movables; the Tudor subsidies of a similar

**Improved
Ground Rents.**

Rack Rents.

**Early Taxa-
tion of Landed
Property.**

(1) *Town Holdings* (Cassell), vol. i, p. 26.

nature, which superseded the poll tax of the 14th century, and continued down to the time of the Parliamentary war (1). But it was not till the year 1656 that a permanent tax was imposed on property, the object being "the maintenance of the " Spanish war, and other necessary services of the Common- " wealth " (2).

By this Act tenants of houses and lands rated to the tax were required to pay the whole of it, a deduction being
First
Permanent
Land Tax. authorized to be made from their rent of so much of the tax as the landlord ought to bear, the payment being considered to be made by the landlord. Differences between the landlord and the tenants were to be settled by "divisional commissioners", who heard appeals against the assessment by the parochial authorities. The rate was to be so much in the pound on the rent or yearly value of land, and 5 per cent per annum on the capital value of money, stock and other goods, a specified sum being charged upon every county or town. This assessment, which was condemned by Halifax "as " impolitic and unreasonable a method of raising great sums of " money as was ever introduced in any nation", was made monthly, and yielded about $1\frac{1}{2}$ millions a year. It was abolished in 1691 (3).

In the following year, which also witnessed the creation of our national debt, a tax of 4s. in the pound was levied on all real estate, offices and personal property, land being rated at its rack rent (4). The object of the tax was that of
Land Tax of
1692. carrying on the war with France, but it continued at different rates for over a century. Personal property appears to have constantly evaded assessment, and the tax gradually became known as an annual land tax. It was declared by Walpole to be "the most grievous, the most unequal, and the most oppressive tax ever raised in this country" (4).

In 1797 Pitt introduced the "triple assessment", which consisted of a charge varying from three to five times the taxes payable by persons of the richer classes, according to a fixed scale.

(1) Dowell, *History of Taxation*, vol. iii, ch. i, sect. i.

(2) Dowell, vol. iii, ch. i, sect. ii.

(3) Dowell, vol. iii, ch. i, sect. iii. See also Stephen, *Commentaries of the Laws of England*, vol. ii, ch. vii, p. 567.

(4) The inequalities of this tax were due partly to the double taxation of land belonging to Roman Catholics, and partly to the fact that after the Revolution owners returned their property at a high or low value, according as they supported or opposed the new Government.

Pitt's
Schemes.

This tax, however, proved an utter failure, and was repealed in the following year, a uniform property and income tax of 10 per-cent being substituted; this continued in operation till 1815, when, the French war having been brought to a successful conclusion, its existence could no longer be justified.

In the same year as that in which the triple assessment was levied, the land tax was declared to be perpetual, the basis remaining the same as in 1692. At the same time facilities for its redemption were provided by the Act 38 Geo. III, cap. 60, the object being to diminish the pressure of the public debt in the market by the absorption of a large amount of stock.

Land Tax of
1797.

Although the land tax eventually falls upon the landlord, it is actually payable by the tenant, for the reason, as laid down by Lord Mansfield, that "the parish cannot tell who is the landlord, or who has a rent charge; it is upon the occupier that the officer of the Government takes his remedy; and though the landlord is directed to allow the sum paid out of the rent, the tenant is to be considered as the person first liable, the parish having nothing to do with transactions between landlord and tenant." The tax is levied on each county separately, the counties being composed of several divisions, and each division being under the control of a body of local commissioners. In these divisions the contributions vary

Incidence
in London.

considerably according to the amount of increase in the value of the land since 1692, which still continues as the standard of assessment, the counties contributing from 8*d.* to 1*s.* 6*d.* in the pound. London is, for the purposes of this tax, divided into parochial districts, which present most striking differences in the amount of their contributions, the assessment in St. Pancras being only $\frac{1}{4}$ *d.* in the pound, while in parts of the City which have not risen in value during the last 200 years, it is as much as 3*s.* in the pound. Various provisions as to the appointment of commissioners and the collection of the tax are contained in the Taxes Management Act, 1880. The tax on personal property was abolished in 1833 ⁽¹⁾.

In 1842, the income and property tax, which had been dis-

⁽¹⁾ For a detailed history of the Land Tax during the last two centuries see *Report of the National Debt Commissioners* (1891), p. 199.

Re-imposition of the Income Tax. continued for nearly 30 years, was re-imposed by Sir Robert Peel, not, however, as a war tax, but as a temporary substitute for the various Customs duties which were repealed in that year. At first the rate was 7*d.* in the pound on all incomes of £150 and upwards; in 1853, however, Mr. Gladstone, with the object of abolishing a further large number of Customs duties, levied a tax of 5*d.* in the pound on all incomes between £100 and £150. Between 1853 and 1876 other revisions were made in the limit of exemption which was finally settled in the latter year by Sir Stafford Northcote, a total exemption being made of all incomes under £150, and a partial exemption of those between £150 and £100.

The tax known as the inhabited house duty was first imposed in 1778, but repealed in 1831, and afterwards re-imposed in 1851 as a substitute for the old window duty. Till quite recently it was at the rate of 9*d.* in the pound for all houses **The Inhabited House Duty.** valued at £20 and upwards, and 6*d.* in the pound for shops and other buildings occupied for the purposes of trade. In 1890, however, a sliding scale was introduced by Mr. Goschen which has had the effect of considerably reducing the tax in the case of small houses and shops.

We now come to the only remaining tax which is levied on landed property, namely, that derived from the death duties ⁽¹⁾. This term is used to designate five distinct duties to which property may become liable on the death of the owner, namely, probate duty, account duty, legacy duty, succession duty and estate duty. Of these various duties, only the last two are payable in respect of real property, which up to 1888 was taxed at a lower rate than personal property; under the Customs and Inland Revenue Act of that year, however, it was provided that probate duty on personal property amounting to £1,000 and upwards should for Imperial purposes be reduced from 3 to 1½ per-cent, the remaining 1½ per-cent being devoted to the relief of local taxation. By the same Act succession duty, which, as well as legacy duty, varies in amount according to a fixed table of consanguinity, was raised by ½ per-cent in the case of

(1) See Dowell, *History of Taxation*, vol. iii, ch. ii: also Buxton and Barnes, *Handbook to the Death Duties*, where their distribution between real and personal property is discussed.

lineals—who are exempted altogether from legacy duty on personal property if probate duty is paid—and by $1\frac{1}{2}$ per-cent in the case of collaterals. By this means real and personal property, so far as the incidence of the death duties in respect of Imperial taxation was concerned, were placed upon an equal footing, since probate and legacy duty together are now equal to succession duty alone. There is, however, one distinction to be noticed in the mode of treatment of real and personal property respectively, which is, that while legacy duty and probate duty are payable in a lump sum before distribution, succession duty is spread over a period of between four and five years, being divided into eight half-yearly instalments. A curious anomaly has arisen under the Act of 1888 with regard to leasehold property, since for purposes of probate this class is regarded as personalty, and thus becomes liable to probate duty, whereas for purposes of distribution it is included amongst realty, and hence, is subject to the higher scale of succession duty. Account duty was imposed under the Act of 1881, and was designed with the object of rendering liable to taxation property which had hitherto escaped probate duty, in consequence of its transfer to the beneficiary having been effected by means either of a *donatio mortis causa*, or of a voluntary settlement. This duty was made more comprehensive in its scope in 1889, in which year Mr. Goschen established the estate duty of £1 per-cent on all estates of a value exceeding £10,000, subject to some modification in the case of settled property.

In the above cursory review of the various taxes which affect landed property, we have spoken only of those devoted to Imperial purposes. This class of property is, however, subject, and almost exclusively subject, to taxation for local purposes, and it is only since the year 1888, when an improved system of Local Taxation. Local Government was introduced into this country, that a definite share of personal property, other than buildings erected on land, has been contributed to local taxation. This mode of taxation is of a somewhat complex nature, owing to the large number of authorities exercising administrative functions, and to the different areas over which these functions are performed. In order, therefore, to throw some light upon the development of the system of local taxation now in operation, as well as upon the real incidence of rates upon the owner and the occupier respectively, it will be useful to trace

briefly the different stages of its progress in the history of this country.

The first period may be said to commence in the reign of Richard II, when we first hear of a Statute being passed with the object of granting licenses to beggars under certain conditions. In the reign of Henry VIII it was further enacted that the several hundreds and parishes should sustain the poor with such charitable voluntary alms as might prevent the necessity for begging, the poor being directed to abide in the places where they had been born or had dwelt for the past three years. A later Act in this reign required the churchwardens of every parish church to make collections every Sunday for the relief of the poor; but until this Act, such relief was left to the humanity of the more prosperous inhabitants ⁽¹⁾. In 1531 the Commission of Sewers was established for the whole country—thus superseding the old elective officers called dyke-reeves—and contributions for the purposes of arterial drainage, and the cleansing of rivers, streams and ditches, were levied upon all persons whose property derived any advantage from the works undertaken by the commissioners, “after the quantity of their lands, tenements and rents by the number of acres and perches.”

The second period, which marked the development of our present Poor Law System, extended from 1601 to 1782. In the former year the Statute for the relief of the poor authorized the overseers of every parish to raise, weekly or otherwise, by the taxation of every inhabitant and every occupier of lands, houses, &c., in such parish, a stock of flax, hemp, &c., to provide work for the poor, and also competent sums of money for the relief of those unable to work, and for putting out children as apprentices, such relief being given according to the ability of the parish. Under this system, the burdens imposed were comparatively light, averaging about £700,000 per annum ⁽²⁾.

The third period dates from the close of the American war in 1782, which had caused a general rise of prices and hence, also, of wages; and in this year an Act, called Gilbert's Act, was passed

⁽¹⁾ Burns, *Justice of the Peace*, vol. iv, ch. i, “The Poor Laws.”

⁽²⁾ For a description of the burdens of a London householder in the middle of the 18th century see Besant, *London*, ch. ix, p. 363.

authorizing the adoption of out-door relief; in other words, wages were supplemented out of the rates. "In consequence of this system", says Mr. Dudley Baxter, "and the immense rise during the American and French wars in the prices of corn and all the

Third Period. "necessaries of life, the poor rate increased, quite out of proportion to the increase of population, from £700,000 in 1750 to £2,000,000 in 1785, then to £4,000,000 in 1803, to £8,500,000 in 1814, and even to £9,300,000 in 1818, or more than fourfold in 33 years. . . . As a natural consequence the ratepayers were weighed down with the burden, and the nation seemed approaching a period of social disorganization and ruin" (1).

The fourth period was one in which many of the evils caused by the above pernicious system were successfully remedied, the instrument being the New Poor Law Act of 1834, which provided for the abolition of the grant of out-door relief to the able-bodied, and substituted the workhouse test in its place; the Act also introduced a more thorough system of supervision and accounts. The result of this reform was the reduction of the poor rates by 36 per-cent in three years. Up to 1840 it had been customary

Fourth Period. to levy rates upon personal property as well as upon houses and lands; in that year, however, an Act was passed making it unlawful "to tax any inhabitant in respect of his ability derived from the profits of stock-in-trade or any other property for or towards the relief of the poor." This Act was only a temporary one, but its principle has been acknowledged ever since by its inclusion, up to the present time, among the Acts in the annual Expiring Laws Continuance Act. In 1855 the Metropolitan Board of Works was established for the purpose of consolidating the different local boards and creating a single rating authority for the whole of the Metropolis.

The fifth period commenced with the passing of the Union Assessment Act, 1862, by which the standard of rateable value outside London, which had hitherto varied from 10 to as much as 70 per-cent below the real net value of the property assessed, was placed upon a sound and equable basis, and, as a result, this value rose from £76,000,000 in 1863 to £90,000,000 in 1867. This standard was extended to the Metropolis by the Valuation Act of 1869.

The sixth period dates from 1888, when the Local Government

(1) *Transactions of the Social Science Association* (1873), p. 535.

Act was passed. The principal provisions of this Act may be summarized as follows :

- The Local Government Act.**
1. The jurisdiction of the county magistrates, so far as related to the administration of county affairs and the assessment of county rates, was transferred to County Councils elected by the ratepayers of each county, the term including not only the 52 counties of England and Wales, but a large number of the principal towns.
 2. The Metropolitan Board of Works ceased to exist, its place being taken by the London County Council, to which was granted the same powers with the addition of that of granting licenses. The ratepayers constituted the electors, the former system of election by the Local Boards of Works being abolished.
 3. Grants in aid of local taxation from the National Exchequer were discontinued, and instead, two-fifths of the probate duty, as well as the bulk of the license duties, were set aside for this purpose.
 4. Power was given to the County Councils to borrow money upon the security of their rates, repayment to be made by means of a sinking fund spread over a period of 30 years, or, in the case of the London County Council, of 60 years.
 5. Some slight alterations were made in the manner of the assessment of the local rates affected by the Act, one result being that the sewers rate, which, up to the date of the Act, was levied by Commissioners of Sewers and assessable upon the owner and occupier according to their respective interests, was converted into a poor rate, and thus became payable exclusively by the occupier without any right of deduction of his share from the rent ⁽¹⁾.

Its effect on Local Taxation.

The effect of the substitution of the probate and license duties for the previous grants in aid will be seen from the following table, which shows the amount distributed between the Metropolis, the

⁽¹⁾ Evidence of Mr. C. Harrison before the Town Holdings Committee (1890), 3,548. See also Goschen, *Reports and Speeches on Local Taxation*, p. 161.

boroughs and the counties for the year ending 31 March 1890 ⁽¹⁾ :

How Distributed	Estimated Rateable Value	Amount of Discontinued Grants	Substituted Revenues	Excess of New Revenues	Additional Revenue per pound of Rateable Value
	£	£	£	£	d.
Metropolis .	31,000,000	628,000	814,000	186,000	1·44
Boroughs .	27,000,000	592,000	1,060,000	168,000	4·16
Counties .	92,000,000	1,640,000	2,919,000	1,279,000	3·33
Total .	150,000,000	2,860,000	4,793,000	1,933,000	3·09

The principal rates that contribute towards local taxation are the poor rate, the general district rate, the burial board rate, the sewerage rate, the towns improvement rate, the school board rate, and in some districts the public libraries rate, the two last-named being of recent growth. In the area of the London County Council there is also the consolidated rate administered by that body for the improvement of the Metropolis, and outside this area county and borough rates are levied for similar purposes. Of these rates two fall directly upon the owner, and then only under special circumstances ⁽²⁾. The poor rate is paid by him, when, under the Assessed Rates Act, 1869, he is rated by resolution of the vestry, or agrees to pay it instead of the occupier. Under the same Act occupiers with a term not exceeding three months may deduct from their rent the poor rate paid by them. The general district rate is, in the case of tenements of small annual value, levied directly upon the owner, who is allowed a small commission for paying it.

Parochial and
other Rates.

Under section 214 of the Public Health Act, 1875, provision is made for the levy of a private improvement rate upon the occupier, who, unless he has stipulated to pay the rate himself, is entitled to deduct three-fourths of the amount from his rent, the same privilege being extended to every intermediate lessee, whose term unexpired is not less than 20 years. This rate has been extended in operation by the Artizans' Dwellings Acts, and also by the Housing of the Working Classes Act, 1890, which repeals those Acts.

Private
Improvement
Rate.

⁽¹⁾ See *Economist*, 11 October 1890.

⁽²⁾ Owen, *Manual for Overseers*, p. 105 *et seq.*

The annual value of the property upon which local rates are levied is the net value which, as a rule, is determined by a fixed deduction from the assessed gross annual value. Outside London this deduction is defined by the Act of 1836 as the probable average cost of repairs, insurance and other expenditure, if any, necessary to maintain the house so as to command a fair rent. Within the Metropolitan area the deduction is a certain proportion of the gross rental, and varies from 5 to 33 $\frac{1}{3}$ per-cent, according to the nature of the building and to its annual value. The lowest deduction is made in respect of lands without buildings, and the highest in respect of mills and manufactories.

Assessment of
Annual Value
for Rating
Purposes.

The various local authorities having power to levy rates are County Councils, Town Councils, Sanitary Authorities, Local Boards of Health, Boards of Guardians, Burial Boards, Highway Boards, School Boards, Lighting Inspectors and Overseers.

Various
Rating
Authorities.

These bodies are elected almost exclusively by the occupiers. In the case of Local Boards and Boards of Guardians, however, owners of property within the district possess from one vote to six votes apiece, according to the amount at which their property is rated, the number being regulated by a graduated scale up to £250 annual value. If they are occupiers as well as owners, they have double voting power on the same scale. Owners are also entitled to record their votes by proxy ⁽¹⁾.

There are thus two distinct systems of taxation existing in this country, each possessing its own laws and levied without regard to the other. Imperial taxes are either direct or indirect in their nature. Income tax, the house duty and the death duties are direct, being actually borne by the person paying them.

Direct and
Indirect
Taxation.

Customs and excise duties, as well as taxes liable to be deducted from rent, are indirect, since the person paying them does not bear the whole of the tax. In local taxation, however, such a distinction is more obscure, and political economists have held divergent views as to the real incidence of the burden which, in the first place, is borne by the occupier of the house for which he is assessed.

Professor Fawcett ⁽²⁾, for example, held that there was a fundamental distinction between the incidence of rates when

⁽¹⁾ Chalmers, *Local Government*, ch. ii, p. 30; Brodrick, *Essay on Local Government in England*.

⁽²⁾ *Manual of Political Economy*, ch. vii.

Views of
Professor
Fawcett on
the Incidence
of Local
Taxation.

imposed on land and when imposed on houses and buildings. He maintained that in the case of an agricultural tenancy, or of a house possessing exceptional advantages of situation, the rent would be increased by the amount of the rates, if none were levied. The rent of an ordinary urban house, however, he considered to be determined partly by the cost of production and partly by the laws of supply and demand, and in such a case the incidence of the rates was altered.

Mr. John Stuart Mill ⁽¹⁾ divided the proposed tax into two parts—that on the ground rent and that on the building rent.

Of J. S. Mill. The former, he thought, would be borne by the occupier only if it were a fixed sum per square foot, irrespective of the rent paid for a more valuable site; if, however, it were a proportion of the rack rent it would fall on the owner. The tax on the building rent, he considered, must ultimately be borne by the occupier.

Professor Sidgwick ⁽²⁾ has expressed the decided opinion that although such a tax would at first be paid by the owner, the bulk of it would in the end fall upon the occupier, *Of Professor Sidgwick.* owing to the fact that its effect would be to stop the building of new houses, and hence, to raise the prices and rents of existing houses to an extent sufficient to cover the portion of the tax which was paid in respect of the building rent.

Professor Cliffe Leslie ⁽³⁾ also adopted the theory that, as rents are regulated by supply and demand, so if rates were abolished rents would be raised, and that, to the extent *Of Professor Cliffe Leslie.* of the additional amount which might then be exacted, the tax falls upon the owners. He believed, however, that the owner is, as a rule, the stronger party to the contract, and hence, is able to throw any increase in the rates upon the occupier.

The subject seems to have first engaged the serious attention of Parliament in 1866, when it was recommended by the Select Committee on Metropolitan Government “that in any arrangement of the financial resources of the Metropolitan Board of Works, a portion of the charges for permanent *Report of the Select Committee of 1866.* “improvements and works should be borne by the “owners of property within the Metropolis, the rate

⁽¹⁾ *Principles of Political Economy*, People's Edition, book v, ch. iii, sect. vi.

⁽²⁾ *Principles of Political Economy*, ch. viii, sect. viii.

⁽³⁾ Art. on “The Incidence of Taxation on the Working Classes”, in the *Fortnightly Review* for February 1874.

“being in the first instance paid by the occupier, and subsequently
 “deducted from his rent, as is now provided in regard to the
 “general property tax.” They also suggested that owners, on
 becoming liable to such a rate, should be properly represented in
 the Metropolitan Board of Works.

In the following year two Bills having reference to this
 subject were introduced into the House of Commons—one by
 Mr. Ayrton, the Chairman of the Select Committee, to raise a
 special rate for improvements not exceeding 3*d.* in the pound, to
 be deducted from the occupier’s rent, a special provision being
 made against the right of the landlord to contract himself out of
 his liability; and the other under the auspices of the Corporation
 of the City of London, to empower them to levy a
 rate of 6*d.* in the pound upon owners of property in
 the City for the purpose of effecting improvements
 within its area. This last-mentioned Bill was referred to a Select
 Committee, who reported against it on the ground that the
 Metropolitan Board of Works was the only proper authority in
 the Metropolis to whom such a power should be entrusted,
 provided that owners were adequately represented on the Board.
 Neither Bill was proceeded with, and the subject appears to have
 been dropped for three years.

On 3 March 1870, the matter again received the attention
 of Parliament, another Select Committee being appointed “to
 “inquire and report whether it is expedient that the charges now
 “locally imposed on the occupiers of rateable property should be
 “divided between the owners and occupiers, and what changes in
 “the constitution of the local bodies now administering rates
 “should follow such division.” Mr. Goschen, then President of
 the Poor Law Board, was appointed Chairman, the Committee
 including such men as Mr. Ward Hunt, Mr. Ayrton, Sir Massey
 Lopes, Sir Thomas Dyke Acland, Mr. Rathbone and
 Mr. W. H. Smith. The principal points upon which
 evidence was given were the following:

1. The various kinds of local rates leviable.
2. The purposes for which they are levied.
3. The authorities by whom they are levied and expended.
4. The manner in which those authorities are constituted.
5. The persons by whom the several rates are paid.
6. The difference in the scale of contribution.
7. The incidence of the rates with respect to occupiers and
 owners generally.

After hearing the evidence, a Draft Report was drawn up by Mr. Goschen, which dealt in an exhaustive manner with each of the above points. As expressing Mr. Goschen's personal views upon the subject of the incidence of the rates between owners and occupiers, the following passage is of interest :

"To sum up the case of house property generally, it appears that the owners of building land, like the owners of other land, have to submit to a reduction of rent equivalent to the average amount of rates which the builder or other lessee calculates that he would have to pay according to the average of past rates; and that as regards any subsequent excess after the owner of the soil has made his bargain, it is borne, in the case of leases, exclusively by the occupier. When leases expire, and in the case of short tenancies, in fact, whenever re-adjustments take place, the above excess is borne sometimes by the leaseholder or owner of the house, sometimes by the occupier, according to the state of supply and demand.

**Draft Report
drawn up by
Mr. Goschen.**

"A distribution of rates between owner, leaseholder and occupier, coupled with an enactment prohibiting contracts by which either party could exempt himself by anticipation from all future local taxation, would probably be to the pecuniary advantage of the occupiers of that class of houses which are usually held under leases, and would, at all events, relieve them from what is now considered a grievance, even under their bargains to undertake all risks, *i.e.*, their liability to pay for permanent improvements, which, while made at their expense, they themselves will only enjoy for a limited time.

"As regards property held under tenures which admit of frequent adjustment of rents, it may be fairly argued that the question as to the person from whom the rate is levied will not much affect the incidence of the rate. In short, a system of rack rents and frequent re-adjustments diminishes the importance of the division of rates to either party" ⁽¹⁾.

Mr. Goschen then proceeded to advocate the importance of the ground landlord having a direct influence in the election of the local authorities and in the assessment of the rates, and suggested that the owners should be constituted into a separate electoral body to vote by themselves and to elect a certain number of members to represent them on all boards, administering rates to which they contribute.

The Draft Report was not, however, adopted by the committee, it being determined to proceed by resolutions; these were 11 in number, and may be summarized as follows :

1. That, the existing system of local taxation being contrary to sound policy, it is expedient that owners

(1) *Reports and Speeches on Local Taxation*, p. 168.

Summary of
Report of the
Committee.

should be liable for a certain proportion of the rates; and that, subject to equitable arrangements as regards existing contracts, the rates should be collected from the occupier, power being given him to deduct from his rent the proportion of the rates to which the owner may be made liable.

2. That owners should be entitled to a direct representation upon the local bodies administering the rates.
3. That, in the case of long leases still unexpired, the owners of property held under lease should be exempted from payment of rates for three years only, after which period, the occupiers should be entitled to deduct from the rent the proportionate part of the rates to which the owner might be liable, power being given to the owner at the same time to add to his rent a sum equivalent to the like proportionate part of the rates calculated on the average annual amount of the rates paid by the occupier during the above period.

These recommendations were followed up in the next session of Parliament by the introduction by the Government into the House of Commons, of a "Rating and Local Government" Bill, which contained *inter alia* the following provisions:

1. "With regard to the liability of owners for a part of the consolidated rate, it is enacted that, where an occupier, who has paid any instalment of the rate, pays a rent in respect of the property on which he is rated, not less than the rateable value, he shall be entitled to deduct from the rent one-half of the sum which he has

The Rating
and Local
Government
Bill.

paid as rate. If the rent paid is less than the rateable value, he may only deduct from each pound of the rent one-half of the sum in the pound which he has paid as rate. Again, where any person, receiving rent in respect of any rateable property, also pays rent in respect of the same, he is authorized to deduct from the rent which he pays, a sum bearing such proportion to the amount of rate deducted from the rent received by him, as the rent paid by him bears to the rent received by him.

"All future contracts between owner and occupier, by which the latter is deprived of this right of deduction, shall be void. But no deduction shall be made by any occupier in respect of such portion of the rate as he has, before the passing of the Bill, contracted with his landlord to pay, or has paid in satisfaction of the precept of a Town Council."

2. "For the purpose of making a register of owners, the collector shall, at or before the prescribed time, enquire from the occupier of

every rateable hereditament, whether he is himself, or if not, who is, the owner of it. Having thus learnt the name of the owner, the collector shall enter it in his rate book, and the entry so made shall be *prima facie* evidence of his being an owner for the purposes of the Act. If the same person is owner and occupier of the same hereditament, he shall, in respect of it, be entitled to vote as occupier only."

This Bill, which it may be observed excluded the Metropolis from its operation, was eventually withdrawn at an early stage of its progress, in consequence of the great press of other business at that period, and also of the hostility which was manifested towards some of its features on both sides of the House. The transfer of Mr. Goschen to the Admiralty, together with the change of Government which occurred three years later, prevented its re-introduction at a subsequent period.

We next hear of any public reference to the incidence of local taxation in the Reports which were furnished by the Commissioners on Agriculture appointed in 1881, who collected evidence from agriculturists and others with reference to this subject.

In the course of his evidence before the Commission, the late Sir James Caird stated as his opinion that ultimately the rates must fall upon the landlord, and that it would be better that they should fall directly upon him, as this would leave the tenant entirely free for the single consideration of the cultivation of the land; he would then know exactly what his responsibilities were, and would have nothing to do but attend to his farm. The burden, he thought, would be lessened, because the landlords or their agents would have the administration of the rates, and thus superior intelligence would be brought to bear upon the subject ⁽¹⁾.

In their final Report the Royal Commissioners say: "Whatever change may be made in the incidence of local taxation, we are of opinion that, without disturbing existing contracts of tenancy, all rates should in future be borne equally by owners and occupiers" ⁽²⁾.

In each of the subsequent years resolutions have been brought forward in the House of Commons relating to the incidence of local taxation, and in 1885 an amendment to the Budget

⁽¹⁾ Evidence before the Royal Commission, 62,663.

⁽²⁾ Page 26.

Resolutions in the House of Commons. proposals in favour of measures for the relief of the ratepayers was carried against the Government, and resulted in a change of Ministry. In the early part of 1886 a Select Committee on Town Holdings was appointed, and on 16 March Mr. Wm. Saunders moved that the reference to that Committee be extended to an enquiry into "the question of imposing a direct assessment on the owners of ground rents and on the owners of increased values imparted to land by building operations or other improvements", this motion being agreed to.

A week later, Professor Thorold Rogers, after an interesting speech giving the principal points in the history of local taxation, moved the following resolution :

"That the present system under which, in England and Wales, the first incidence of local taxation (with some slight exceptions) falls on the occupier and not on the owner of lands and tenements, is unjust ; that such owners ought in equity to bear at least a moiety of those charges ; that the system under which country mansions are rated is unfair ; and that the owners of ground rents in towns are liable to no part of those charges, the outlay of which is essential in order that the property may possess any marketable value whatever."

To this resolution the following amendment was moved by Sir Richard Paget :

Sir R. Paget's Amendment. "That while the apportionment of the payment of rates between landlord and tenant may be desirable, as part of a complete scheme for remedying the admitted inequalities of the incidence of local taxation, this House is of opinion that the financial injustice complained of can only be removed by a comprehensive measure, and that an equitable re-adjustment of taxation as between real and personal property is urgently required."

On a division the amendment was negatived by a majority of 19, and the original motion affirmed by a majority of 40 (1). The Dissolution of Parliament in the following June prevented the motion from being carried into effect.

The next allusion to the subject was made at the committee stage of the Local Government Bill, 1888, when Mr. F. A. Channing proposed the addition to the Bill of the following clause :

"Enactments with regard to levying of rates under Act—

"In respect of any rate made, assessed or levied for the purposes and under the provisions of this Act by the County Council or

(1) Hansard, *Debates*, 23 March 1886.

by a District Council as hereinafter provided, the following enactments shall be made (that is to say):

Mr. Channing's
Amendment to
the Local
Government
Bill.

- “(1) Where the occupier of any rateable property who has paid any quarterly, half-yearly or other instalment of such rate or rates, pays a rent in respect of the property on which such rate is made and assessed, he may, if the rent paid by him is not less than the rateable value of the property, deduct from such rent one-half of the sum which he has paid as rate; but if the rent paid by him is less than the rateable value, he may deduct from each pound of the rent which he is liable to pay, one-half of the sum in the pound which he has paid as rate, and so on in proportion for any less sum than a pound.
- “(2) Where any person receiving rent in respect of any rateable property also pays a rent in respect of the same, he may abstract from the rent so paid by him a sum bearing such proportion to the amount of the rate deducted from the rent received by him as the rent paid by him bears to the rent received by him.
- “(3) Where any person receiving rent in respect of any rateable property has agreed to pay rates, or is rated under the Poor Rate Assessment and Collection Act, 1869, or Acts amending the said Act, he shall, for the purposes of the provisions of this Act, be deemed to be the occupier.
- “(4) Every person receiving, in respect of any rateable property, rent from which a deduction has been made on account of rate, shall be deemed to be an owner for the purposes of this Act.
- “(5) Any contract made by an occupier after the passing of this Act, by virtue of which he is deprived of his right to make a deduction from his rent, which he would otherwise be entitled to make under this section, shall, so far as relates to such deduction, be void both at law and in equity” (1).

In the course of the debate which took place on this amendment, Mr. Ritchie, who had charge of the Bill, said that the Government did not desire to complicate an already overweighted Bill by including in it a burning question of this kind, which could only be discussed and settled when they were dealing also with the question of representation. Mr. Goschen expressed himself in sympathy with the resolution, but pointed out that it was the essence of his own proposals that the owner should be represented in virtue of a division of the rates, his object being to enable landowners to exercise

Views of the
Government.

(1) Report of Town Holdings Committee (1890), App. 2.

a direct and powerful influence in local administration. He also reminded the House that in his own Bill this division only applied to future rates. On behalf of the Government a promise was given by Mr. W. H. Smith that the matter should receive their early attention, and with this assurance the amendment was negatived by a majority of 55 ⁽¹⁾.

In the sessions of 1889 and 1890, Mr. Samuel Montagu brought in a Bill to provide for the amendment of rating in respect of permanent improvements in the Metropolis, by means of a "Metropolis Improvement Rate", to be levied every year by the London County Council for the repayment of all capital loans. The rate was to be first charged upon the occupier, who was to be entitled to deduct the amount from his rent, similar power being also given to successive tenants, so that eventually the rate would be borne by the ground landlord. In cases where the ground rent was nominal, or where the rate amounted to at least one-fourth of the ground rent, the amount of such rate was to be refunded to the occupier paying the same, and to be charged with compound interest at 3 per-cent per annum upon the freehold under an order of the Court of Quarter Sessions; such charge to have priority over all other incumbrances and interests other than succession or legacy duty, and the freehold to be absolutely mortgaged to the London County Council as security for the payment of such rate and interest when the reversion accrued. Upon the expiration of the lease, it was provided that no rent should be paid by the occupier until the amount of the charge had first been paid by him to the London County Council, such payment to be a discharge of an equivalent amount of rent due to the superior landlord.

This Bill was considered by the Local Government and Taxation Committee of the London County Council very shortly after the formation of that body, who reported against it on the grounds that the repayment of the capital of improvement loans in the manner proposed would tend to delay the requisite arrangement, that intermediate lessees who would gain most from the improvements were excluded from contributing, and that the accumulation of the charge at compound interest was both complex and cumbrous, and in cases of distant reversions would swamp the property. This Report was adopted by the Council on 21 May 1889, and in

Opposition by
the London
County
Council.

⁽¹⁾ Hansard, *Debates*, 18 July 1888.

consequence the Bill was opposed by their representatives in Parliament. It has been adduced by Sir Thomas Farrer as "an excellent illustration of the difficulty of combining taxation of rent or income with the taxation of freehold or ground values" (1).

But although the Council could not see their way to support this Bill, they still approved of the principle upon which it was based, and proposed as a substitute—

"That every Act empowering the County Council to raise loans shall contain a provision that the repayment of such loans and interest shall as between the various classes of persons interested in London property, be borne in such manner as Parliament shall, either by the same or any other Act, direct, and a further provision that no contract shall take effect, whereby the proportion of expense directed by law to fall on any person, shall be shifted to another person.

Substituted
Proposals of
the Council.

"That Parliament shall be moved (at least so far as regards London), to enact that, for the future, no contract shall take effect, by which it is provided that one person shall pay rates which, independently of contract, the law casts on another."

This resolution was succeeded by a further one on 9 July 1889, which declared that all private contracts as to payment of rates by the occupier should be void, and that, until this principle was adopted by Parliament, no further improvements should be made. In opposition to this second resolution, a memorial was signed by 30 members of the Council addressed to the Chairman, protesting most strongly against the resolution, this memorial being followed by a discussion in the newspapers.

As a result of these resolutions, the Council presented a petition to the House of Commons containing an epitome of the Parliamentary history of the subject since the year 1843, and praying as follows—

Petition of
the Council to
Parliament.

"(1) That your honourable House will proceed as speedily as may be to consider how the burden of the rates may be more fairly adjusted.

"(2) That a portion of the rates may be thrown on the owner of the land as distinguished from the occupier thereof.

"(3) That an owner shall not be able by contract to throw such portion on the occupier.

"(4) That in the meantime, while the proper mode of relief is being settled, and in order to lessen the difficulty arising from contracts existing when the

(1) Evidence before the Town Holdings Committee (1890), 1,257.

law is changed, a Suspensory Act may be passed, enacting that the burden of the rates shall be borne by such persons or classes of persons as Parliament shall direct, notwithstanding any private contract made after the enactment now asked for.

- “(5) That, in order to prevent charges for new improvements falling under existing contracts, it may be enacted that the burden of all loans raised for permanent improvements after the enactment now asked for, shall, notwithstanding any private contract, be borne by such persons or classes as Parliament shall direct” (1).

Two Bills embodying these proposals were subsequently introduced by the Council, but were not proceeded with.

The subject of the incidence of local taxation was again brought before the London County Council on 13 November 1891, when Mr. Stuart, M.P., moved the following amendment to the Report of the Local Government and Taxation Committee:

Final Resolution of the Council. “That the Council is of opinion that the existing system of local taxation in the Metropolis unduly favours owners at the expense of occupiers, and that it is urgently necessary that this injustice should be redressed.”

This amendment was, on a division being taken, carried by a majority of 42 votes, two further amendments relating to existing contracts, and to the direct representation of owners on the bodies administering the rates, being negatived by large majorities.

It will be gathered from the above account of the proceedings of the London County Council with reference to this burning question, that the subject has been regarded by that body as of supreme importance in the interests of the inhabitants of the Metropolis. Nor has the matter been viewed as merely a periodic

subject of debate in Spring Gardens. In 1887 some of the most active supporters of the movement, who have since become prominent members of the London County Council, formed a society called the “United Committee for Advocating the Taxation of Ground Rents and Values”, with the object of influencing public opinion in London and elsewhere, in favour of substituting the taxation of land for the taxation of buildings. The society has since its formation circulated a large number of pamphlets setting forth the reasons

Formation of a Society for Advocating the Taxation of Ground Rents.

(1) Evidence of Sir T. H. Farrer before the Town Holdings Committee (1890), 1405; also App. S of Report (1890).

for such a reform, and many lectures and public meetings have been held under its auspices.

The subject had not been debated in Parliament since the rejection of Mr. Channing's amendment to the Local Government Bill in 1888, when in February 1891 Mr. A. D. Provand moved the following resolution in the House of Commons:

"That, in the opinion of this House, the proportion of taxation which falls upon and is payable by land and its rentals is insufficient, and ought to be increased."

It may be mentioned that the original motion contained the additional words: "that land should be assessed on
Mr. Provand's Resolution. " its value, whether occupied or vacant, and that " taxation which falls upon food, industry and commerce, should " be repealed to a corresponding amount." These words were, however, omitted, in order that the opinion of the House might be taken solely on the incidence of local rates.

A long debate took place on the motion, in the course of which Mr. Gladstone made the remark: "It appears to me beyond " all doubt that under our present system ground rents enjoy " undue exemption; and in the Metropolis in particular the owner " of property has long enjoyed at the expense of the ratepayers " privileges which are really unjust " (1).

The motion was defeated by a majority of 51 votes.

In the following month Mr. J. Stuart moved:

"That, in the opinion of this House, the freeholders and
Mr. Stuart's Motion. owners of ground values in the Metropolis ought to contribute directly a substantial share of local taxation."

After a long debate, the motion was negatived by 149 to 123 votes (2).

Outside Parliament and the London County Council, the subject has evoked much discussion in several of the London vestries, as well as in provincial town councils, particularly those of Birmingham, Manchester and Bury, where resolutions have been passed during the last few years in favour of the scheme of dividing the rates equally between the
Feeling in the Provinces and the London School Board. owner and the occupier. Shortly before its dissolution in November 1891, a debate took place at the London School Board upon the expediency of levying rates on the owners of ground values instead of exclusively on the industrial classes

(1) Hansard, *Debates*, 27 February 1891.

(2) Hansard, *Debates*, 13 March 1891.

as at present, a resolution to this effect being negatived by 19 to 11 votes.

The foregoing description of the progress of the agitation on behalf of a revision of the present system of local taxation which has existed during the last few years, contains the principal points in its history; and although, up to the present time, no legislation has actually taken place in this direction, the subject is receiving the very careful consideration of the Select Committee

on Town Holdings, whose final Report, based upon a review of the expert evidence which, in the course of the enquiry, has been laid before them, may be expected during the session of 1892 ⁽¹⁾. Upon the publication of this Report, the views of the Committee will doubtless be embodied by the Government in a measure providing for the further development of Local Government, and dealing with the incidence of local taxation in a manner at once effective and equitable. Such a measure has already been foreshadowed by members of the Government, and in a recent speech Lord Salisbury referred to the subject in the following terms:

“I have always entertained the opinion that our rating system is singularly ineffective and imperfect, and that we do not draw into the taxations all kinds of property. I should be very glad to see that anomaly corrected. The only thing you have to guard against is that this particular grievance, which is the real one, should not be made the occasion or opportunity for the gratification of some particular class or political antipathy. If the question of all the interest of land being brought under the net of the rate collector is raised, it will be a very difficult and thorny one. There is a great deal of logic on that side, but you must bring in the mortgagee and the debenture-holder as well as the ground rent owner, or else you will not do justice” ⁽²⁾.

In connection with the last sentence in the above quotation, attention may be drawn to an Act passed by the Legislature of

(1) Since the above was written, this Report has been presented to Parliament. The following is a brief summary of the recommendations of the Select Committee:

- (1) That the real and apparent incidence of local taxation should be as identical as possible, and no one should be able to contract himself out of any tax imposed upon him.
- (2) That existing contracts should remain unaltered.
- (3) That under all future contracts, local rates should be equally divided between occupiers and owners, and that each owner should have the right of making a proportionate deduction from his superior owner.

(2) Speech at Birmingham, 24 November 1891. Since this was written a change of Government has taken place.

Taxation of
Mortgages in
New Zealand.

New Zealand in 1891, which contains provisions for the taxation of mortgages of land subject to a deduction by way of exemption of from £500 to £2,500, according to the amount of the mortgage, and to a further deduction when the owner of a mortgage, yielding more than £120 per annum, is "incapacitated by age, ill-health, or "other cause from earning further income from business or "employment" ⁽¹⁾.

A method has been proposed by Lord Hobhouse of a similar character to that which formed the basis of Mr. Channing's amendment to the Local Government Bill, with the exception that instead of dividing the rates equally between the owner and the occupier as proposed by Mr. Channing, he takes the proportion of one quarter for the owner and three for the occupier, giving the following illustration of the working of the principle:

"Take a house assessed to the rates at £200 a year. Let A be the freeholder, M a mesne lessee paying rent £50 a year to A, and Z, the occupier, paying rent £100 a year to M. Assume the rate at 4s. in the pound. The parish will receive from Z 4s. for each of his £200, just as happens now. But when Z pays his rent to M, he will deduct the fair proportion, 1s., from each of M's £100, and M will deduct the same amount from each of A's £50. Thus, to the £40 taken by the parish, Z will contribute £35 for his occupancy within the parish and for his profit value of £100, M will contribute £2. 10s. for his profit rent of £50, and A £2. 10s. for his ultimate rent of £50. Of course the same principle is applicable whatever may be the number of persons interested in the house and whatever the proportions of their interests" ⁽²⁾.

It is claimed for this method that it combines simplicity with universal application; that the principle is already adopted in Schedule "A" of the Income Tax Act, with which everyone is familiar; and that its adoption would ensure that the actual beneficiaries would contribute their proper share to the assessment.

In spite, however, of the apparent fairness and simplicity of this scheme, it is liable to three practical objections which apply equally to the proposals of the London County Council. In the first place it must be remembered that it is customary to levy rates twice a year and to vary the amounts, which are often fractional, at each assessment. At one period the rate might be

⁽¹⁾ Land and Income Assessment Act, Schedule A.

⁽²⁾ Art. on "The Local Taxation of Rent in London", in the *Contemporary Review* for July 1888.

2s. 3½*d.* and at another 2s. 7¾*d.* in the pound, and to calculate the proper deduction to be made from the rent would cause an immense amount of inconvenience and trouble as well as possible error, which would be repeated through successive tenants. Some rates, too, might be levied at one time and others at another,

while rack rents and ground rents frequently differ in the mode of payment. Hence, endless confusion would arise, and few people would care to undertake the management of house property, which would, consequently, become depreciated in value. Secondly, it would be necessary for the tenant, in order to claim his right of deduction, to produce the receipt or receipts for the rates, with the double object of showing the proper amount to be deducted from the rent, and of proving that the payment had been duly made. It is not improbable that receipts would be sometimes mislaid, and that their non-production would be a source of annoyance and unnecessary friction to all the parties concerned. Thirdly, it may be remarked that if instead of the ground rent being £50, as given in Lord Hobhouse's illustration, it were only a peppercorn, such as 1*s.*, the ground landlord would be liable to contribute, as his share of the rates less than 1*d.*, while the mesne lessee, whose interest is limited to a terminable annuity and who has thus no reversion to be benefited from the rates, would have to pay the sum of £4. 19*s.* 11*d.* in respect of the rent which he receives from the occupier. Such an anomaly cannot fairly be considered an improvement upon the existing system, since its only effect would be to shift the injustice from the shoulders of the occupier to those of the leaseholder, who, in the course of time, might be in a position to shift it back again to the occupier by means of the simple process of raising his rent by the amount of the rate. It must, however, be conceded that, in spite of the existence of these objections to Lord Hobhouse's scheme, his principle remains unassailed, and at some future time might be developed in another and better form.

It will be observed that both in the scheme of the London County Council and in that of Lord Hobhouse, the assessment on owners is to be made by way of deduction from rent. For this method there seem to have been several precedents in former legislation. During the period which elapsed between 1675 and 1734, various local Acts were passed for the erection of churches, which empowered tenants to deduct a fixed proportion of the statutory rate from their rents.

**Practical
Objections.**

**Precedents for
the Deduction
of Rates from
Rent.**

Similar deductions were authorized by the Irish Poor Laws of 1838 and 1843, the Metropolis Management Acts, 1855 and 1862, as well as by the Rating Act, 1874, so far as the levy of poor rates on certain mines is concerned ⁽¹⁾. Provisions relating to the deduction of a private improvement rate from the rent are, as we have already noticed, contained in the Public Health Act and in subsequent Acts dealing with the housing of the working classes.

But besides this method of levying rates on owners of ground values, a scheme for their direct assessment has been proposed by Mr. Fletcher Moulton, Q.C., who maintains:

- Mr. Moulton's Scheme.**
- (1) That in towns the land and buildings should be assessed separately according to their respective annual values.
 - (2) That the rates levied upon the land and buildings respectively in accordance with these assessments should be separate and distinct.
 - (3) That the rate levied upon the land should be borne by the owners of the ground values, each such owner paying personally the rate upon the ground values owned by him; and that to effect this in cases where the rate is collected from the occupier, each occupier or lessee should be entitled to deduct it from the rent payable to the landlord, and that all arrangements to the contrary should be illegal.
 - (4) That, subject to modifications in special cases, existing leases should be included in the assessment.

His grounds of contention in favour of these proposals are the injustice of the existing system of rating, the great increase in value of urban land, the present immunity of landowners from payment of rates, and the fact that the scheme is warranted by precedent ⁽²⁾.

Summary of Proposals. We have now discussed three schemes which have been proposed for revising the incidence of local taxation as it at present exists. (1) The scheme brought forward by Mr. Channing in the House of Commons, and afterwards adopted by the London County Council. (2) Lord Hobhouse's scheme, based on similar principles, and differing from Mr. Channing's only in the proportion to be levied on the

⁽¹⁾ See notes to Lord Hobhouse's Article.

⁽²⁾ *The Taxation of Ground Values*, pp. 8-14.

owner. (3) Mr. Fletcher Moulton's scheme for taking the ground value as the proper basis of the division of rates.

The effect of each scheme upon the different parties interested in the property subject to assessment will be best seen by the following illustration.

A house is assessed at £500 a year to the occupier D, the rates being 4s. in the pound and the ground value £350; D pays to C, a leaseholder, a rent of £400; C pays to B, a superior leaseholder, a rent of £250; and B pays to A, the freeholder, a ground rent of £50.

Effect of each
Scheme upon
Owner and
Occupier.

By whom Rate borne	INCIDENCE OF RATES UNDER								
	Scheme (1)			Scheme (2)			Scheme (3)		
	£	s.	d.	£	s.	d.	£	s.	d.
A	5	0	0	2	10	0	10	0	0
B	20	0	0	10	0	0	40	0	0
C	15	0	0	7	10	0	20	0	0
D	60	0	0	80	0	0	30	0	0
Total	100	0	0	100	0	0	100	0	0

A fourth scheme for rating ground rents has been suggested by Mr. Robert Hunter⁽¹⁾. He contends that, inasmuch as the increase of buildings destroys open spaces and pollutes the air, means should be found to make this increase supply the funds to

provide open land and pure air. He therefore
 Mr. Hunter's
 Scheme. proposes that the additional value acquired by the land by the erection of buildings upon it should be taxed by the levy of a rate of 2s. in the pound on all ground rents or increased annual land values derived from building operations. The tax would be levied on the occupier in the ordinary way, a deduction being made of the amount from his rent as in the case of property tax. The principle, it will be observed, is the same as that adopted by Mr. Montagu in his Metropolis Rating Amendment Bill. Mr. Hunter estimates that the area of land annually built upon in the Metropolis is about 1,100 acres, yielding a total increased rental of fully £120,000, which is equal to about an eighth of the total additional assessment of Greater London. On this estimate the proposed rate would amount to £12,000

⁽¹⁾ Art. on "A Fair Taxation of Ground Rents" in the *Nineteenth Century* for July 1891.

in the first year, £24,000 in the second, and so on, till at the end of 20 years it would be £240,000, which sum would go a long way towards the object he has in view. He suggests that the rate should be levied by the County Council, and should cease at the end of 50 years.

In a paper read before the Surveyors' Institution in 1889, Mr. George Beken contends that the proportion payable by the owner should be calculated as falling due at the end of the lease when the full benefit of the improvement is reaped by the owner, and he therefore proposes, in the case of existing leases, the following mode of procedure :

**Mr. Beken's
Proposals.** "Eliminate the proportion of the rate per pound appropriated to paying off the improvement loan raised since the date of the lease.

"Debit the lessor with this proportion of the rate, probably a fractional of a penny in the pound on the rateable value.

"Credit the lessor with interest on all previous contributions, they being so much capital sunk by him, in which the lessee takes the present benefit.

"Credit the lessor for waste.

"Credit the lessor, if the works prove a failure or become obsolete, with a refund in respect of any payments that may have been made on the assumption that he would reap a benefit from them at the end of the term" ⁽¹⁾.

In another paper read before the same Institution by Mr. Wm. Mathews, the following principles which should govern local taxation are laid down :

**Principles
laid down
by Mr. Wm.
Mathews.** 1. That owners of property, upon whom so large a proportion of local taxation is ultimately incident, should have an equitable share in the election of those who levy the taxes and in administering the expenditure.

2. That some of the taxes should be levied on the landlord and others on the tenant, a division of the same tax being a clumsy expedient. The county rate might with advantage be levied on the land, and the maintenance of main roads thrown entirely upon it. In other areas all those taxes which are levied for the purpose of contributing directly to the improvement of property, such as sanitary and improvement rates, should be charged upon the land, subject to proper regulations for preventing injustice as between lessor and lessee.

(1) *Transactions of the Surveyors' Institution*, vol. xxi, p. 359.

3. In every taxable area there should be one valuation list only for all purposes of taxation, whether local or Imperial. The present system of charging income tax on gross rentals should be abandoned, and taxation levied on the rateable value as set forth in the valuation lists.
4. The assessment of personalty for local purposes would no doubt be attended with difficulties, but there is no reason to think they would prove insuperable. A large amount of property of this description producing profit escapes all contribution to local taxation, and it is difficult to see on what ground of reason or justice it should continue exempted. In the United States realty and personalty are both assessed to state, county and city taxes, the assessments being made on capital, not on the annual value—a method of assessment which avoids many of the inequalities of the English system ⁽¹⁾.

The difficulties, both theoretical and practical, which surround the different proposals which have been put forward, have been very ably and impartially summed up by Sir Thomas Farrer in the following words:

“First, the relation between the occupier who pays the rates and the owner or owners is not a simple one, especially in London and other great towns. There is often lease behind lease, and the complete ownership is made up of a series of different interests, beginning with the actual occupier and ending with the freehold reversioner. This makes it necessary, in providing for the incidence of taxation, to spread it as fairly as possible over all the interests concerned. It is not only the ultimate freehold, but intermediate beneficial interests also, which ought to bear their fair share of the burden of local taxation.

“Secondly, the taxes, the burden of which has to be disturbed, differ widely in their application, and in the way in which their expenditure benefits different interests in the land; but they differ in this respect by varying degrees, so that it is impossible to say that one tax ought to fall on one particular interest, whilst another ought to fall on a different interest. The effect of the expenditure of a lighting or paving rate is comparatively transitory, and that of a rate for the purposes of main drainage or of the Thames Embankment, is comparatively permanent. But the benefit of neither of them is coincident with any particular interest of occupation, lease, or ownership.

Difficulty of
fairly Assessing
Intermediate
Lessees.

Varying
Benefits
Derived from
Rates.

⁽¹⁾ *Transactions of the Surveyors' Institution*, vol. xviii, p. 291.

Moreover, besides such rates as these, there are a number of rates, *e.g.*, poor rate, school rate and police rate, which benefit all the successive interests in the land. This consideration seems to me to make it impossible to say with the Bill which bears Mr. Montagu's name, that certain special rates for permanent improvements ought to be charged on the freeholder, and certain others on the occupier.

"Thirdly, there exists at present in this country an established system of valuation and rating, moderately well understood by the assessment committees, whose business it is to make the valuations. It is an assessment of annual value; it is avowedly based on rent, and has, in the actual rents obtained in the market, a certain basis and test. When it is suggested that rates shall be based on assessments of capital values by taking the whole capital value of the land and houses, and then charging them according to a given rate of interest on the capital value, it is no doubt conceivable that such a system might be adopted. But it would be a complete subversion of the system which now exists, and would need a very different machinery. This objection appears to be fatal to many of the proposals which have been made for taxing capital values by means of annual rates.

"Fourthly, a similar objection seems to be fatal to Mr. Moulton's ingenious plan of assessing and taxing the land and the buildings upon it separately. Some valuers say they can do it. But the only ultimate bases of a valuer's knowledge are actual market values, and as the land and the houses upon it are sold and let together, no such basis can exist for a separate valuation of the two things. Moreover, the question is not so much a question between the owner of the house and the owner of the land, as a question between the owner and occupier of both house and land.

"Fifthly, there is, nevertheless, a sound foundation for the contention of the many persons who are dissatisfied with a mere taxation of rents such as was proposed by Mr. Goschen in 1870. Rent and capital value are two different things; and rent is often no test of capital value. In the case where a lease has been paid for by a premium; in the case of land which can be, but has not been, built on; in the case of an old lease in an improving quarter; in the case of land in the neighbourhood of towns, and in all the pleasant residential parts of England; the actual rent, which is often a mere agricultural rent, bears no proportion to the real value; and if rent alone were taxed in proportion to its amount, there would be a large quantity of real property which would escape local taxation altogether.

"Sixthly and finally, there is the consideration that we have to deal, not only with new contracts, but with existing contracts. As regards the latter, it is true, on the one hand, that many, if not most, existing leases contain stipulations that the occupier shall pay the rates; and it is also true, on the other, that we are certain, in the case of existing contracts, that the incidence of new and unforeseen burdens falls on the occupier and not on the owner. This sixth and last consideration is one which raises

**The Taxation
of Capital.**

**Separate
Assessment of
Land and
Buildings.**

**Rent no test of
Annual Value.**

**As to existing
Contracts.**

great difficulties—especially in cases where the interest behind the occupier consists of a fixed rent with little or no reversion, and where, consequently, such interest is not benefited by the expenditure of new rates ”⁽¹⁾.

The following suggestions as to legislation have been made by Lord Hobhouse, and approved by Sir Thomas Farrer and the Local Government and Taxation Committee of the London County Council:

I.—In the case of Future Contracts.

- | | |
|---|---|
| Recommenda-
tions of the
Local
Government
Committee of
the London
County Council. | <ol style="list-style-type: none"> 1. Some portion of the burden of the rates should be thrown on owners as distinct from occupiers. 2. All owners of whatever tenure, whether for years or for life, or in fee, should bear their due share. 3. Each owner should be charged upon the present amount of annual benefit accruing to him from the property assessed. 4. Each owner should pay part of the rate collected from the occupier by means of a deduction from his rent, according to the method used in the case of property tax. 5. Owners should contribute to the rates, not only in respect of improvements, but of other purposes. 6. Owners should pay, not different proportions of rates levied for different objects, but a fixed proportion of the whole. 7. A fair proportion should be charged upon owners. |
|---|---|

II.—In the case of Existing Contracts.

1. New taxes may be imposed without regard to private contracts.
2. Future increases of old rates may be treated as new taxes.
3. Rates existing at the date of a contract should be left under the operation of the contract; but new rates, or increased rates, imposed after the date of the contract and before the change of law, require special treatment ⁽²⁾.

⁽¹⁾ Art. on “Imperial and Local Taxation” in the *Contemporary Review* for December 1890.

⁽²⁾ Evidence of Sir T. H. Farrer before the Town Holdings Committee (1890), 1243–70, also 1332–45.

With regard to such of these suggestions as affect future contracts, it may be remarked that no exception can be taken to the principle that owners of property should be required to pay for any improvement which enhances the value of such property, provided—(1) that they are allowed to have a voice in the administration of the necessary funds which they are to contribute to such purposes; (2) that the proportion which is to fall upon them is fixed at the time the contract is made and guaranteed to remain unalterable during the continuance of such contract; and (3) that fractions of a shilling are eliminated from the owner's rate, which should be levied not more frequently than once a year.

Conditions
under which
Owners should
be taxed.

At the same time, it may be doubted whether any permanent advantage would be secured to the occupiers by shifting the burden of a proportion of the rates from their shoulders. In the first place, the deduction would cause some trouble, as has been already pointed out. Secondly, their share of the rates would probably remain but little changed. And thirdly, it is more than likely that their rents would be increased so as to include the owner's rate.

Doubtful
advantages
of proposed
change.

As regards existing contracts, however, it is difficult to understand the soundness of the principle underlying these suggestions. It is true that a modification of such a principle is to be found in the Income Tax Act, the Ground Game Act and the Agricultural Holdings Act, where any contract inconsistent with the provisions of these Acts is declared to be void. But it may be observed that, inasmuch as any injury which may be caused by the operation of these Acts is comparatively insignificant, and in the last two only of partial scope, the analogy which is thus sought to be drawn falls to the ground. In support of this contention it may be easily proved that the

As to the
treatment of
existing
Contracts.

insertion of any provision rendering existing contracts nugatory, would produce an immense amount of injustice and hardship upon mortgagees and purchasers of ground rents. In this class may be included municipal corporations, insurance companies ⁽¹⁾, hospitals, public schools and charitable institutions, who invest largely in this class of security, under the belief that the returns which it yields are perfectly secure and of fixed amount.

(1) A summary of the amounts invested in ground rents by the various insurance companies is given in an Article in the *Bankers' Magazine* for August 1892.

In considering all these various schemes it should not be overlooked that it would be obviously unfair to include in the assessment such property as chief rents and leasehold ground rents to which no reversion is attached. This description of property is of a similar character to that of debentures and terminable annuities, being of the nature of a rent-charge on the estate, no improvement of which can be of any possible benefit to the holder, whose income, if such a tax were levied, would be reduced by one-eighth, or whatever proportion might be fixed as the rate. This is quite contrary to the principle which constitutes the basis of a tax on ground rents, namely, that the owner should contribute towards the cost of improvements which will enhance the value of his reversion. It has, indeed, been suggested that a distinction might be drawn between property retained by the original landowners and that purchased in the market, the latter being exempted from assessment; but the difficulties attending such a distinction are too great to admit of its being adopted.

It is a favourite argument with the advocates of a division of the rates between the owner and the occupier, that this system is already in force in Scotland, and has been found to work very satisfactorily. On a close examination, however, of the Scotch system, it would seem that the parallel is very imperfect, and that, owing to the peculiar form of tenure which exists in that country, to reproduce this system in England would be both difficult and costly.

There is, in the first place, an important distinction to be observed in the use of the term "owner" in Scotland and in England. In the former country this term is applied, for purposes of rating, only to the "feuar" who feus the land of the superior lord, paying him for the use of the land a feu duty, similar to a chief rent in England, in perpetuity. The feuar sublets the ground when he has built upon it, and it is between him and the occupier that the division of rates is made, the feuar having no right to make any deduction except property tax from his payment to the superior or ground landlord.

Secondly, it may be noted that the division is an irregular one and only affects certain rates. For instance, the expense of sewers and drainage falls wholly on the owner, the police rate exclusively on the occupier, and the poor and school rates are divided equally between both parties. On the average, two-thirds of the rates are payable by the occupier and one-third by the owner.

The Assessment of Chief Rents and Leasehold Ground Rents.

Incidence of Rates in Scotland.

Thirdly, it should be mentioned that prior to the Education Act of 1872, the entire burden of providing a parish school in Scotland lay upon the owners alone, but since that Act came into operation, half the rate for educational purposes has been shifted on to the shoulders of the occupier.

Lastly, the method of imposing rates upon owners is to charge them direct with their share of the assessment ⁽¹⁾.

As an illustration of the incidence of local taxation between owner and occupier in Scotch Burghs, the following table relating to the assessment of Greenock for the year 1890-91 is given ⁽²⁾:

Practical
Illustration.

PAYABLE BY OWNER	
Particulars of Rate	Amount
	<i>s. d.</i>
Statute Labour	0 1
Lands Valuation	0 0 $\frac{1}{4}$
Registration of Births	0 0 $\frac{1}{2}$
Court House	0 0 $\frac{1}{4}$
Registration of Voters	0 0 $\frac{1}{2}$
Animals' Diseases	0 0 $\frac{1}{8}$
School Rates	0 5
Poor „	0 5
Water „ (Public)	0 1 $\frac{1}{2}$
Total	1 1 $\frac{3}{8}$

PAYABLE BY OCCUPIER	
Particulars of Rate	Amount
	<i>s. d.</i>
Assessment for Water, Watching, &c.	1 9
Prison Assessment (Municipal)	0 2
Statute Labour—Road Assessment (Maintenance)	0 0 $\frac{1}{2}$
Poor Rate	0 5
Lands Valuation	0 0 $\frac{1}{4}$
Registration of Births	0 0 $\frac{1}{2}$
„ Voters	0 0 $\frac{1}{8}$
Artizans' Dwellings Rate	0 5
Public Health	0 3
Animals' Diseases	0 0 $\frac{1}{8}$
School Rates	0 5
Water „ (Domestic)	0 9
Total	4 3 $\frac{1}{8}$

⁽¹⁾ See evidence of Messrs. Harrison and Logan before the Town Holdings Committee; also Mr. Wm. Macdonald's essay on "Local Government and Taxation in Scotland."

⁽²⁾ See Mr. Cameron's evidence (1891), 4,492.

We will now consider three other methods which have been proposed for the taxation of owners of ground values, namely, (1) the rating of vacant building land; (2) the levy of a municipal death duty; and (3) the taxing of betterment.

The first method was brought into prominence by the Royal Commissioners on the Housing of the Working Classes who, in the course of their Report, remark:

“In connection with any such general consideration of the law of rating, attention would have to be given to the following facts. At present, land available for building in the neighbourhood of our populous centres, though its capital value is very great, is probably producing a small yearly return until it is let for building. The owners of this land are rated not in relation to the real value, but to the actual annual income. They can thus afford to keep their land out of the market, and to part with only small quantities, so as to raise the price beyond the natural monopoly price which the land would command by its advantages of position. Meantime, the general expenditure of the town on improvements is increasing the value of their property. If this land were rated at say 4 per-cent on its selling value, the owners would have a more direct incentive to part with it to those who are desirous of building, and a two-fold advantage would result to the community. First, all the valuable property would contribute to the rates, and thus the burden on the occupiers would be diminished by the increase in the rateable property. Secondly, the owners of the building land would be forced to offer their land for sale, and thus their competition with one another would bring down the price of building land, and so diminish the tax in the shape of ground rent, or price paid for land which is now levied on urban enterprise by the adjacent landowners—a tax, be it remembered, which is no recompense for any industry or expenditure on their part, but is the natural result of the industry and activity of the townspeople themselves. Your Majesty’s Commissioners would recommend that these matters should be included in legislation when the law of rating comes to be dealt with by Parliament” (1).

The subject also formed part of the enquiry before the Town Holdings Committee, who have the matter still under consideration (2).

Opinions adverse to the proposal have been expressed by Lord Salisbury, Lord Cross and Mr. Goschen, on the grounds that (1) a new principle would be introduced into the law of rating by taxing capital instead of annual value;

(1) Report of the Royal Commission on the Housing of the Working Classes, p. 42. A minority of the Commissioners, however, dissented from this view.

(2) The Committee have now reported on this subject as follows: “The proposal to rate vacant building land on its capital value is a total departure from the existing basis of local taxation, and would be practically very difficult in operation.”

(2) the law would be successfully evaded; (3) it would produce injurious sanitary effects by reducing the area of open spaces near large towns (1).

In addition to those objections it may be urged that the change is unnecessary from the fact that, as a rule, owners are only too eager to attract builders to their property, which thus acquires a greatly enhanced value; and, further, that it would be unfair to levy a tax on property which is at present yielding no return to the owner.

The second scheme is thus described by Mr. Sidney Webb :

**A Municipal
Death Duty.**

“ A simple method of providing funds for London improvements by a death duty would be to empower the County Council to call upon the Inland Revenue Department to collect a ‘ local real estate duty ’ of so much per-cent upon the capital value of all interests in land and house property in the Metropolitan area, on each occasion when such interests pass by death, irrespective of whether such interests are freehold, copyhold or leasehold; whether the succession be absolutely terminable or in trust; whether the successor be related to the deceased or not. It would be advisable, both for the sake of avoiding trouble in collection and as part of the general fiscal policy to the time, to exempt all estates of which the aggregate value does not exceed, say £1,000. And, as the intention of the proposal is rather to reclaim for the public a portion of the property left by the deceased than to tax his heirs or legatees, the ‘ local real estate duty ’ should be made payable by the executor or administrator in the same manner and at the same time as the existing probate and estate duties ” (2).

It is estimated by Sir Thomas Farrer, who is in favour of the principle of the scheme, that the average length of life of a London ground landlord is about 25 years, and that the capital value of the land in the Metropolis is approximately £750,000,000. Hence, on these assumptions, the annual value to be taxed would be £30,000,000, and the tax taken at 3 per-cent would amount to £900,000 per annum. It is, however, suggested that the taxation should be on a graduated scale, so that at present it is impossible to arrive at any definite conclusion as to the amount which such a tax would produce (3).

**Estimated
Revenue.**

(1) See Supplementary Reports appended to the main Report of the Royal Commissioners.

(2) *The London Programme*, ch. xix, p. 202.

(3) Evidence before the Town Holdings Committee (1890), 1,428.

The third scheme, and one that within recent years has obtained considerable support in this country, is that of levying a direct assessment upon owners of reversions in respect of the increased value which the property assessed is expected to receive on account of a present expenditure on the part of local authorities.

Such an assessment is known as a betterment tax, **Betterment.** and is of common occurrence in many parts of the United States, as well as in some of the colonies (¹). The principle is not new to the English law, having been recognized in several important Statutes, including the Sewers Act of Henry VIII and the Act for the Rebuilding of London in 1667, both of which have already been noticed. A similar principle is the basis of the compensation clauses of the Artizans' Dwellings Acts and the Housing of the Working Classes Act, 1890, which charge the neighbouring houses with the amount of compensation for the demolition of an obstructive building.

The subject is also dealt with in the following passage from the Report of the Royal Commission on the Housing of the Working Classes:

"In this country the principle has been, to a small extent, adopted in the Acts of 1879 and 1882 (the Artizans' Dwellings Acts). The former provides that the arbitrator is to take into account, in estimating the amount of compensation to be given to an owner, any additional value given to the adjoining property of the same owner by reason of the destruction of his house which is in a bad condition. **Its adoption in the Artizans' Dwellings Acts.** This to some extent incorporated the principle, but only as regards the same owner. There is a clause in the Act of 1882, in which the provision is introduced that where an obstructive building is taken for the purpose of improving adjacent property, the improvements given to that property may be charged upon it in the shape of a rate in aid. The advocates of betterment think that in every case an adjoining property, if it can be shown to be improved, should pay more than the general rate of the town towards the improvement scheme" (²).

The Royal Commissioners believe the principle of betterment to be that "the rate should be levied in a higher measure upon property which derives a distinct and direct advantage from "an improvement instead of upon the community generally,

(¹) The working of this principle in the United States is very ably discussed by Mr. A. A. Baumann in his recent book on "Betterment." See also Ely, *Taxation in American States and Cities*, part 3, ch. iv.

(²) Report of the Royal Commission on the Housing of the Working Classes, p. 47.

“who have only the advantage of the general amelioration in the health of the district.”

In the United States of America various local Acts contain powers authorizing the construction of sewers or other improvements at the expense of the owners of the property benefited, no portion being paid by a tenant for years, even though he may possess the right of purchasing the fee at a specified price. In Pennsylvania a lease subject to a ground rent is taxed as an unincumbered estate in fee-simple, and if the grantee of such lease covenants to pay taxes and ground rents, such ground rent estate is not liable to taxation. The tax on the ground rent is assessed on the value of the ground rent estate, and not merely on its annual value ⁽¹⁾.

The system is also in force in the colonies of New South Wales and Victoria. In the former colony an Act for the improvement of Moore Street, Sydney, contains provisions of which the following is a summary:

1. A schedule shall be published showing the nature and extent of the proposed improvement, together with its estimated cost, and giving the names of the owners of the property adjoining on whom the contribution is to be levied.
2. The amount so contributed by the owners towards the improvement scheme shall be at least one-half of its cost, the balance being defrayed out of the special street improvement rate.
3. The share of each owner shall be determined by the position of his property, and the degree of permanent enhancement in its capital or annual value which the improvement is expected to produce.
4. Appeals against the assessment may be made to the Supreme Court, whose judgment shall be final ⁽²⁾.

It was on lines similar to the above that the London County Council proceeded, when they drafted their well-known “London Streets (Strand Improvement) Bill.”

Under Section 28 of this Bill, it was proposed that an arbitrator should be appointed by the Secretary of State for the purpose of making a report on the land and premises in the neighbourhood of the improvement, the area of which extended

⁽¹⁾ See Articles on “Betterment” in the *Contemporary Review* for May, June and July 1890; also in *Times*, 27 January 1890.

⁽²⁾ Webb, *The London Programme*, ch. xvi, p. 167.

The Strand Bill. from Lincoln's Inn Fields on the north to the Thames Embankment on the south, and from the Law Courts on the east to Somerset House on the west, but excluded all Government buildings. A list of owners of adjoining property was to be made on whom not more than one-half of the capital value of the total contribution was to be levied. The share to be paid by each owner was to be fixed by the arbitrator in proportion to the benefit estimated to accrue to the property on account of the improvement, and was to be in the form of an annual rent charge.

The Bill was introduced into the House of Commons by Sir John Lubbock early in 1890. It was read a second time on 18 March, when Mr. Ritchie, as representing the Government, made the following observations :

“ The principle involved is, no doubt, of very great importance, but it is one not unknown to the law at the present time, although only to a very limited extent. The cost of removing obstructive dwellings may be placed either partially or entirely on the remaining houses in the particular areas which are benefited by the removal of the obstructive dwellings. That is a betterment that does not apply to the whole of a town like London, or even of a locality. However, so far as it goes the principle of this Bill is recognized, although the Government consider that a sound principle may be so applied as to become oppressive and unjust” (1).

After the second reading the Bill was referred to a hybrid Committee of nine members, five being nominated by the House, and four by the Committee of Selection. The Committee thus appointed reported against the Bill, on the ground that no appreciable increase of value would accrue by carrying out the proposed improvement to any houses other than corner houses and the actual frontage of the improved and widened street, forming a very small portion of the betterment area. They suggested that the Council would attain their object in a more effectual manner by taking compulsory powers to buy up the land and houses adjoining the proposed improvements. A practical illustration of the wisdom of the course recommended by the Committee may be seen in the policy adopted in the construction of Queen Victoria Street by the Corporation of the City of London, who bought up the land on each side of the new street, and are now, in the greatly-increased ground values caused by the improvements, reaping the fruits of their sagacity.

Reception by the Government.

Report of the Select Committee.

Different Policy of the Corporation of London.

(1) Hansard, *Debates*, 18 March 1890.

CONCLUDING REMARKS.

In the foregoing pages an attempt has been made to consider the questions of leasehold enfranchisement and the taxation of ground rents in the light of history, political economy and social science, and to assign to the principal arguments for and against the different schemes such positions as their importance and interest demand. Many of these arguments may be said to rest upon the broad issues arising out of the principle of freedom of contract. The supporters of the proposed reforms contend that if a contract has been made with respect to one condition of things, and in course of time that condition of things entirely changes, then the contract should also be changed. On the other hand, there are those who hold that men of full age and competent understanding should have the utmost liberty of contract, and that this liberty should not be lightly interfered with. Why, they ask, should one of the parties be relieved of his responsibilities when rates increase or property rises in value, and the other party be deprived of any power to re-adjust his part of the contract when the reverse takes place?

Hitherto, it may be noted, these two proposals have been treated upon entirely different lines. But, inasmuch as they now occupy a prominent place in the authorized programme of one of the great political parties of the State, it may well happen that both may be included in the legislative measures of the same Parliament. In such an event the consideration of their relations to each other, and the order in which they should be taken, would form a necessary prelude to the satisfactory solution of the problems which have already been indicated in these pages. At the present time, however, in the absence of any definite schemes before Parliament, it would be premature to devote more than a passing allusion to these details. Three practical difficulties, nevertheless, suggest themselves for our notice. (1) If compulsory powers were given to tenants to purchase their holdings, upon what principles of justice could owners of ground values be taxed for improvements of which they might never reap the benefit? (2) If a tax on ground rents were imposed upon freeholders, would not enfranchisement be absolutely necessary in order that, when a re-adjustment of contracts took place, the leaseholders might not be compelled to recoup the freeholders for the additional burdens which the latter had borne during the continuance of the leases?

(3) In view of the imposition of such a tax having taken place, on what terms could enfranchisement be fairly carried out ?

Here, then, we must leave our subject, the future consideration of which will doubtless be renewed at no distant date both in Parliament and elsewhere, with the result, let us hope, that the practical wisdom and sense of fairplay by which our statesmen have been guided in the past, may secure great and lasting benefits to succeeding generations of the English people.

DISCUSSION UPON THE ESSAYS OF MR. KYD AND MR. TARN.

The PRESIDENT (Mr. A. Hendriks) said that whilst the enfranchisement of leaseholds was looked for by a large section of the public, there was a great division of opinion as to the mode in which it should be carried into effect. Whilst it was the wish of the meeting that the subject should be discussed with the greatest freedom, they should, so far as possible, confine themselves to personal and general views rather than trench upon anything approaching to politics. The papers had, according to custom, been confided to two referees, Mr. Browne and Mr. R. P. Hardy, whom he would ask to address the Institute.

Mr. T. G. C. BROWNE said they were to be congratulated upon having under consideration a subject of wider interest and less technical nature than those which generally came before them. It was, however, a matter of regret that no essayist had come forward to advocate what he must call the popular view of this question, and it was somewhat significant that, from a body which could claim to have some logical and accurate training in dealing with questions of finance, no member had been found bold enough to advocate leasehold enfranchisement or the taxation of ground rents. The writers had shown that no case had been made out by the advocates of either of these proposals, and the time during which the leasehold system had existed, and its wide diffusion, afforded presumptive evidence that it had been found of use among mankind. This view was practically taken by the Select Committee on Town Holdings of the House of Commons in 1889, and he thought the very crude notions of leasehold enfranchisement which had been embodied in such Bills as that introduced by Colonel Hughes could not be held to have damaged the deliberate conclusions of the Select Committee. The one argument in favour of interfering with leasehold contracts was that they did, in some cases, give rise to the perpetuation of unwholesome and unsanitary property. The proposal of Colonel Hughes distinctly did away with any power to deal with the fag-end of leases, just the period when the property would presumably be at its worst. During the last 20 years of a lease there was to be no power to buy up the freeholder's interest. If the argument that this reform was needed in order to do away with unsanitary property was to hold

good, the freeholder should have power to buy up the sag-ends of those leases when the property had become unsanitary. Assuming that the property had risen in value it would be for his interest to do so, but it would also be for the public interest, and he could in many cases afford to pay more than the value that the leaseholds would fetch in the market in order to effect what would be a public improvement. As things stood at present, if one leaseholder held out for an unfair value the whole plan was frustrated, and in many cases useful improvements thereby prevented. The taxation of ground rents as at present suggested was rather a crude attempt to alter the incidence of taxation, though he did not say that the incidence should not be carefully considered and re-adjusted from time to time. Much has been said against landowners whose land had gone up in value since their grandfathers bought it, but very little was heard of compensating those landowners who 25 years ago bought land in the eastern counties which would now sell for two-thirds or one-half of its cost. He should advocate neither the robbing of the one nor the compensating of the other. The question arose, if they were to deal with this class of property where were they to stop? Take, for instance, debenture stock of railways, which might be said roughly to represent the land upon which the railway ran. If 25 years ago the 4 per-cent stocks which were now selling at about 135 could be bought at par, or under par, was there any good reason for taxing land that did not also apply to taxing the increment in the value of debenture stocks? The incidence of taxation required constant reconsideration, because the composite parts of the national wealth of the country were constantly changing. Professor Thorold Rogers pointed out that in the middle ages land was of little marketable value, only a small portion was cultivated, and the chief value of a farm was in the buildings upon it, with the cattle and implements. Later on land acquired a greater relative value, but of recent years it had depreciated relatively, owing not only to agricultural depression, but also to the creation of other forms of wealth only remotely connected with the land. With regard to taxation of ground rents, it assumed an increase in the capital value. But in England they had always taxed income. In the United States the endeavour was made to tax capital, but that had been altogether a failure, the evasions being endless. It was very much easier to get at a man's income than at his capital. The main objections to the present system were that temporary incomes, either for life or during good health, and for terms of years, were taxed at the same rate as perpetual incomes. If there was to be any great change in taxation, he should advocate, instead of the wholesale breaking of contracts, that they should seek a remedy in a properly adjusted income tax.

Mr. R. P. HARDY said that looking at the highly debatable nature of the subjects before them, and bearing in mind their wide range, he must admit the almost impossibility of doing justice to them, or to the writers who had exercised such care and ability in marshalling the facts brought forward. He felt that they were precluded from fully discussing the question on the ground that it would be impossible to do so without disclosing their own political

proclivities, which he thought would be undesirable in that room. The various questions which had been made the subject of the Institute prize essays had been well selected. None of them had been made vehicles for the exhibition of mere actuarial gymnastics, or for the indulgence of the fascinating pursuit of mere problem solving. No better instance could be afforded than the two papers under discussion, embracing, as they did, an extensive survey of facts, patiently collated and thoroughly digested. Upon reading the papers he was struck with two ideas. The first was, how admirably contrived a piece of state-craft was the feudal system, and how it must have played no inconsiderable part in welding together the heterogeneous populations of these islands, as well as in exercising a strong formative influence upon the characters of its component units. It was also a fair inference that the various arrangements connected with real property, some of which were so often objects of ignorant merriment, arose solely and wholly in response to the demands for human convenience, and that they were in no sense Machiavelian contrivances for subjecting the people to the sway of the territorial lords. No doubt many of those arrangements had outgrown the dimensions of their original intentions, as well as a large portion of their political usefulness; but the remedy, in the case of the abuse of the leasehold system, was a more rigid enforcement of the municipal law of sanitation. The exaggerations upon both sides had been enormous. The landlords had, on the one side, stretched the case of their legal rights close up to the point of human endurance; and, on the other side, the tenants misrepresented their case. There was, however, a real difficulty in effecting any fundamental alteration in the leasehold system. That system must arise wherever lands were so costly that the builder could not afford to embark the additional capital required. The remedy was a modification of the Scotch system of feuing. With regard to the taxation of ground rents, he objected in principle to any endeavour, whether under shelter of a contract which the courts would enforce or not, to shift that personal obligation which should always attach to citizenship on to the shoulders of another. It was almost impossible to find a tax which would fall upon the ground landlord and not be thrown back in some shape upon the tenant, with the exception of a death duty. That could be wholly avoided by the tenant, and it was in that direction that the taxation should be imposed.

Mr. A. G. MACKENZIE considered that as questions of politics often involved questions of finance, the Institute, which was a body of financiers, contained many members who might assist the public in counting the cost of some of the schemes proposed for adoption. Of the questions with which the papers dealt the one most in the public mind was that of taxation of ground rents. He did not know what proportion the ground rents bore to the rateable value of the Metropolis, but assuming on a liberal basis that it was 20 per-cent., and that the rate charged on the present rateable value amounted to 30 per-cent., a similar rate upon ground rents would produce 6 per-cent., or a trifle over one shilling in the £, while nearly one-third of the landlord's income would disappear. The ambitious schemes which had been suggested could not be carried out under a charge of

three or four shillings in the £ upon the rateable value, so that if the ground landlord were to pay for them his income would be swept away altogether. Leasehold enfranchisement, if carried out upon a fair principle, must be made voluntary and a matter of contract. If these proposals were dealt with by Parliament, extreme care would have to be taken in any alteration not to introduce greater anomalies than at present existed.

Mr. C. F. DOWSETT (a visitor) thought, with regard to the propositions put forward for the enfranchisement of leaseholds, that to make them equitable the following conditions should be imposed, and these would render them practically futile. First, the full market value of the ground rent must be paid. Next, there must be the payment of at least 10 per-cent for compulsory purchase together with all costs. Liberal and ample compensation must also be made for severance, because estates would be very much prejudiced unless that were done. Again, in any power of purchasing ground rents, the ground rent ought to be assessed at its then value, not at the value being paid for it, but at the value which the land would fetch if vacant and in the market. Lastly, the freeholder should have the same privilege as the leaseholder. If power was to be given to the leaseholder to buy out the freeholder, power should be given to the freeholder to buy out, if necessary, the leaseholder. The question of the taxation of ground rents had been thoroughly discussed. Ground rents were taxed already, and to add an additional tax would be an injustice.

Mr. E. SMYTH remarked that improvement loans were usually to be repaid in a materially shorter period of time than that for which the improvements effected might reasonably be expected to last. It was this discordance which had given rise to the objection quoted in Mr. Kyd's Essay that "Rates are raised not merely for current expenses, but to pay off outlays incurred in improvements which will outlast existing leases, and benefit the freeholder when the reversion falls in." It had been proposed that leaseholders should be relieved of this hardship by making the reversioners pay their proportion of cost at the present time. The awkwardness of this proposal was that it required reversioners to pay at a time when they were deriving no income whatever from the improvement that had been made. The repayment of loans should be prolonged for the length of time that the improvements themselves might reasonably and moderately be expected to last. Again, the mode of repaying loans should be not by a terminable annuity with equal annual payments, but by one of diminishing payments, as, for example, that wherein each year's payment contained an equal amount of capital. Supposing the improvement was a structure, it became of diminishing value, because it was subject to continuously increasing decay, the repairs of which would fall upon the future owner. If it was merely a street improvement its importance was very likely to become reduced in course of years in consequence of alterations in the locality. Such a plan as he suggested would be just, because it would call upon the ratepayer in any year to pay only such proportionate part of the cost as he enjoyed himself during that year. It would be far better to adopt that plan than to attempt to cope with the bewildering intricacies which were involved in any plan for apportioning the rate. With regard to

Mr. Kyd's remarks on copyholds, it should be understood that there was no power on behalf of copyholders to enfranchise. There were two notoriously different classes of copyholds—one where the copyholder had a right of renewal, and in those cases the copyholder could compel the lord to grant enfranchisement, or the lord could compel the copyholder to accept enfranchisement; but in a vast number of other copyholds there was no right of renewal, and, therefore, no power was given to either party to force the other. The latter class of copyholds was exactly on a par with leaseholds, of which it was, most improperly as he considered, sought to obtain compulsory enfranchisement.

Mr. A. H. BAILEY, speaking on the question of the taxation of ground rents, said he was informed that the whole of the money for the erection of the Thames Embankment was advanced by the Bank of England for a term of forty years, repayable not by way of annuity, but by annual instalments of principal, a process involving, he presumed, a decreasing charge upon the ratepayer. Seeing that it had materially improved the property of several freeholders, was it right that the latter should go entirely scot free? He recognized the extreme difficulty of apportioning between the one and the other, but at present the charge was levied entirely upon the occupier. Other instances could be given. The rates under the School Board Act came entirely upon the occupiers of houses. They all admitted the need of more thoroughfares between the Strand and the northern railway stations; it would be a very expensive operation, and who was to pay for it? Under present arrangements the money would be borrowed, and a charge levied upon the rates. It was vain to say that the occupier was prepared for such a charge. It came upon him, and it materially improved some of the property of the owners. It was said taxation should be on income, but it must be remembered the value of the reversion was materially improved. It was not true that it was merely taxing income if the property was so improved. He admitted the extreme difficulty of apportioning the burden between the landowner and the tenant, but he had given those illustrations to show that there was a word to be said on the tenant's behalf.

Mr. A. G. MACKENZIE explained that he referred to the injustice of taxing ground rents: he did not at all object to taxing reversions.

Mr. C. D. HIGHAM pointed out that if they once began breaking contracts, it might lead them further than they anticipated.

Mr. H. W. MANLY, as the proposer of the particular subject for the prize essays, expressed his gratification at the result of the competition. He was a little disappointed that the essayists had not discussed more fully the difference between ground rents and other classes of property, and the reasons why they should or should not be dealt with differently. He would first consider the question of national taxation. Land was subject to special laws, and for a great number of years there had been preferential laws as regards taxation. As the owners of land had made the laws in the past, so they had relieved themselves of many of the burdens, especially the death duties, which had been imposed upon other classes of property. He thought the taxation of land should follow exactly the same lines as the taxation of other classes of property, and when property in land contributed the same death dues as other kinds of property, the

plea for taxing land, so far as national taxation was concerned, would at once be removed. With regard to municipal or local taxation, the principle of local government was that all improvements were for the benefit of the community, and that each member of the community should contribute according to his power of paying. It was roughly estimated that each person would pay such a rent as he could afford, and that, therefore, the rent of his house was the measure of his power to pay. A man whose income was derived from ground rents contributed to local taxation according to his power measured by the rent of his house. The difference between this man and one whose income was derived from consols was that the latter did not get his consols increased in value while the former found his reversion improved. If, therefore, they were to tax the landowner for permanent improvements (and they could not justly tax him for anything else), they must tax his reversion.

The PRESIDENT, in proposing a vote of thanks to the authors, said the rule of the Institute that subjects should be discussed from a non-political point of view had been admirably maintained in the discussion. There were few questions with which Parliament would have to deal in the coming Session of greater importance than that of the enfranchisement of leaseholds, and he hoped that whenever this was approached, and by whatever party, there would be justice done as between landlord and tenant. The increment in the value of property did not invariably come from the tenant. A certain property in the heart of the City of London, which was sold recently, realized a sum greater than anything that had been reached in that particular locality, namely, the rate of £62 to the superficial foot. Supposing the ground rent had been fixed 30 years ago at a rate corresponding to the then value of the property, it would be unjust to indemnify the freeholder upon such a basis with no allowance for increment. It might be clearly demonstrated, particularly in large cities, that the increment arose quite as often from the efflux of time and the improved value of City property generally as from any act of the tenant. He thought that the system of greatly extending the term of payment for present improvements was one that ought to be deprecated. Only a few years ago the extreme number of years for which they were asked to lend money to local boards for local improvements was 30 years, and the term was more often 20 years. Now they had applications for loans extending over 40, 50, and even 60 years, every endeavour being made to throw the obligation upon posterity.

Mr. TARN having briefly acknowledged the vote of thanks on behalf of Mr. KYD and himself, the meeting terminated.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Progress of Profit in a Life Assurance Fund. By
THOMAS J. SEARLE, A.I.A., *Accountant.*

[Read before the Institute, 27 February 1893.]

I. ACTUARIAL BOOK-KEEPING.

THE principal difference between the accounts of an insurance company and those of an ordinary trader lies in the fact that the trader buys before he sells, while the insurance company sells before it buys, that is to say, the insurance company first receives the money on any transaction, and afterwards pays out. From this it arises that the trader keeps account of his liabilities by book-keeping and ascertains his assets by taking stock, while the insurance company keeps account of its assets by book-keeping and ascertains its liabilities by taking stock. There are certain classes of trade, chiefly brokerage, where there is either no stock, or a stock not needing stock-taking, in which the trader can keep account both of his liabilities and of his assets by book-keeping, and has thus a far better knowledge than he otherwise would of the sources and the progress of his profits. The purpose of this paper is to suggest that an insurance company, including even a life office, may keep accounts of both assets and liabilities by book-keeping methods, and may thus have clear information as to the sources and progress of its profits.

With a fire or accident company there is no difficulty. The difference is one rather of principle than of practice. All that is necessary is to separate the premiums when received into two portions, one belonging to the current financial year and the other to succeeding years. When preparing the accounts of an accident company, I have always held that the "unearned premiums" should be reserved, and not an estimate for "unexpired risks." The latter is the stock-taking method, which is convenient by its elasticity and therefore dangerous. All stock-taking in trade is elastic, and therefore convenient and dangerous. The book-keeping method of ascertaining the liability of a life office need not destroy the elasticity, but it will mitigate the danger, because it will require that the profit and loss account shall contain a distinct entry showing the profit or the loss from any change of basis.

For convenience, we may call the life accounts, as at present kept, the manager's accounts, and the accounts now proposed the actuary's accounts. It will be best to keep the two sets of accounts separate, and they will be at their points of contact counterparts of each other—where the manager leaves off the actuary will begin. The manager's revenue account begins with an amount to the credit of the life assurance fund, that is, to the credit of the actuary. The account is credited with premiums, interest, and so on, and debited with claims, surrenders and so on, and closes with a balance to the credit of the fund, that is, to the credit of the actuary. The actuary, on the other hand, commences with a balance to the debit of the fund, that is, to the debit of the manager; he *debts* the manager in his revenue account with premiums and interest, and *credits* the manager with the amounts paid for claims, surrenders, and other outgo, leaving the balance to the debit of the manager. The manager's balance sheet has, as at present, assets to the debit side, and the fund to the credit. The actuary's balance sheet will have the fund to the debit, and the assurance liability, undivided profit, and other liabilities, if any, to the credit.

The actuary will commence his journal and ledger with his balance sheet at a given date, and, as this involves an entry of the life reserve or assurance liability, he must perforce begin, except in the case of a new company, at a valuation date. He will then put his revenue account through, premiums going to the debit of the fund, and to the credit of "office premiums", and so on. Premiums will then be transferred to the credit of "net

premiums", "extras", and "loading" respectively. Commission and expenses will come to the debit of loading, and profit on loading will appear as a balance, waiting to be transferred to "profit and loss." A sum of interest (to be ascertained as hereinafter explained) will be transferred to the credit of "assurance liability", and the interest left will be interest profit. The "assurance liability" will be debited with the year's contribution to mortality and the reserve values of the policies become claims, both of which will be carried to the credit of the claims account, which account will then show the profit on mortality. The reserve values of new policies, surrendered and lapsed policies, and some minor matters will be carried from the "liability" account to the debit or credit of their respective accounts, and the balance of the "liability" account will show the "assurance liability" at the end of the year. The above is a very rough sketch of the entries to be made, and not sufficient to work upon. In practice there will be many refinements, which cannot be discussed here within any reasonable space. Some of the points raised will be considered at greater length presently.

The actuarial ledger having been thus posted up, it will be closed, and a profit and loss account and balance sheet will be made out in the usual way. The profit and loss account will be as follows:

<i>Dr.</i>	<i>Profit and Loss.</i>	<i>Cr.</i>
	£ s. d.	£ s. d.
To Policy Alterations	By Balance brought down (undivided Profit)	
„ Bad Debts	„ Mortality	
„ Changes of Basis	„ Interest	
„ Balance (profit to date)	„ Loading	
	„ Surrenders	
	„ Dropped Policies	
	„ Bonus Surrenders	
	„ Policy Alterations	
	„ Extra Premiums	
	„ Fines and Fees	
	£	£

The balance sheet will be as follows:

<i>Dr.</i>	<i>Balances.</i>	<i>Cr.</i>	
	£ s. d.	£ s. d.	
Assurance Fund . . .		Assurance Liability . . .	
Interim Bonuses . . .		Profit and Loss (as above)	
Other Accounts (if any) .		Other Accounts (if any) .	
	£		£

It will be noticed that the balance sheet is the same as the valuation balance sheet of Schedule V, which has hitherto been a phantom balance sheet, having no ledger behind it, but which would become a real balance sheet of a real ledger.

The method is not proposed as a substitute for the periodical valuation. If it were, it would be liable to criticism, because any errors (in taking out the reserve values of the claims, for instance), would be compounded and carried on for ever. It will be checked by the periodical valuation, and will in return form a very stringent check upon the valuation, for which, I believe, there does not exist at present any precise and thoroughly independent check. Without waiting for the valuation, the method proposed will ascertain from year to year the amount and sources of profit.

The obvious difficulty in the way of its application is the computation of the annual contribution to mortality, which will be shown to be the same thing that Mr. C. D. Higham has brought into familiar use as the "expected strain." No plan has been yet suggested for obtaining the "expected strain" without first making the valuation at the end of the year, a course which would be fatal to some of the objects now contemplated. The solution of the difficulty lies so near the surface that any actuary looking for it would have found it. The valuation at the beginning of the year can be easily accumulated by the help of what I have called "mortality multipliers", which will give the "contribution to mortality", and thence the valuation at the end of the year by means of the ledger account already described.

The method does not exist merely in theory. Through the very great courtesy of Mr. James Chisholm, and his well-known interest in the practical application of scientific developments, I

am indebted to the Directors of the Imperial Life Office for the highly valued opportunity of professionally applying it to the participating policies of that company for a period of five years between two valuations, completely reconciling the earlier valuation with the later one, and dividing the profit into all its various sources with the certainty which attaches to a book-keeping method.

It may be well to state that this paper has nothing whatever to say to any particular modes of valuation. The only duty of the new method to the valuation is to interpret carefully the bases adopted and processes employed, whether these relate to mortality tables, rates of interest, premiums to be valued, assumed ages, terms still to run, formulas or anything else, and to work on exactly the bases employed in the valuation, any difference between the methods of the two valuations being allowed for in the accounts as "change of basis."

II. MORTALITY MULTIPLIERS.

The factors thus called are the values of

$$k_x = \frac{1}{p_x} - 1 = \frac{1 - p_x}{p_x} = \frac{q_x}{p_x},$$

whence

$$(1 + k_x) = p_x^{-1}.$$

The following special notation will also be convenient; as we shall not be concerned with policy-values in any connection with the ages at entry, but only in connection with the valuation age, let V_x = the value of any number of policies at the valuation age x , say the values of all the policies on a particular page of the class list: let S = the sums assured and bonuses, and P = the premiums to be valued on the same policies. Let $(V_x + P)(1 + i) = V'_x$.

The purport of the notation of k_x may be seen in its simplest form, as follows:

$$a_x = v \cdot p_x(1 + a_{x+1}),$$

$$\therefore a_x(1 + i)(1 + k_x) = (1 + a_{x+1}) \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and} \quad P \cdot a_x(1 + i)(1 + k_x) = P(1 + a_{x+1}) \quad . \quad . \quad . \quad . \quad (2)$$

where i , the interest, and k_x , the mortality, are similarly involved.

$$\text{Also} \quad A_x = v \cdot q_x + v \cdot p_x A_{x+1}$$

$$\therefore \quad A_x(1+i)(1+k_x) = k_x + A_{x+1} \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and} \quad S \cdot A_x(1+i)(1+k_x) - Sk_x = S \cdot A_{x+1} \quad . \quad . \quad . \quad (4)$$

then, since

$$V_x = SA_x - P(1+a_x) = S \cdot A_x - Pa_x - P$$

$$S \cdot A_x - P \cdot a_x = V_x + P,$$

and, subtracting (2) from (4), we have

$$(V_x + P)(1+i)(1+k_x) - Sk_x = V_{x+1} \quad . \quad . \quad . \quad (5)$$

$$V'_x(1+k_x) - Sk_x = V_{x+1}$$

$$\therefore \quad V'_x - (S - V'_x)k_x = V_{x+1} \quad . \quad . \quad . \quad . \quad (6)$$

It will be noticed that the formula does not require any previous relation between the S and the P , both of them being only class-book totals for policies of many different durations, the S including bonuses, and the P being office premiums, reduced premiums, or net premiums, whatever premiums are valued to get the V_x . Further, the future status of the policies is quite immaterial, provided only that V_{x+1} is ascertained on the same principles as V_x , and that S is the amount at risk throughout the year, and P the annual premium payable at the commencement of the year, or with slight modification, as will be shown, at any time or times during the year. The formula therefore applies to assurances, premiums or annuities, immediate, deferred or limited, increasing or decreasing, or varying after the current year in any possible way. It has always been the practice, in taking out expected claims, to include all policies on single lives in one calculation, and the new formula will do the same, not only for the contribution to mortality, but also for carrying on the valuation from one year to the next.

A numerical illustration will be useful.

A_{gc} $_x$	V_x	P	$V_x + P$	$(V_x + P)i$	$\frac{(V_x + P)(1+i)}{= V'_x}$	S	$S - V'_x$	$(S - V'_x)k_x$	V_{x+1}
50	16,315	865	17,180	515	17,695	50,266	32,571	528	17,167
49	11,823	1,153	12,976	389	13,365	52,739	39,374	609	12,756
48	9,972	931	10,903	327	11,230	44,837	33,607	492	10,738
...	38,110	2,949	41,059	1,231	42,290	147,842	105,552	1,629	40,661

The interest is 3 per-cent, and the k_x values are those of the H^M mortality, from a table appended to this paper. The figures given, in point of fact, include ordinary whole-life assurances, increasing premiums, limited payments and endowment assurances, less premium-reductions and re-assurances. They also provide for the whole-life loading on the limited payments, because, in making up the premium total, the office limited premiums were included and the whole-life loading was deducted.

A table, such as the specimen, for every valuation age on the books, gives the whole history for twelve months of all the policies on single lives continuing through that period, and dispenses with any necessity for a yearly valuation of limited payment and endowment assurances. The totals, besides checking the work, give the material for the ledger entries, thus:

<i>Dr.</i>		<i>Liability.</i>		<i>Cr.</i>
	£		£	
To Mortality	1,629	By Balance from previous		
„ Balance to following Year	49,661	Year	38,110	
		„ Net Premiums	2,949	
		„ Interest	1,231	
	<u>£12,290</u>		<u>£42,290</u>	

The equation on page 321 of the *Text-Book* (Part II, —

$${}_nV_x + P_x = v(q_{x+n} + p_{x+n} {}_{n+1}V_x)$$

becomes, in the notation we are using,

$$V_x + P = v Sq_x + p_x \cdot V_{x+1},$$

and from it, as Mr. Meikle has kindly pointed out to me, my formula (6) can be readily obtained. The formula of the *Text-Book* is the same as that on which Mr. Higham founded, (*J.I.A.* xx, 154,) his formula involving the “expected strain”, which, changing the notation, is:

$$(V_x + P) (1 + i) = V_{x+1} + q_x (S - V_{x+1}) \quad \dots \quad (7)$$

It is strange that, until Mr. G. F. Hardy suggested it last session, so little attention should have been paid to the possibility

of using the left-hand side of the equation. I am not aware of it being used, otherwise than by myself, except by Mr. T. G. C. Browne, whose plans I had the privilege of inspecting when I began to study the subject for Mr. Chisholm.

Comparing equations (6) and (7), and remembering that the left-hand side of (7) is to be called V'_x , it is evident that

$$q_x(S - V_{x+1}) = k_x(S - V'_x) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

that is, that Mr. Higham's "expected strain" and my "contribution to mortality" are the same thing. Mr. Higham's "expected strain" has not been treated with due respect, in that it has been used comparatively only, to set against the "actual strain" as a piece of statistics, instead of being used positively as a step towards obtaining a profit and loss account. The latter view of it may be shown thus:

$$\begin{aligned} (\text{Mortality Profit}) &= (\text{Expected Strain}) - (\text{Actual Strain}) \\ &= (\text{Expected Strain}) - (\text{Claims} - \text{Claim Policy Reserves}) \\ &= (\text{Expected Strain}) + (\text{Claim Policy Reserves}) \\ &\quad - (\text{Claims}) \\ &= (\text{Contribution to Mortality}) + (\text{Claim Policy Reserves}) - (\text{Claims}) \end{aligned}$$

Equation (7) may be modified as follows:

$$\begin{aligned} V'_x &= V_{x+1} + q_x(S - V_{x+1}) \\ \therefore S - V'_x &= S - V_{x+1} - q_x(S - V_{x+1}) \\ &= (1 - q_x)(S - V_{x+1}) \\ &= p_x(S - V_{x+1}) \quad . \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

from which, by assuming $k_x = \frac{q_x}{p_x}$, as we have done, equation (8) immediately follows; or, if we had not yet given any value to k_x , and were in search of a factor which would produce the relation in equation (8), and so enable us to obtain the "expected strain" from V_x instead of from V_{x+1} , equation (9) enables us to do it. Thus:

$$q_x(S - V_{x+1}) = k_x(S - V'_x) \\ = k_x \cdot p_x(S - V_{x+1})$$

$$\therefore q_x = k_x \cdot p_x$$

$$k_x = \frac{q_x}{p_x} = p_x^{-1} - 1.$$

A numerical illustration of equation (8) may also help to fix the idea. Using the same figures as before—

Age x	S	V'_x	$S - V'_x$	$(S - V'_x)k_x$	S	V_{x+1}	$S - V_{x+1}$	$(S - V_{x+1})q_x$
50	50,266	17,695	32,571	528	50,266	17,167	33,099	528
49	52,739	13,365	39,374	609	52,739	12,756	39,983	609
48	44,837	11,230	33,607	492	44,837	10,738	34,099	492

The "Retrospective Method" of valuation has been discussed in the *Text-Book* and elsewhere, specially by Mr. Meikle, but I think always on the assumption that the mortality in the calculation was the expected tabular mortality, as for instance in the accumulation on page 316 of the *Text-Book*, exactly similar up to column 6 to that now proposed. The effect of variations has also been considered on the hypothesis of a percentage increase or decrease of the tabular mortality, but not on the basis of the ever-varying actual mortality experience. Recurring again to a numerical illustration: by the new method the accumulation runs thus:

Age x	V'_x	$(S - V'_x)k_x$ $= (S - V_{x+1})q_x$	V_{x+1}
50	17,695	528	17,167
49	13,365	609	12,756
48	11,230	492	10,738

where V'_x is the fund arising from previous reserve values, premiums and interest, which is sufficient to provide reserve values for the *whole* of the policies, and the contribution to mortality for those expected to become claims. By the retro-

spective method, as hitherto explained, the accumulation would run thus :

Age x	V'_x	$S.q_x$	$p_x.V_{x+1}$
50	17,695	802	16,893
49	13,365	803	12,562
48	11,230	647	10,583

where the same fund V'_x provides the sums assured and bonuses for the expected claims, and the reserve values for those policies only which are expected to survive.

The difference between the two accumulations is distinctly laid down by Mr. Higham (*J.I.A.* xx, 154), both in words and in algebraic symbols. All that is now new on this point is the numerical illustration, and the attaching of Mr. Higham's form of the calculation to an accumulation under the retrospective method such as shown by Mr. Meikle, and at page 316 of the *Text-Book*.

It is clear that any deviation from expected mortality upsets the usefulness of the latter form, but it is provided for by the former form, because the values of $(S - V_{x+1})q_x$ are the contribution to mortality, to be carried to the credit of the mortality fund, whatever happens. If there should be no deaths at all, those values would be all profit: whatever deaths there are, the reserve values of the policies become claims will be separated from V_{x+1} , and carried also to the credit of the mortality fund, and the reserve values of the surviving policies will be left untouched.

This then is the sum of the whole process: the retrospective method is amended by the introduction of the "expected strain" instead of the "expected claims", and the difficulty thus introduced, which seemed to require the calculation of V_{x+1} , is avoided by obtaining the "expected strain" from S and V'_x by means of the factors k_x .

Tables are appended to this paper, giving the values of k_x for the following mortality tables: Carlisle, H^M , H^{M^5} , Sprague's Select, and the Government Annuitants, 1883, Males and Females. Mr. L. Stahlschmidt, and Mr. C. Pearl Dawson, both of the Imperial Life Office, have very kindly relieved me of the labour and responsibility of preparing these tables, and to those gentlemen I tender my best thanks.

III. JOINT LIVES AND SPECIAL CASES.

The tabulated values of k_x apply to all cases of policies on single lives, that is, to all policies which are regularly valued by mortality and interest. In cases such as short-terms and issue risks, which are not regularly valued, but for which a reserve is made consisting, for instance, of the whole or a portion of the premiums paid, the accumulation by k_x is not wanted, but the indication of the valuation must be followed. The value at the beginning of the year can be credited with premiums received, if any, and with interest, and the excess over the value at the end of the year can be carried away to mortality or loading at discretion.

For Joint Lives, the accumulation is the same as for single lives, but values of k_{xy} will be wanted. They are very easily calculated:

$$1 + k_{xy} = \frac{1}{p_{xy}} = \frac{1}{p_x} \times \frac{1}{p_y} = (1 + k_x)(1 + k_y) \\ = 1 + k_x + k_y + (k_x)(k_y)$$

$$\therefore k_{xy} = k_x + k_y + (k_x)(k_y).$$

Using II^M Table, to find k_{xy} for ages 79 and 74.

$$k_{79} = .153481$$

$$k_{74} = .100346$$

$$k_{79} \times k_{74} \left\{ \begin{array}{r} 15348 \\ 46 \\ 6 \\ 1 \end{array} \right\}$$

$$k_{79,74} = .269228$$

The multiplication is done by supposing k_{74} to be inverted, and the top line multiplied by it.

For assurances on joint lives and survivor, the policy may be treated as if it were three separate policies, all having the same sum assured and bonuses, and the same annual premium. For

$$S.A_{\overline{x},y} = S.A_x + S.A_y - S.A_{xy},$$

and $P(1 + a_{\overline{x},y}) = P(1 + a_x) + P(1 + a_y) - P(1 + a_{xy})$

$$\therefore V_{xy} = V_x + V_y - V_{xy},$$

and the accumulation may proceed for the three supposed policies separately. For instance:

Ages	V_{xy}^-	P	$V_{xy}^- + P$	$(V_{xy}^- + P)i$	V'_{xy}^-	S	$S - V'_{xy}^-$	$(S - V'_{xy}^-)k$	$V_{x+1, y+1}^-$
79	858	3	861	26	887	1023	136	21	866
74	814	3	817	25	842	1023	181	18	824
79·74	903	3	906	27	933	1023	90	24	909

where the bottom line has to be subtracted from the total of the other two.

For contingent policies it must be noticed that the sum assured depends on one risk, that of x dying before y , but the reserve value and the premiums depend on a different risk, that of the cessation of the joint lives. The reserve value and the premium must, therefore, form the subject of one accumulation, and the sum assured of a separate accumulation. Thus, using the Carlisle table:

Ages	V_{xy}^1	P	$V + P$	$(V + P)i$	V'	S	$S - V'$	$(S - V')k$	$V_{x+1, y+1}^1$
30·60	11·40	10·00	21·40	0·64	22·04	...	-22·04	-0·99	23·03
30 r. 60	1,000	1,000	10·38	-10·38

The mortality multiplier for the top line is that for joint lives, k_{xy} : we will not stay to demonstrate that that for the bottom line is $k_x(1 + \frac{1}{2}k_y)$. The two lines have to be added together for the complete result.

IV. MODUS OPERANDI.

In dealing with the business of a life office, the first step is to write up the class-lists for the period to be observed upon. This is not to be done by ruling out cancelled policies, but by debtor and creditor on opposite pages, so as to show not only the result at a given date, but the added, cancelled and altered policies, which have contributed to the bringing about of the resulting totals. It will be convenient that the "Cancelled Policy Book" should be a book of alterations as well as cancelments, in order that the posting of the class-lists may be as complete as possible.

The endowment assurances will be classified under the years of birth which were used for them at the last valuation. It will probably be more convenient to have a separate series of years of birth, and not actually to mix them in the class-lists with the whole-life cases.

In every case it must be seen that the particulars written off for a cancelled policy are the precise particulars that were on, and for this reason a record should appear in the class-lists of the term yet to run in endowment assurance and limited payment cases.

The class-lists having been posted up for a financial year, the new policies for that year will be summarized in a schedule giving the total of them for each year of birth, and the like for claims, surrenders, bonus surrenders, dropped policies, alterations, and so forth. The total of sums assured and bonuses in the claim schedules should agree with the amounts actually payable, and the policies taken off as surrendered should correspond with the surrender-values paid through the office accounts. The columns in these summaries will be as shown in a specimen immediately following, and their totals will be combined in a general summary, of which a specimen is also shown.

I have adopted the assumption that all policies coming under observation do so at the end of the financial year of entry, the odd time of the year of entry being dealt with separately. Similarly, that all cancelled policies pass from observation at the beginning of the financial year, the odd time of the year of exit being dealt with separately. The chief exception to this rule is for the claims, the policies for which are always supposed to run right through the year of exit, and are therefore retained under the general observation until the close of the year.

On these assumptions the general summary will have no transactions to record during the course of the financial year, because every entry and exit will take place, so far as regards the general observation, on 31 December or 1 January. Leaving out decimals, an extract from an actual general summary now follows:

1 January 1888.

Cr.

	No. of Policies	Sums Assured	Bonuses	Office Premiums	Net Premiums	Reductions	Extras	Values (1 + a)
		£	£	£	£	£	£	£
Balance from 1887 . . .	3143	1,926,216	326,873	59,135	46,098	669	2,022	745,274

Dr. 1 January 1888.

	No. of Policies	Sums Assured	Bonuses	Office Premiums	Net Premiums	Reductions	Extras	Values (1 + a)
		£	£	£	£	£	£	£
New Bonus on Re-assurances	958	534
Surrenders, 1888 . . .	30	13,700	2,212	562	462	...	6	5,482
Cropped, 1888 . . .	37	16,700	62	603	477	...	5	356
Transfers, 1888 . . .	1	3,500	...	147	123	1,142
Alterations, 1888	175	175	...	5	-741
Bonus Surrenders, 1888	1,582	1,086
Extras off, 1888	8	...
Balance, 1888 . . .	3,075	1,892,316	322,059	57,648	44,861	669	1,998	737,415
	3,143	1,926,216	326,873	59,135	46,098	669	2,022	745,274

31 December 1888.

Cr.

	No. of Policies	Sums Assured	Bonuses	Office Premiums	Net Premiums	Reductions	Extras	Values (1 + a)
		£	£	£	£	£	£	£
Balance	3,075	1,892,316	322,059	57,648	44,861	669	1,998	773,881
New, 1888	303	155,814	...	7,422	6,453	10	289	2,959
Transfers, 1888	7	3,650	...	150	126	...	1	60
Alterations, 1888	2	500	...	99	93	1	...	-312
Extras on, 1888	80	...
Claims, 1888, Re-assurances	...	3,500	102	127	102	...	2	(Contra)
	3,387	2,055,780	322,161	65,446	51,635	680	2,370	776,588

Dr. 31 December 1888.

	No. of Policies	Sums Assured	Bonuses	Office Premiums	Net Premiums	Reductions	Extras	Values (1 + a)
		£	£	£	£	£	£	£
New, 1888, Re-assurances	...	8,217	...	1,210	1,123	...	10	(Contra)
Claims, 1888	79	51,207	12,691	1,685	1,332	...	74	29,738
Balance to 1889	3,308	1,996,356	309,470	62,551	49,180	680	2,286	746,850
	3,387	2,055,780	322,161	65,446	51,635	680	2,370	776,588

In practice the statement for 1 January 1888 is not separated from that for 31 December 1887, the two dates being in fact the same, and the new policies of the previous year come on at the same moment that the cancellments of the succeeding year go off from the general observation. The exception is at the beginning or end of the quinquennium, when the new policies of the previous year come on before the valuation, and the cancellments of the following year go off after the valuation, and the figures in force at the moment of the valuation have to be ascertained.

In the above account, the line "Surrenders 1888" is the total of the summary of surrenders, the details of which are arranged according to dates of birth, to correspond with the class-lists. The following is an extract from a summary of surrenders, say, for 1888 :

Born	Class	No. of Policies	Sums Assured	Bonuses	Office Premiums			Net Premiums	Reductions	Extras	Values 1/1/88 (1 + a)
			£	£	£	s.	d.	£		£ s. d.	£
1847-8	...	4	1,700	30.05	54	7	0	40.714	...	2 9 2	7.14
1848-9	...	3	1,200	11.75	16	1	6	8.468	45.11
"	L. P.	17	7	11	17.396	-243.37
"	E. A.	1	2,000	90.00	92	0	0	70.700	249.44
1849-50	...	1	100	1.90	3	2	0	2.265	3.75
"	E. A.	1	1,000	30.00	46	0	0	35.350	...	10 0 0	85.76
1850-1	...	3	1,250	17.75	31	5	8	22.181	211.18

In the limited payment case the whole-life loading is deducted from the whole-life premiums of the same year of birth. The value of the limited premiums is necessarily a negative liability.

Now, if the balance of the general summary at any given date consists of, and agrees with, the total of the balances of the separate years of birth in the class-lists, inasmuch as every entry in the general summary is the total of a summary of entries under the separate years of birth, it is only a matter of careful book-keeping to make the balance of the general summary agree with the balances of the separate years of birth at any later given date. This agreement of the general summary with the age-classes does subsist at the beginning of the quinquennium, and can therefore be carried on from year to year to the end of the quinquennium. If on the quinquennial revision of the class-lists any error should be discovered, such error must affect one of the age-classes as well as the general summary, and its cause can be easily traced. The full policy particulars can thus be reconciled

from one valuation to the next, both in the general summary and in the age-class for every year of birth, the details of the reconciliation being contained in the yearly summaries of entries and exits from various causes.

So much for the particulars: now for the values. It will be noted in the specimens above that I have adopted the suggestion to base all the values, in the first instance, on annuities due, adjusting afterwards to policy-values depending upon $(t + a_x)$, where t is a proper fraction. All the summaries of entries and exits must have the policy-values calculated for each year of birth; for instance, in the above specimen summary of surrenders, 74.14 is the value, by one calculation, of sums assured of 1,730.05, subject to annual premium of 40.714, by H^M 3 per-cent at age 40, four surrendered policies being valued together. Limited payment policies can be reckoned in with the whole-life so far as regards sums assured, bonuses, and loadings, but the limited premiums and endowment assurance policies cannot very often be thus condensed.

The values ascertained are not the values at the moment of entry, or the moment of exit, but the values at 31 December, the date of coming under, or going away from, the general observation, and the adjustment for odd time will be made separately. The new policies have to be valued, as well as the cancelled ones, because in these days of charging for quarter ages and valuing by nearest birthday, very few policies start with a value of *nil*, and the actuarial ledger will contain an account of profit or loss on new policies.

The values inserted in the summaries of entries and exits will be summed and their totals carried into the general summary, as shown. Starting from the valuation, we thus obtain (see specimen) a value at, say, 1 January 1888 of £737,415, corresponding to the policy particulars at that date. These are the particulars of policies running the whole year from 1 January to 31 December 1888, and the particulars at the latter date are therefore the same as at the former, but the value in the meantime has increased from £737,415 to £773,881. The increase has been calculated by the formula for accumulation and the mortality multipliers.

	Policies Valued	Policies not Valued
Values, 1 January 1888	737,326	89
Net Premiums, less Reductions Interest	11,192 23,416	29 ...
Mortality	804,964 31,201	118 ...
Values, 31 December 1888	773,763	118

The "Policies not valued" were some in which the reserve was taken to be the amount of premiums paid, and net premiums for them were not stated in the valuation schedules.

To obtain the contribution to mortality, and the values at the end of the year, as just shown in gross, the operations of the last paragraph must be carried out in detail for each age-class. It will not be quite convenient to carry the values from the summaries of entries and exits into the ordinary class-lists, and a digest of them can be prepared in some such form as the following :

Dr. *Values, 1 January 1888 (1 + a).*

	Born 1846-7	Born 1847-8	Born 1848-9	Born 1849-50	Born 1850-1
New Bonus on Re-assurances					
Surrenders					
Dropped					
Transfers					
Alterations					
Bonus Surrenders					
Balance					
Total					

This will give all the details of the "Values" column of the General Summary, and the balances will therefore give all the details, age by age, of the balance of £737,415 in the general summary. The balanced class-lists already give the sums assured and bonuses, and the net premiums, making up the corresponding items of the balance of the general summary, and the materials

are thus all prepared for the accumulation by mortality multipliers. The accumulation will be made in schedule form, and its totals have been already given.

As with the particulars, so with the values, the two valuations can be completely reconciled. Throughout the general summary the values are the totals of the corresponding values of the separate age-classes, and the same thing therefore holds in the final result. If that final result differs from the result of a fresh independent valuation at the end of the quinquennium, the difference of the general summary from the new valuation must also appear in one or more of the separate age-classes. The bulk of the values of the age-classes will be right, and the errors have to be looked for only under the years of birth where they severally occur. By hypothesis, the policy-particulars will have been already reconciled, so that the mistake under any particular year of birth can only arise in calculations, that is, either in the initial or terminal valuation, in the policy-values coming on or going off, or in the mortality multiplications, all other matters having been self-checked in the progress of the work. Mistakes are therefore found and rectified without any serious difficulty.

By the processes suggested I have reconciled two valuations, at an interval of five years, involving a liability of £800,000, within a difference of £12, and have proved that even that difference was due to the rejection of fractions of £1. With two decimals uniformly throughout, the reconciliation could have been effected to the nearest sovereign.

Before leaving the general reconciliation, there are some adjustments to be arranged. We have throughout used valuations by $(1 + a_x)$, whereas the usual valuation is by $(t + a_x)$, t being a fraction. $P(t + a_x)$ is generally interpreted to mean that P is on an average payable $(1 - t)$ portion of a year hence, but mathematically it does not mean that, and we are bound to interpret the expression strictly. It means that Pt is payable at the beginning of each year, and $P(1 - t)$ at the end of each year, for

$$P(t + a_x) = Pt(1 + a_x) + P(1 - t)a_x.$$

$$\text{Let} \quad \frac{V_x}{(1 + a)} = SA_x - P(1 + a_x)$$

$$\text{then} \quad \frac{V_x}{(t + a)} = SA_x - P(t + a_x)$$

$$\text{and} \quad \frac{V_x}{(1 + a)} + P = SA_x - Pa_x = \frac{V_x}{(t + a)} + Pt.$$

We have been using

$$\frac{V_x}{(1+a)} + P + \text{Interest} - \text{Mortality} = \frac{V_{x+1}}{(1+a)}$$

which has to be converted into

$$\frac{V_x}{(t+a)} + Pt + \text{Interest} - \text{Mortality} + P(1-t) = \frac{V_{x+1}}{(t+a)}$$

The sums of the first two terms in each case have been proved to be equal, the interest is therefore the same interest, and the P is equally required by each expression. In the event of death the $P(1-t)$ at the end of the year would not be paid, so that the reserve value of a claimed policy is V_{x+1} under either expression, $(1+a)$

and the contribution to mortality in both cases is the same. It may be objected that in claim cases, if $P(1-t)$ is not paid at all, Pt should not be considered to be payable at the beginning of the year, but to do so is at least as correct as to assume that Pt is payable in the middle of the year under the same circumstances, and the former is the inevitable interpretation of the valuation formula. It results that to transform a valuation by $(1+a)$ into a valuation by $(t+a)$ for policies in force throughout the year, no change whatever is required in the figures of premium, interest, or mortality, but only an addition of $P(1-t)$ to the initial and terminal values.

The other important adjustment is that for immediate payment, which on a 3 per-cent valuation adds $1\frac{1}{2}$ per-cent to all values. The following is a specimen of the application of these adjustments:

Dr.

1 January 1888.

	Values (1+a)	Net Premiums, less Reductions	55 Per- cent thereof	Values (1.45+a)	1½ Per- cent for Immediate Payment	Final Values
	£	£	£	£	£	£
New Bonus on Re-assurances	534	534	8	542
Surrenders, 1888 . . .	5,482	462	254	5,736	86	5,822
Dropped, 1888 . . .	356	477	262	618	9	627
Transfers, 1888 . . .	1,142	123	67	1,209	18	1,227
Alterations, 1888 . . .	-741	175	96	-645	-10	-655
Bonus Surrenders, 1888 . . .	1,086	1,086	16	1,102
Balance (valued) . . .	737,326	44,192	24,306	761,632	11,424	773,056
„ (not valued) . . .	89	29	...	89	1	90
	745,274	45,458	24,985	770,259	11,552	781,811

Year 1888 (1 January to 31 December.)

Cr.

	Values (1 + a)	Net Premiums, less Reductions	55 Per- cent thereof	Values (.45 + a)	1½ Per- cent for Immediate Payment	Final Values
	£	£	£	£	£	£
Balance (valued) . . .	737,326	44,192	24,306	761,632	11,424	773,056
„ (not valued) . . .	89	29	...	89	1	90
Premiums (valued) . . .	44,192	44,192	663	44,855
„ (not valued) . . .	29	29	1	30
Interest (valued) . . .	23,416	23,416	352	23,798
„ (not valued) . . .	3	3	0	3
	805,085	44,221	24,306	829,391	12,441	841,832

Dr.

Year 1888 (1 January to 31 December.)

	Values (1 + a)	Net Premiums, less Reductions	55 Per- cent thereof	Values (.45 + a)	1½ Per- cent for Immediate Payment	Final Values
	£	£	£	£	£	£
Mortality (valued) . . .	31,201	31,201	468	31,669
Loading (not valued) . . .	3	3	0	3
Balance to 1889 (valued) . . .	773,763	44,192	24,306	798,069	11,971	810,040
„ „ (not valued) . . .	118	29	...	118	2	120
	805,085	44,221	24,306	829,391	12,441	841,832

There are no other adjustments, except the reserve for extra premiums, which is a small matter of account, and the final values shown above bring down the adjusted reserve for liability from year to year, from the final figures of one valuation to the final figures of the next. These supply material for entries in the actuarial journal and ledger.

There are many instances in which negative values arise: a loading is a negative premium, a bonus reduction is also a negative premium, the value of premiums is a negative liability. I found it convenient to abandon the use of the *minus* sign, and to write all negative amounts in red ink.

V. THE YEARS OF ENTRY AND EXIT.

For both entries and exits the ordinary accumulation is made for the current year, say, from V_x to V_{x+1} , in order to take a proportion of the interest and mortality according to the time,

i.e., fraction of a year, at risk. In these accumulations, as in the general accumulation, the limited payments and endowment assurances are all reckoned in one total with the whole-life policies of the same year of birth. In both entries and exits it is convenient to start with V_x and work forward to V_{x+1} : this was foreseen and so arranged when working the reserve values in the last column of the summaries of new and cancelled policies, already explained, for compiling the general summary. For the new policies the values calculated were those of V_x , the values for the beginning of the financial year of entry, generally negative values, and these were accumulated by the formula into V_{x+1} values, for the end of the year of entry, at which point they fall into the general observation. For surrenders and other cancellments the values were equally V_x , but these part from the general observation at the beginning of the year of exit, and run to V_{x+1} for the sake of the odd time only. Take first the case of the new policies, and let it be given that the following are the totals of the summary for the year of entry:

V_x (1 + a)	P	Interest	Mortality	V_{x+1} (1 + a)
-1,101	5,320	127	1,387	2,959

Now, if the annuity in the valuation be $t + a$, this implies that the inception of the new policies took place t of a year before 31 December, and that $127 t$ will be the proportionate interest in the above case, and $1,387 t$ the proportionate contribution to mortality. Further, being new policies, P will be due at the moment of entry, and the then policy-value will be $t.V_x + (1-t).V_{x+1}$. Assuming the valuation to have been made $(1+a)$ $(1+a)$ by $(.45 + a)$, the experience of the year of entry will be:

Value at moment of Entry	P	Proportion of Interest	Proportion of Mortality	V_{x+1} (.45 + a)
1,132	5,320	57	624	5,885
1,149	5,400	58	633	5,974

The top line contains the figures from the data, and the bottom line the same with $1\frac{1}{2}$ per-cent added for immediate payment.

It will be seen that the figures balance, and can be journalized thus:

New Policies <i>Dr.</i> . . .	£1,149	Mortality <i>Cr.</i> . . .	£633
" " " " . . .	5,400	Liability <i>Cr.</i> . . .	5974
Interest <i>Dr.</i> . . .	58		

The credit to liability account falling into its proper place among the items journalized from the adjusted general summary.

If all the new policies were granted by annual premiums, the first debit (£1,149) would be £0, and the second debit (£5,400) would be the exact amount of the first premiums, and could be debited to premium account. But there are half-yearly and quarterly premiums on the one hand, and single and limited premiums on the other, and it therefore becomes necessary to debit the valuation premiums to New Policies account, and to credit that account with the new premiums actually received.

The cases of premiums payable fractionally dispose of themselves in a curious manner. The policy-value at moment of entry would be $S.A - \left(\frac{m+1}{2m} + a\right)P$, and the premiums receivable on the average before the close of the year of entry would be $\frac{m+1}{2m}P$. The two debits to New Policies account would be

$$S.A - \left(\frac{m+1}{2m} + a\right)P + \frac{m+1}{2m}P = S.A - a.P = S.A - (1+a)P + P,$$

which is precisely the same as when the premiums are payable yearly, and the figures relating to yearly premiums may properly be debited in every case. The credit to New Policies account, for premiums received, would be less by $\left(\frac{m-1}{2m}\right)P$ in fractional than in yearly cases, and if the P be the same, that is, if the yearly net premiums are valued when they are payable fractionally, there will be a loss of $\left(\frac{m-1}{2m}\right)P$ on the granting of the policy, which agrees with Mr. Sprague's note in *J.I.A.*, xxiii, 264.

A point also arises on the limited payment policies. At the commencement of the assurance the net limited payments equal in value the net whole-life annual premiums. It is customary to reserve the whole-life loading: therefore, unless the office limited payments are mathematically equivalent to the whole-life net premiums, plus the whole-life loading, that is to say, to the office whole-life premiums, there will be a capitalized profit or loss

on the granting of limited payment policies, which may be considerable.

There may, therefore, be a profit or loss on "New Policies" account from three causes; from the valuation ages nearest birthday not corresponding to the office ages next birthday, from yearly net premiums being valued in half-yearly and quarterly cases, and from office limited payments not being equivalent to office whole-life premiums.

Turning now to the year of exit, the claims present no difficulty, as they are assumed to be at risk till the close of the financial year, and their reserve values are debited to the general liability and credited to mortality account, without any adjustment for odd time. The only adjustment is for the proportion of premiums for the year of death, which, as we have seen, when valuing by $(t + a_x)$, will be Pt payable, and $P(1-t)$ not payable. The transfers to mortality account will therefore be:

<i>Dr.</i>	<i>Mortality</i>	<i>Cr.</i>
$P = Pt + P(1-t)$	$V_{x-1} = V_{x-1} + P(1-t)$ $(t+a) (1+a)$	

The general liability account, in the course of the accumulation, has been credited with P , which, in the case of the claims, cannot be debited to premium account, because it is not premiums actually received. So the P for the claims is debited to mortality account; a portion of it $(1-t)P$ occurs on both sides of the account and cancels itself; the remaining portion tP will be more or less nearly balanced by the portion of the year's premiums paid before death, which premiums actually received will be carried to the credit of the mortality account for the purpose. There will then be, to the credit of the mortality account, the reserve values of the claim policies, V_{x+1} , and when $(1+a)$

all the contributions to mortality come to the credit, and the claim payments to the debit, the profit on mortality will appear.

The other exits (lapses, surrenders, and so on) will run for a portion of the year of exit, and not necessarily all for the same portion. When valuing by $(t + a_x)$, it will probably be right to assume that policies lapse at the average renewal date, and therefore run for $(1-t)$ of the year of exit and go off with the premium just due and with a reserve value of $t \times V_{x+1} + (1-t) \frac{V_{x+1}}{(1+a)}$.

The surrendered policies may perhaps be assumed to run on an average for six months of the year of exit. They would then go off with a premium due in $(.50-t)$ of a year hence, and with a reserve value of $\frac{1}{2}(V_x + V_{x+1}) + (.50-t)P$. The whole process

$$(1+a)(1+a)$$

is much easier to follow in figures. Let $t=.45$, and let the following be the accumulations for the complete year of exit :

	V_x (1+a)	P	Interest	Mortality	V_{x+1} (1+a)
Lapses	356	477	25	173	685
Surrenders	5,482	462	178	290	5,832

The values at the beginning of the year, at the moment of separation of the values from the general liability account, are $(.45+a)$ values, that is, £618 for the lapses and £5,736 for the surrenders. The lapses run .55 of the year, and incur .55 of the year's interest and mortality. The surrenders run .50 of the year, and incur .50 of the interest and mortality. The values at the moment of exit will be for the lapses—

$$(.45 \times 356) + (.55 \times 685) = 537$$

and for the surrenders—

$$.50(5,482 + 5,832) + (.5 \times 462) = 5,680.$$

These may be tabulated:

	Values beginning of Year	Proportion of Interest	Proportion of Mortality	Values at moment of Exit
Lapses	618	14	95	537
Surrenders	5,736	89	145	5,680
	6,354	103	240	6,217

The corresponding journal entries would be:

General Liability <i>Dr.</i>	£6,354	Mortality <i>Cr.</i>	£240
Interest <i>Dr.</i>	103	Lapsed Policies <i>Cr.</i>	537
		Surrenders <i>Cr.</i>	5,680

It is unnecessary to consider whether any proportion of the premiums for the year will be payable before the assumed average surrender date, because any such assumption has no effect at all on the surrenders account. The proportion of premium assumed to be paid would be debited to the surrenders account, and would, to precisely the same extent, increase the reserve value at moment of exit, which is on the other side of surrenders account. Any such assumption would, therefore, be useless. This is a different thing from the premiums actually paid, if any, on surrendered policies during the year of exit, which premiums will be credited to surrenders account, and will increase the profit on surrenders.

All other questions relating to the years of entry and exit, will be solved on similar lines. It must be remembered to multiply all values by $\left(1 + \frac{i}{2}\right)$ before journalizing, to provide for the condition of immediate payment of claims, which affects all transactions throughout.

VI. THE PREMIUM ACCOUNT.

To those who have followed the argument thus far, it will be becoming apparent that the right mode of dealing with the premium account will be an important factor in the probability of success in reducing the transactions of a year into the form of an actuarial profit and loss account. It has been assumed, not alone in this paper, that the amount of net premiums will be ascertainable. The office premiums, that is to say, will need to be divided into net premiums, loading and extras. Further, the net premiums will need to be divided into premiums on policies in force throughout the year, and premiums attaching to the years of entry and exit. Further still, the premiums of entry and exit will need sub-division under their several categories, as premiums paid on new policies, claims, surrenders, lapsed policies, or otherwise. Furthermore, the net premiums on policies in force throughout the year, as ascertained from the accounts, will need to agree with the net premiums in force throughout the year, as ascertained from the valuation class-lists.

To fulfil all these functions, easily and simply, the following form of premium book has been devised:

Premium Account.

Policy No.	FINANCIAL YEAR, 1891				ALTERATIONS, 1891				Income in force 1 January 1892
	Income in force 1 January 1891	Fol.	Debited	Fol.	Credited	Deductions from Income in force	Premiums of Years of Exit or Entry	Additions to Income in force	
	£ s. d.		£ s. d.		£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
1	10 9 2	11	10 9 2	10 9 2
2	11 4 7	12	11 4 7	11 4 7	11 4 7	...	Claim
3	7 3 1	14	7 3 1	13	7 3 1	7 3 1	Surrendered
4	5 6 4	15	2 13 2	5 6 4	2 13 2	...	Lapsed
5	31 12 7	17	0 16 3	16	31 12 7	0 16 3	30 16 4
001	18	65 6 8	...	65 6 8	65 6 8	65 6 8
2	19	3 7 4	...	3 7 4	6 14 8	6 14 8
	65 15 9		7 19 4		131 16 7	24 10 3	82 11 9	72 1 4	113 6 10

The first column contains, in numerical order, all policies whatever in existence at any part of the year. The next column is posted up from the class-lists, and its total has to be agreed with the total of office premiums in the valuation summary. This can be done by posting some hundreds of premiums, and proving their total up to a point, then ticking the premiums already posted, posting some hundreds more, again agreeing the total, and repeating the process until all the premiums are in and the gross total proved. The account premiums will need to be reconciled with the valuation office premiums once only, when first beginning: the principle of the book will keep them in agreement ever after. Totals will, of course, not be carried on from page to page, but will be summarized.

The next two columns contain the premium transactions of the year posted from the office accounts, and their total will be agreed with the premiums in the printed revenue account. The cause of cancelment will be noted in the right-hand column from the office cancelment books, and the alteration columns, and income in force at the end of the year, can be filled up on the face of the book as it stands, proving page by page.

The gross totals will then prove thus:

Income in force, 1 January 1891 . .	£65	15	9
Deductions during the year	24	10	3
In force throughout the year	41	5	6
Additions during the year	72	1	4
Income in force, 1 January 1892 . .	£113	6	10

The deductions, that is, the income gone off during the year, can be reconciled with the summaries of claims, surrenders and other exits which we have previously discussed, and the additions to income with the similar summaries of entries. The income at the beginning of the year, the deductions and the additions, in the numerical order of the premium book, thus agreeing with the income in the class-lists and summaries according to years of birth, the income at the end of the year will do the same, and the premium book will be kept permanently in strict accord with the accounts on one side and the valuation class-lists on the other.

The income in force at the 1st January is to be clearly distinguished from the income of the year. The book shows also the latter, as follows:

Income in force throughout the year,			
as just ascertained	£41	5	6
Premiums of years of exit and entry . .	82	11	9
	£123	17	3

agreeing with the account book columns:

Premiums credited	£131	16	7
Less debited	7	19	4
	£123	17	3

The income of the year having been thus divided into that on policies running through the year, and that on entries and exits, the division of the latter into the several groups of new policies, claims, surrenders and the rest can be readily made, and it is not a long step afterwards to divide them into net premiums, loading

and extras. The great bulk of the premiums are those running through the year, for which the net premiums, loading and extras already appear in bulk, which is all that is wanted, in the general summary of the class-lists.

The next most numerous group are the new policies, but here we have already the net premiums corresponding to the permanent annual premiums, and it is only necessary to notice where the premiums received differ from the permanent premiums. In the claims and other smaller classes the premiums received in the year of exit will have to be split up individually.

There are four main purposes served by this premium book.

- (1) It furnishes the means of dividing the whole premium income into net premiums, loading and extras, as just explained.
- (2) It enables the premiums of entry and exit to be carried to their proper accounts. This point has already arisen incidentally, it may now be looked at specifically. Supposing we value by $(.45 + a_x)$, and therefore expect 45 per-cent of the annual premiums to be paid in the year of death, the premiums actually paid will not be exactly 45 per-cent, and there will be a profit or loss from this cause, a point to which I do not remember that attention has ever been drawn. We provide for it by debiting the mortality account with the 45 per-cent, and crediting it with the premiums received. The same remark applies to the other entries and exits: in lapsed policies, the premiums received may be negative, through receipts debited to agents one year coming back the next.
- (3) The premium book columns of additions to, and deductions from, income form a strong check upon the posting of the class-lists, and enable the policy-particulars to be kept essentially right without waiting for the quinquennial revision. There is scarcely anything that can happen to a policy that does not affect the premiums. Paid-up policies are an exception to the rule, but nothing happens to them beside the payment of claims or surrenders, which can also be discovered from the cash accounts.

- (4) The premium book brings a check upon the premiums themselves, more efficient than any audit. Every practical auditor knows that checking the premiums is the most difficult, and therefore the weakest, part of an insurance audit. For this reason alone, even if without the other columns, it is always worth the trouble to keep an account of premiums received in strict numerical order.

VII. GENERAL REMARKS.

It will be very properly asked, what are the uses of a general investigation, such as now proposed, into the sources and progress of profit? The investigation itself, without its results, is a powerful instrument for throwing a light into many dark corners. This is so, as we have noticed, in the premium account and in the charging of limited premiums, where the whole-life loading is intended to be reserved. It would be more so than it is in bringing the sums assured, net premiums, and ages correctly into the class-lists, if matters were so arranged that all new policies started with the value *zero*. But as it is, these particulars are tested to a certain extent, and could easily be more so, by looking into the causes of the profit and loss on new policies. The method makes a thorough proof of the valuing, on such points as the casting of the class-lists, the assurance and annuity-values used, and the multiplications. It searches keenly into the effect of the formulas, discovered two errors to be caused by misprints in *Orchard's Tables*, and has a generally bracing effect all round.

Some instances of the effect of multiplying the liability by $1 + \frac{i}{2}$, to provide for the immediate payment of claims, are curious as showing how everything is called to account for itself. If the sum assured is payable at the instant of death, so are the bonuses, but how many actuaries, besides Mr. Sprague, have allowed for this in valuing the reversionary bonuses about to be declared? The premium reductions by bonus are also multiplied by $\left(1 + \frac{i}{2}\right)$, whence it results that the net reductions are larger than the office reductions. Some of the net premiums, which are multiplied by $\left(1 + \frac{i}{2}\right)$, are not paid to provide for sums assured, but to provide for future loading, so that, if a single

premium were exactly sufficient to pay for the assurance, and for the future annual loading, $\frac{i}{2} \times$ that single premium would have to be found from other funds, to be repaid gradually out of the annual loading.

As to the usefulness of the results, it is sometimes too hastily assumed that the only purpose of an enquiry into sources of profit is to allot bonuses in proportion to their sources. It might be that the mortality profit would be reserved and not divided at all, or that there were two sections, as in a case mentioned by Mr. Frankland (*J.I.A.*, xxx, 208), the only distinction between which was in respect of mortality. Apart from questions of distribution, most actuaries like to form some rough idea beforehand of how the next valuation is likely to turn out, and, by the means now suggested, the actuary may have an accurate knowledge from year to year of the profit up to date and the sources from which it is flowing. Such knowledge may be very useful in guiding the lines of active competition. It has been said that you cannot alter the past, but you can learn lessons from it which will improve the immediate future. Knowledge is power. Suppose that an actuary finds from his mortality account in the actuarial ledger that he ought to pass from H^M to $H^{M(5)}$, and from the other accounts in the same ledger that he can afford to do it, how much better off he is than if he has to wait for the knowledge until he has closed the valuation on the old basis, and then in the hurry, and under pressure from directors, to decide what change to make, and to recalculate the greater part of his values.

All the processes proposed for finding the "expected strain" involve a good deal of work, and it might therefore be concluded that the plans now suggested, having so much wider a scope, would be still more laborious; but it is claimed that the contrary is the case. It must be remembered that the new scheme finds not only the mortality profit, but the profit from all sources, expected and unexpected, and balances up the total with the ultimate divisible profit.

The work of finding the "expected strain" ranges itself under four principal heads:

- (1) Finding the net premiums.
- (2) Writing up the class-lists.
- (3) The reserve values of the entries and exits.
- (4) The complete valuation at the end of the year.

The work under (4) has hitherto been the main difficulty, especially with the ever-increasing numbers of limited payments and endowment assurances, and this, under the new scheme, is entirely abolished. The policy-values under (3), it may be noticed, under the new scheme, will now serve a double purpose: they are carried out of the liability under the general observation, thus adjusting that liability for all the additions and cancellments, and they are carried into the claims, surrenders, and other accounts, to help find the profit under those several categories. I have invented a new formula for calculating policy-values, which has made an appreciable saving in the work: this I will communicate in a note to the *Journal*. The work under (2) is common to all systems of taking out mortality profit, and I do not see that it can be otherwise. The work under (1), the finding of the true net premiums, has been hitherto lightly taken for granted, or set aside, the inference being that some approximation is generally considered sufficient. The question is to some extent separable from the other questions involved: I have, under the previous section, given my reasons for recommending the use of a special form of premium book, in connection with, or apart from, profit investigations, and these reasons are respectfully submitted to the judgment of the Institute.

The plan suggested by Mr. G. F. Hardy at the last April sessional meeting (*J.I.A.*, xxx, 205) is a very valuable one for certain purposes. I have had it in use for the last nine years, and published it by showing it to Mr. Chisholm two years ago. It had much to do with encouraging the wider enquiry now reported upon. I had used it when valuing the business of a small company, having fewer than 40 policies and no new business, in order to divide the profit from interest and loading, and to leave the profit from mortality, lapses and surrenders untouched as "suspended mortality." Mr. Hardy suggests it as a means of ascertaining the mortality profit *after* a quinquennial valuation, but surely this is to rob such an enquiry of the major part of its uses. If used annually it would involve all the work of headings (1), (2) and (4), and all that of (3), except for the claims. It would then show the profit from mortality *et ceteris*, the *cetera* including the profit from many miscellaneous sources, known and unknown. Suppose, for instance, that the actuary omitted to remember that he had converted his last surplus into reversionary bonuses by a different rate of interest from that used in the valuation: what effect would this have upon the ascertained

mortality profit? If, to save trouble, the policy-values at entry were not calculated, or the net premiums were assumed by using a percentage loading, there would be still further elements of uncertainty in the result. The suggestion, however, like so many others proceeding from Mr. Hardy, was well worth bringing under the notice of the Institute.

VIII. OTHER APPLICATIONS.

Apart from the immediate purpose of an enquiry into sources of profit, the accumulation by mortality multipliers will have other interesting applications. I ventured to make one such, some months since, in the pages of the *Bankers' Magazine*, applying them to the individual policy, in an attempt to make clear to an intelligent portion of the public, who, though not actuaries, might like to understand the subject, how one portion of their premiums goes to provide for current risk, and another portion to build up reserves.

The mortality multipliers may also become useful in enquiries into the after effects of a change in the valuation basis, for instance, from 3 to $2\frac{1}{2}$ per-cent. The immediate effects of such a change can be assessed in pounds sterling, but the after effects form a very complicated problem, depending upon the action and re-action of reserves, premiums, interest, and contributions to mortality, in the solution of which there may be room for the humble additional instrument of enquiry now provided.

The contributions to mortality seem to have some relation to the "normal future yearly costs of insurance", the present value of which is called the "insurance value", and is made the basis of surrender charges by the Massachusetts law. Questions concerning the "insurance value" and the "self-insurance value" have been discussed before the Actuarial Society of America on the initiative of Mr. Sheppard Homans, but I regret that I have not had time to give them the attention they deserve.

Since writing all the foregoing, I have lighted upon a paper, at page 33 of No. 3 of the Transactions of the Actuarial Society of America, by Mr. Asa S. Wing, strongly advocating the preparation of an annual balance of profits and their sources, incidentally touching upon many of the points herein discussed, but not avoiding the necessity of making the valuation at the close of the year. Mr. Wing speaks highly of the advantages

he has found to accrue to his company from "knowing surely from year to year exactly where the gains are coming from."

At page 22 of No. 4 of the same Transactions, I have, at the last moment, made the very curious discovery that the function, which I have called k_x , is known by precisely the same designation in the official notation of the Actuarial Society of America. I do not know to what purposes it is applied, but I feel sure that there are important improvements yet to be made in the scheme for analysis of profits which I have ventured to submit for the kindly consideration of the Institute.

MORTALITY MULTIPLIERS.

Carlisle.

Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x
0	·181893	35	·010363	70	·054457
1	·087672	36	·010665	71	·062529
2	·069425	37	·010974	72	·073110
3	·039439	38	·011293	73	·084736
4	·029572	39	·012020	74	·099104
5	·018125	40	·013176	75	·105610
6	·012436	41	·013968	76	·114790
7	·008874	42	·014583	77	·120363
8	·006623	43	·014798	78	·122108
9	·005108	44	·015020	79	·134313
10	·004509	45	·015032	80	·138590
11	·004843	46	·015039	81	·154482
12	·005025	47	·014820	82	·163724
13	·005209	48	·014132	83	·177694
14	·005556	49	·013873	84	·188764
15	·006230	50	·013601	85	·212533
16	·006753	51	·014499	86	·239865
17	·006962	52	·015436	87	·275862
18	·007011	53	·016413	88	·281769
19	·007061	54	·017187	89	·274649
20	·007111	55	·018250	90	·352380
21	·006995	56	·019368	91	·399999
22	·007043	57	·021343	92	·388889
23	·007093	58	·024807	93	·349999
24	·007144	59	·029096	94	·333333
25	·007368	60	·034619	95	·304347
26	·007423	61	·037112	96	·277778
27	·007829	62	·038862	97	·285713
28	·008775	63	·039771	98	·272728
29	·009026	64	·041418	99	·222222
30	·010206	65	·042848	100	·285713
31	·010311	66	·044388	101	·399999
32	·010234	67	·046450	102	·666667
33	·010153	68	·048713	103	2·000000
34	·010257	69	·051645

MORTALITY MULTIPLIERS.

 H^M

Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x
10	·004924	39	·010185	68	·056227
11	·004906	40	·010413	69	·060828
12	·003330	41	·010598	70	·066316
13	·002924	42	·010851	71	·073019
14	·002770	43	·011252	72	·081011
15	·002879	44	·011691	73	·090346
16	·003258	45	·012343	74	·100346
17	·003897	46	·013110	75	·109091
18	·004815	47	·013893	76	·119034
19	·005778	48	·014652	77	·129543
20	·006369	49	·015455	78	·140528
21	·006771	50	·016209	79	·153481
22	·006831	51	·016953	80	·169115
23	·006810	52	·017863	81	·187699
24	·006683	53	·018953	82	·206785
25	·006674	54	·020132	83	·228279
26	·006730	55	·021485	84	·248247
27	·006952	56	·022969	85	·265639
28	·007224	57	·024580	86	·281485
29	·007489	58	·026300	87	·300779
30	·007783	59	·028319	88	·314579
31	·007980	60	·030586	89	·339040
32	·008171	61	·033099	90	·387833
33	·008354	62	·035879	91	·455047
34	·008577	63	·038950	92	·541578
35	·008852	64	·042112	93	·711678
36	·009193	65	·045103	94	1·029633
37	·009555	66	·048844	95	1·755095
38	·009877	67	·052509	96	4·444458

MORTALITY MULTIPLIERS.

 $H^{M(5)}$

Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x	Age x	Mortality Multiplier, k_x
10	·004016	39	·011316	68	·057585
11	·003426	40	·011146	69	·061704
12	·003031	41	·011147	70	·067050
13	·002940	42	·011712	71	·073605
14	·003049	43	·011986	72	·081725
15	·003264	44	·012404	73	·091084
16	·003788	45	·013113	74	·101602
17	·004422	46	·013848	75	·110487
18	·005480	47	·014613	76	·121260
19	·006765	48	·015553	77	·130564
20	·008397	49	·016536	78	·142373
21	·009751	50	·017414	79	·155955
22	·010388	51	·018335	80	·170642
23	·010825	52	·019146	81	·188659
24	·010944	53	·020316	82	·208168
25	·010618	54	·021389	83	·228154
26	·010166	55	·022691	84	·248186
27	·010043	56	·024072	85	·265985
28	·009799	57	·025719	86	·281967
29	·009548	58	·027303	87	·303419
30	·009289	59	·029188	88	·322034
31	·009257	60	·031606	89	·351147
32	·009314	61	·034037	90	·383618
33	·009311	62	·036895	91	·468750
34	·009521	63	·040012	92	·560976
35	·010103	64	·043525	93	·782614
36	·010455	65	·046697	94	1·090904
37	·010817	66	·050240	95	1·749995
38	·011189	67	·053693

MORTALITY MULTIPLIERS.—*Mr. Sprague's Select-Life Tables.*

Age x	YEARS ELAPSED SINCE DATE OF INSURANCE						Age x
	0 Mortality Multiplier, $k_{[x]}$	1 Mortality Multiplier, $k_{[x-1]+1}$	2 Mortality Multiplier, $k_{[x-2]+2}$	3 Mortality Multiplier, $k_{[x-3]+3}$	4 Mortality Multiplier, $k_{[x-4]+4}$	5 or more Mortality Multiplier, k_x	
15	·003009	·003264	15
16	·003714	·003788	·003788	16
17	·004319	·004422	·004422	·004422	17
18	·004722	·005480	·005480	·005480	...	·005480	18
19	·005025	·006644	·006765	·006765	·006765	·006765	19
20	·005025	·007861	·008397	·008397	·008397	·008397	20
21	·004893	·008624	·009751	·009751	·009751	·009751	21
22	·004773	·008315	·010388	·010388	·010388	·010388	22
23	·004669	·008036	·010121	·010825	·010825	·010825	23
24	·004584	·007794	·009845	·010725	·010914	·010944	24
25	·004520	·007590	·009576	·010505	·010618	·010618	25
26	·004479	·007426	·009328	·010166	·010166	·010166	26
27	·004459	·007302	·009111	·009940	·010043	·010043	27
28	·004469	·007218	·008932	·009678	·009799	·009799	28
29	·004481	·007171	·008796	·009466	·009548	·009548	29
30	·004520	·007159	·008703	·009289	·009289	·009289	30
31	·004575	·007179	·008655	·009245	·009257	·009257	31
32	·004616	·007226	·008649	·009244	·009344	·009344	32
33	·004730	·007301	·008678	·009311	·009311	·009311	33
34	·004824	·007400	·008750	·009521	·009521	·009521	34
35	·004924	·007516	·008859	·009598	·010103	·010103	35
36	·005030	·007643	·008991	·009829	·010251	·010455	36
37	·005142	·007783	·009140	·010064	·010531	·010817	37
38	·005263	·007935	·009308	·010282	·010760	·011189	38
39	·005391	·008101	·009495	·010485	·011054	·011316	39
40	·005530	·008284	·009705	·010673	·011302	·011446	40
41	·005681	·008486	·009939	·010869	·011447	·011447	41
42	·005845	·008710	·010201	·011116	·011686	·011712	42
43	·006024	·008958	·010494	·011143	·011957	·011986	43
44	·006222	·009233	·010822	·011847	·012362	·012404	44
45	·006441	·009542	·011191	·012336	·012894	·013113	45
46	·006686	·009880	·011607	·012898	·013567	·013848	46
47	·006956	·010279	·012080	·013508	·014313	·014613	47
48	·007257	·010718	·012616	·014153	·015159	·015553	48
49	·007591	·011209	·013226	·014845	·015975	·016536	49
50	·007963	·011762	·013915	·015598	·016813	·017414	50
51	·008375	·012380	·014690	·016422	·017681	·018335	51
52	·008831	·013073	·015562	·017344	·018604	·019146	52
53	·009336	·013848	·016543	·018379	·019624	·020316	53
54	·009894	·014713	·017649	·019536	·020774	·021389	54
55	·010509	·015677	·018896	·020814	·022055	·022691	55
56	·011186	·016750	·020296	·022258	·023464	·024072	56
57	·011930	·017942	·021868	·023922	·025048	·025719	57
58	·012745	·019265	·023628	·025844	·026875	·027303	58
59	·013637	·020732	·025581	·028045	·028985	·029188	59
60	·014610	·022358	·027779	·030565	·031403	·031606	60
61	·015675	·024157	·030283	·033336	·034037	·034037	61
62	·016833	·026144	·033141	·036242	·036895	·036895	62
63	·018091	·028338	·036376	·039222	·040042	·040042	63
64	·019455	·030754	·040033	·042314	·043525	·043525	64
65	·020929	·033410	·044066	·045519	·046697	·046697	65
66	·022521	·036328	·050240	·050240	·050240	·050240	66
67	·024236	·039530	·053693	·053693	·053693	·053693	67
68	·026081	·043040	·057585	·057585	·057585	·057585	68
69	·028060	·046881	·061704	·061704	·061704	·061704	69
70	·030184	·051084	·067050	·067050	·067050	·067050	70
71	·032458	·055676	·073605	·073605	·073605	·073605	71
72	·034886	·060693	·081725	·081725	·081725	·081725	72
73	·037480	·066169	·091084	·091084	·091084	·091084	73
74	·040244	·072144	·101602	·101602	·101602	·101602	74
75	·043188	·078660	·110487	·110487	·110487	·110487	75
76	...	·085765	·121260	·121260	·121260	·121260	76

NOTE.—The notation employed in the above table is to be interpreted in the manner suggested by Mr. Sprague.

MORTALITY MULTIPLIERS.—*Experience of the Government Life Annuitants (1883) for Male Lives.*

YEARS ELAPSED FROM PURCHASE						
Age x	0	1	2	3	4 & upw'ds	Age x
	Mortality Multiplier, k_{x-0}	Mortality Multiplier, k_{x-1-1}	Mortality Multiplier, k_{x-2-2}	Mortality Multiplier, k_{x-3-3}	Mortality Multiplier, k_x	
40	0009713	0014075	0014899	0015373	0016684	40
41	0010070	0014580	0015362	0015806	0017159	41
42	0010448	0015115	0015837	0016250	0017646	42
43	0010826	0015662	0016322	0016715	0018154	43
44	0011225	0016229	0016829	0017180	0018672	44
45	0011634	0016818	0017346	0017667	0019202	45
46	0012064	0017429	0017884	0018164	0019753	46
47	0012515	0018060	0018434	0018683	0020325	47
48	0012966	0018714	0019004	0019202	0020898	48
49	0013448	0019389	0019597	0019742	0021503	49
50	0013942	0020096	0020200	0020304	0022119	50
51	0014672	0020790	0020908	0021012	0022746	51
52	0015445	0021325	0021628	0021753	0023405	52
53	0016260	0021972	0022390	0022516	0024076	53
54	0017118	0022631	0023164	0023301	0024769	54
55	0018019	0023311	0023971	0024118	0025473	55
56	0018973	0024013	0024811	0024968	0026200	56
57	0019971	0024748	0025673	0025841	0026958	57
58	0021033	0025494	0026568	0026757	0027739	58
59	0022140	0026262	0027506	0027696	0028531	59
60	0023311	0027053	0028457	0028669	0029347	60
61	0024538	0027876	0029463	0029675	0030195	61
62	0025841	0028722	0030492	0030715	0031066	62
63	0027211	0029590	0031555	0031800	0032220	63
64	0028648	0030482	0032633	0032919	0032893	64
65	0030174	0031406	0033805	0034083	0034103	65
66	0031768	0032422	0035850	0036161	0035095	66
67	0033461	0033291	0040161	0040518	0035503	67
68	0035240	0034648	0043787	0044190	0035720	68
69	0037118	0035910	0047746	0048196	0036332	69
70	0039091	0037306	0052078	0052576	0036984	70
71	0041150	0038876	0056803	0057362	00374529	71
72	0044970	0057451	0061977	0062598	00379179	72
73	0046792	0062677	0067635	0068330	00384669	73
74	0049692	0068887	0073826	0074610	00392371	74
75	0052776	0074633	0080602	0081479	00399626	75
76	0056055	0081479	0088033	0089016	00409312	76
77	0059547	0088969	0096167	0097273	00421655	77
78	0063264	0097189	0105094	0106342	00433787	78
79	0067213	0106195	0114889	0116283	00445903	79
80	0071421	0116084	0125657	0127218	00460955	80
81	...	0126938	0137488	0139251	00475968	81
82	0150510	0152485	00489343	82
83	0167905	00507773	83
84	00522909	84
85	005247583	85
86	005271149	86
87	005300762	87
88	005327387	88
89	005356447	89
90	005383719	90
91	005416671	91
92	005455223	92
93	005505616	93
94	005574109	94
95	005669756	95
96	005807011	96
97	006111182	97
98	006332362	98
99	006886919	99
100	00823497	100
101	008484470	101

NOTE.—The notation employed above is to be interpreted in the manner suggested by

MORTALITY MULTIPLIERS.—*Experience of the Government Life Annuitants (1883) for Female Lives.*

Age x	YEARS ELAPSED FROM PURCHASE					Age x'
	0	1	2	3	4 & upw'ds	
	Mortality Multiplier, $k_{[x]}$	Mortality Multiplier, $k_{[x-1]+1}$	Mortality Multiplier, $k_{[x-2]+2}$	Mortality Multiplier, $k_{[x-3]+3}$	Mortality Multiplier, k_x	
40	·005803	·007465	·009031	·009571	·010938	40
41	·005975	·007618	·009224	·009754	·011163	41
42	·006168	·007760	·009418	·009938	·011398	42
43	·006350	·007922	·009612	·010132	·011634	43
44	·006553	·008075	·009815	·010326	·011869	44
45	·006755	·008237	·010030	·010520	·012115	45
46	·006958	·008400	·010234	·010724	·012361	46
47	·007171	·008563	·010448	·010928	·012617	47
48	·007394	·008736	·010662	·011133	·012874	48
49	·007628	·008909	·010897	·011347	·013140	49
50	·007861	·009082	·011122	·011562	·013407	50
51	·008054	·009591	·011798	·012320	·014271	51
52	·008258	·010132	·012515	·013120	·015321	52
53	·008461	·010703	·013274	·013983	·016518	53
54	·008675	·011306	·014075	·014899	·017770	54
55	·008888	·011941	·014930	·015878	·018465	55
56	·009112	·012617	·015837	·016911	·018890	56
57	·009336	·013325	·016797	·018029	·019805	57
58	·009571	·014075	·017822	·019202	·020846	58
59	·009805	·014878	·018901	·020471	·021367	59
60	·010050	·015713	·020054	·021816	·022390	60
61	·011276	·016591	·021273	·023248	·023877	61
62	·012648	·017532	·022568	·024779	·025410	62
63	·014178	·018527	·023950	·026410	·027337	63
64	·015909	·019576	·025410	·028151	·030068	64
65	·017853	·020679	·026958	·030004	·033037	65
66	·020033	·022830	·029317	·032546	·036033	66
67	·022484	·025210	·031949	·035304	·038983	67
68	·025231	·027834	·034790	·038303	·041992	68
69	·028331	·030737	·037893	·041569	·045369	69
70	·031811	·033955	·041276	·045107	·049032	70
71	·034404	·037506	·044954	·048966	·053852	71
72	·037215	·041439	·048988	·053153	·059906	72
73	·040258	·045795	·053374	·057709	·067133	73
74	·043558	·050630	·058179	·062677	·074911	74
75	·047131	·055977	·063423	·068079	·083529	75
76	·051016	·061909	·069153	·073952	·092096	76
77	·055220	·068479	·075419	·080369	·101443	77
78	·059771	·075778	·082275	·087358	·110667	78
79	·064725	·083893	·089776	·094978	·121680	79
80	·070091	·092908	·097996	·103290	·133286	80
81	...	·102937	·107003	·112372	·147486	81
82	·116882	·122297	·161589	82
83	·133144	·179440	83
84	·197863	84
85	·220971	85
86	·242406	86
87	·272556	87
88	·301846	88
89	·336005	89
90	·364517	90
91	·412350	91
92	·443856	92
93	·488073	93
94	·523229	94
95	·612383	95
96	·668474	96
97	·803914	97
98	·961554	98
99	1·888921	99
100	3·500045	100

NOTE.—The notation employed above is to be interpreted in the manner suggested by Mr. Sprague and adopted in his Select-Life Tables.

DISCUSSION.

MR. G. H. RYAN said that the investigation of the mortality profit arising from the operations of a life office had lately been much discussed, and the author, while throwing new light on that matter, by no means confined his attention to it. His task was rather the quantitative analysis of the profits arising from the working of a life office from all and sundry sources, mortality included. There were two main divisions into which the paper naturally fell, and along which they might usefully conduct their discussion. Dealing with the first section of the paper, Mr. Searle had not only contributed a new formula, but had discovered a new function, which in future might lead to very interesting and novel combinations. By the aid of this function, called k_x , they were enabled to pass from the valuation at the commencement of a year to the mortality profit of the year, and this obviated one of the chief objections to the method proposed by Mr. Higham, involving as that method did the necessity for a valuation made at the end of the year. In this way, under Mr. Searle's plan, the whole of the values operated upon became homogeneous. At the same time he (the speaker) did not hesitate to express his preference for the use of the "continuous system" in dealing with these functions, since by that means they avoided all corrections for disturbances, such as lapses and surrenders, and got, at a given point of time, a true measure of the expected strain. In dealing with premiums for life assurance policies the year as a unit was going out of fashion, and, in truth, it was a trifle arbitrary and unscientific. He did not see why in this matter they should not depart from it and use the continuous basis. Again, he had to take exception to Mr. Searle's reference to the "expected strain" as a mere "piece of statistics." This was surely overstating the case. The "expected strain" formed one side of a profit and loss mortality account, the "actual strain" being on the other side, and the profit or loss forming the balance of the account. To call that a "piece of statistics" was as incorrect as it would be to term the valuation particulars of the fifth schedule "a piece of statistics." True, such an account was, to use Mr. Searle's picturesque description, a phantom balance sheet; but so also, as had been pointed out, was the valuation balance sheet of the fifth schedule. He did not think, therefore, that the author's description could be justified. With regard to the elaboration into which the author would lead them in connection with special policies, it was fair to say that the author stated that his method was not more laborious than the calculation of the "expected death strain" in the usual way. But this must be the partiality of a parent for his offspring. He did not speak on this matter from a merely theoretical standpoint, for he had lately carried through such an operation as that which the author had proposed, and anything more laborious and troublesome it had not often been his lot to undertake. He agreed that the chief difficulty in harmonizing the revenue and valuation accounts was that of accounting for the annual premium revenue. It was exceedingly difficult. In every change that took place in a policy, whether the premium was altered from yearly to half-yearly, or was commuted

for life, or for a series of years, each case had to be separately investigated, and the premium receipts split up into net premium and loading. As would be imagined, that required great care. Another thing was that these changes in the policies were increasing, and that they mainly fell upon the new insurances. He thought the author's scheme, to harmonize the revenue account with the valuation account in the manner proposed, was Utopian rather than practical. Finally, he wished to be allowed to refer to the suggestion made by Mr. G. F. Hardy (*J.I.A.*, xxx, 205) for obtaining mortality profit by operating on the left-hand side of the equation numbered 7 in the author's paper. This plan was put forward as an alternative to the method proposed by Mr. Higham, and adopted with certain modifications by himself (Mr. Ryan) in his recent note. Had Mr. Hardy submitted it merely as a short means of arriving at the result in certain instances, or as a means of verifying the results otherwise obtained, he should have cordially agreed that the suggestion was most valuable and practical, but to put it forward as an alternative scheme did not meet with his concurrence. Mr. Hardy's system brought the whole of the mortality profit on all the policies into hotch-pot along with the profit on surrenders, lapses, conversions, and a variety of other incidental matters, and separate corrections had to be made on all of these accounts before the mortality profit by itself was arrived at. Again, it did not enable them to compare the expectation with the actual experience in ages or groups of ages, and any change of valuation basis rendered it inapplicable without further correction. But, in point of simplicity and shortness of working, it fell far behind its rival, and this was his object in calling attention to the matter on the present occasion: for it would appear that the only way in which Mr. Hardy's formula could be accurately carried into operation was by rigidly adhering to the elaborate system brought before them by Mr. Searle. He thought he was justified in claiming that Mr. Searle's paper succeeded in disposing of Mr. Hardy's contention that his plan was as simple and convenient of application as the established method.

Mr. F. B. WYATT said there was one phrase he should like to call in question. Mr. Searle said, talking of periodical valuations: "I believe there does not exist at present any precise and thoroughly independent check." Upon that matter he thought that when actuaries had to deal with either annual or quinquennial valuations they did see that they had most precise checks on their work. He need hardly dwell upon the great desirability of investigations of this nature. It was some years since Mr. Higham's paper had been read, and it was then looked upon as being rather theoretical. Since then a great many actuaries actually adopted some system of the sort for ascertaining the mortality strain, which was a most important matter. He could not go so far as to say that it was necessary to analyze all the minor sources of profit. It was practically difficult to ascertain the profits from surrenders and lapses, and he did not know that it was necessary to do it. If they knew the profit on mortality they could easily find the product from surplus interest and might throw the rest into hotch-pot. The method by which Mr. Searle had arrived at his result was extremely ingenious: he referred to the use of the function k_x . That function

had, he believed, been adopted before, but it had probably not been put to any practical utility. On this occasion Mr. Searle had made a very great use of it. In these days, when endowment assurances were increasing so much, it was very desirable to arrive speedily at the liability under such benefits from year to year by some approximate method, and this object could be achieved by the aid of Mr. Searle's plan. As to the *modus operandi*, Mr. Searle's explanation was extremely lucid, but at the same time it appeared rather a laborious process. He thought the ordinary plan was quite sufficient for their purposes. Reference had been made to a similar method employed by Mr. T. G. C. Browne, and it would have been interesting had he explained this mode as an addendum to the paper, for he (Mr. Wyatt) had reason to believe that it was excellent and no doubt short.

Mr. C. D. HIGHAM said that the author made too much of the yearly group valuation, and so did other gentlemen who had not tried it. With the modification suggested by Mr. Woolhouse, which he had previously mentioned, they would be surprised how very easily it could be done. He had that afternoon completed some valuation figures. The total sum assured was £9,540,940. The true liability was £4,026,535. The liability brought out by the formula was £4,033,024, a difference of nearly .16 per-cent, which was practically the same thing. Of course, if they admitted that it was no more trouble to write up a class-book year by year than to make out a fresh list at the end of the five years' period, half their work was done. This short modification of Mr. Woolhouse's plan might be worked by almost any one, whereas it would want very skilled labour to work the "in and out" system of Mr. Searle's. As to special policies, he had been accustomed to value them for a period in advance. He obtained the value for five years hence and interpolated for the intervening years. By this means the labour was nothing like so great as was generally thought, and the figures were sufficiently accurate for their purpose. They knew the profit made from mortality, and that, of course, was all that he (Mr. Higham) professed to ascertain from his own paper.

Mr. F. SCHOOLING agreed with previous speakers, that to use Mr. Searle's scheme as a whole would entail enormous labour, especially for companies doing a large business. There was a theoretical objection to applying the exact system of book-keeping, because in place of being able to start upon actual facts they had to start at best only upon an estimate. For the purpose of analyzing profits, the simplest way was to keep a series of class-books by aid of cards and obtain an annual valuation. It must be easier to analyze the profits after a valuation than before, because they had all their facts ready to hand. With regard to k_x , that function could be used in many ways, especially, as Mr. Wyatt had said, with regard to endowment assurances. He did not see that they could use the book-keeping portion as Mr. Searle suggested in its entirety.

Mr. J. A. ROBERTSON would like to have seen a statement somewhat analogous to what was found in treatises on book-keeping, giving specimen entries of transactions such as occur in practice—so many policies in force at the beginning of the year, so many claims, surrenders and lapses going off, and so much new business

coming on; then showing the entries and the calculations required to bring out the reserves at the end of the year and an agreement of these with the results of direct valuation. The author had enumerated the different classes that could be grouped together, but deferred assurances could not be dealt with in the way described, at least during the period of deferment, when there was no current risk. As to increasing and decreasing assurances, he did not exactly see what provision had been made for working them in. The author said that the figures given included ordinary whole-life assurances, increasing premiums, limited payments, and endowment assurances, but in another part of the paper he said it would not be convenient to mix endowment assurances with whole-life policies, and that meant another arrangement. Whether he would start yet another class-list for limited payments he (Mr. Robertson) did not know. With regard to endowment assurances, he had tried grouping a number of the same present age, and found that Mr. Searle's arrangement gave very good results. Turning to the methods of dealing with special cases, he failed to see any material saving of labour in the schedule given for a joint-life and survivor assurance where it was treated as if there were three separate policies. The necessary summation and subsequent deduction were not given, so that in the case of each policy there would be more work than was shown there. He had worked out in full the case given by the author, valuing at the end of each year, and calculating the estimated strain by taking the nearest single age, according to the annuity-value, and had reproduced Mr. Searle's figures with, as it seemed to him, a considerable saving of labour. With regard to contingent policies, the following demonstration might be found of interest:

$$\begin{aligned}
 {}_{n+1}V_{xy}^1 - {}_nV_{xy}^1 &= \pi_{xy}^1 (a_{x+n, y+n} - a_{x+n+1, y+n+1}^1) - (A_{x+n, y+n}^1 - A_{x+n+1, y+n+1}^1) \\
 &= \pi_{xy}^1 \left\{ a_{x+n, y+n} - \frac{a_{x+n, y+n} - {}_v p_{x+n, y+n}}{{}_v p_{x+n, y+n}} \right\} \\
 &\quad - \left\{ A_{x+n, y+n}^1 - \frac{A_{x+n, y+n}^1 + \frac{1}{2} {}_v p_{x+n, y+n} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1)}{{}_v p_{x+n, y+n}} \right\} \\
 &= \pi_{xy}^1 \left\{ a_{x+n, y+n} \left(1 - \frac{1}{{}_v p_{x+n, y+n}} \right) + 1 \right\} \\
 &\quad - \left\{ A_{x+n, y+n}^1 \left(1 - \frac{1}{{}_v p_{x+n, y+n}} \right) - \frac{1}{2} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1) \right\} \\
 &= \left(1 - \frac{1}{{}_v p_{x+n, y+n}} \right) \left\{ \pi_{xy}^1 \times a_{x+n, y+n} - A_{x+n, y+n}^1 \right\} \\
 &\quad + \pi_{xy}^1 + \frac{1}{2} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1) \\
 &= \left(\frac{1}{{}_v p_{x+n, y+n}} - 1 \right) ({}_nV_{xy}^1 + \pi_{xy}^1) + \pi_{xy}^1 + \frac{1}{2} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1) \\
 &= \frac{{}_nV_{xy}^1 + \pi_{xy}^1}{{}_v p_{x+n, y+n}} - {}_nV_{xy}^1 + \frac{1}{2} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1) \\
 \therefore {}_{x+1}V_{xy}^1 &= \frac{({}_nV_{xy}^1 + \pi_{xy}^1)(1+i)}{{}_v p_{x+n, y+n}} + \frac{1}{2} (p_{x+n}^{-1} + 1) (p_{y+n}^{-1} - 1) \quad (1)
 \end{aligned}$$

In Mr. Searle's notation (writing $x+n$, $y+n$ as the present ages) the right-hand member of this identity becomes

$$\frac{V^1}{p_{x+n, y+n}} + k_{y+n}(1 + \frac{1}{2}k_{x+n}) \times S \quad \dots \quad (2)$$

and he might have obtained the first line in his last column by dividing V^1 by $p_{30, 60}$, and thus have saved himself the trouble of calculating $k_{30, 60}$. The corresponding "contribution to mortality" would then have been found by subtraction. This might also be arrived at thus: the expression at the head of the second last column, $(S_1 - V^1)k$, becomes, when $s=0$, $-V^1 \times k$, and the result of subtracting this from V^1 is $V^1(1+k) = V^1 \times p^{-1}_{x+n, y+n}$. Expression (1) might be re-written

$$p_{x+n, y+n} \times {}_{n+1}V^1_{xy} = ({}_nV^1_x + \pi_x)(1+i) - \frac{1}{2}(1-p_{x+n})(1+p_{y+n}),$$

which is exactly analogous to the well-known form for whole-life assurances:

$$p_{x+n} \times {}_{n+1}V_x = ({}_nV_x + \pi_x)(1+i) - q_{x+n},$$

but is less traceable owing to the fact that the probabilities of survivance and of claim are not complementary, the events themselves not being mutually exclusive. Under the head of the "modus operandi", the author said that "it will be convenient that the cancelled policy book should be a book of alterations as well as cancelments, in order that the posting of the class-lists may be as complete as possible." He (Mr. Robertson) supposed that the best way to ensure that being done was to make the cancelled policy book a record of signatures, by initial or otherwise, to the various endorsements on the policy. He had found it of very great service in getting out the balance of the class-book every year, to make up first of all the lists of claims, surrenders, &c., from the entries as they appeared in the class-book, and to agree those with the separate records of claims, surrenders, &c. The premium account might be improved by grouping the premiums according to where and when they were payable, as it would then serve as the source whence the branch and agency debits might be made up.

The PRESIDENT (Mr. A. Hendriks) in proposing a vote of thanks to the author, said it had been remarked that the paper was a scientific one, but it was also equally a practical one. He had read with much attention the formulas contained in the paper, and thought that for simplicity and thoroughly exhausting a very difficult subject, great thanks were owing to Mr. Searle. Whether the system was too complicated or not to be drafted into their everyday operations, it was one which was very interesting from a scientific point of view, and might lead later on to Mr. Searle finding possibly some shorter method. With regard to the investigations into mortality, he agreed that given the profits from mortality and rate of interest those really were the two points upon which they ought to be satisfied and which required most careful watching, but the other sources from which they obtained profit were practically of much smaller moment. All companies now met their policyholders fairly from the point of surrender-value. Very few valuable policies

were allowed to lapse, so that they might be justified if they shrank from so laborious a process as that suggested by the paper in order to arrive at the actual small percentage of profit derived from causes outside the two main sources of profit.

Mr. SEARLE, in reply, said he wished to echo the expectation of the President that there was great probability of further improvement being made in this matter, which would shorten some of the processes. There were some which it seemed to him could not be shortened. He did not see how they could do anything without writing out their class-lists or calculating the value of the policies which went off. Mr. Ryan had suggested that by a continuous method, questions relating to values of lapsed and surrendered policies did not arise, but if they were not required for one purpose it seemed to him that they were wanted for another. If they desired to know the profit from surrenders, they must have the reserve values to set against the values actually paid. The point where there might be some improvement was in the question of dealing with the year of entry and the year of exit. That section contained some interesting investigations. At the same time, he thought, the processes might be very much amended on the lines that Mr. Ryan had suggested in his paper. With regard to the premium book, he did not see why anyone should say it was difficult. It was one of the most simple matters to arrange. The explanation might seem long, but a newcomer in an office could be put upon that book, and he would keep it perfectly. He recommended the premium account to be kept in numerical order, not in months or agencies, nor in any other form. Mr. Ryan had said that he ought not to have multiplied endowment assurance values by $1 + \frac{i}{2}$ to allow for immediate payment of claims. With regard to that, he would call attention to the sentence at the end of the first section of the paper, in which he stated that he had nothing to do with the basis of valuation. Whatever had been used in the valuation he had to deal with. It was not for him to settle whether it should be adopted. The question was, whether the company he was dealing with employed it. If they did he also ought to use it. He was very pleased that Mr. Robertson had thoroughly tested his formulas, as Mr. Robertson had had to do with the investigations in which Mr. Browne was chiefly concerned.

Contingent Reversions, Reversionary Life Interests, and Life Interests in Possession. By WILLIAM BROCKIE PATERSON, F.F.A., A.I.A., of the Norwich Union Life Insurance Society.

[Read before the Institute, 27 March 1893.]

IT has been, I believe, considered an axiom that payment of Life Assurance premiums should be made in advance, or in other words, that under any sound system of Life Assurance, the premiums paid must never be less than the value of the risk

incurred from the outset upon the basis of calculation adopted, and this appears as a fundamental principle in all the published formulas for the determination of the values of any interests involving Life Assurance. The methods that are described in the following pages necessitate a departure from the existing practice in this respect, and it is therefore essential, in the first place, to justify so important a change.

No difficulty presents itself in the calculation of premiums amply sufficient to recompense an office for the risk it has to run, though these are made payable only after a fixed term of years, or after the failure of another life than the life insured, or on the happening of other contingencies, and it therefore appears that the existing practice being so universal, is the outcome of the absence in ordinary Life Insurance contracts of any security for the payment of the premiums. The presence of this security will remove the necessity for payment in advance, and the cases in which this condition is most likely to be found occur in connection with contingent reversions, reversionary life interests, and life interests in possession.

I propose to consider, from this new standpoint, the arrangements of premiums that would enable reversioners to insure to their estates the values of their interests without outlay until their reversions come into possession, the effect upon the values of their interests of so arranging the payments of their premiums, and lastly, an application of a similar method of deferring premiums to the case of a life interest.

A reversioner who is entitled to a contingent reversion, or a reversionary life interest, and has no wish to anticipate it in any way, may at the same time have an object in securing its value to his estate. He is not likely to be in a position to pay an annual premium sufficient to insure its full value, and will probably content himself with a small insurance, the amount of which bears no relation to the loss his estate will sustain by his early death. By any method now existing he can only secure the value of his reversionary interest, by borrowing a sum sufficient to commute the premiums during the joint lives of himself and the life tenant, and for this advance he must either give a reversionary charge upon his reversionary interest and insurance policy, or execute a mortgage of the same securities for the sum borrowed and accumulations of interest. The expense to which he is put by these methods is illustrated by the examples on pages 543 *et seq.*, and no one is likely to see

much inducement in the results to insure the value of his interests.

The case might, however, be dealt with in a way that would entail less expense if it is admitted that an insurance can be granted for a premium or premiums payable only when the reversionary interest comes into possession. To a proposer entitled to a contingent reversion an insurance might be offered, the consideration for which would be a single premium, payable only in the event of the life tenant predeceasing the reversioner. The net premium for this insurance would be $\frac{A_{xy}^1}{A_{xy}^1}$, or $\frac{P_{xy}^1(1+a_{xy})}{P_{xy}^1(1+a_{xy})}$, or $\frac{P_{xy}^1}{P_{xy}^1}$, the last form being the most suitable basis for the calculation of the office single premium. Care must be taken to obtain a single premium at least as remunerative to the office as an annual premium would be, and theoretically this can be accomplished by taking an office annual premium for P_{xy}^1 and a net premium at the rate of interest assumed to be realized for P_{xy}^1 . An excess of one per-cent in the rate of interest employed in the calculation of P_{xy}^1 over that upon which the office annual premium, P_{xy}^1 , is based, would probably be agreed upon as a practical interpretation of the theory, regard being paid to the fact that the profit from this excess of interest is secure whether or not the funds of the office realize one per-cent more than the rate upon which its premiums are based.

In the following table the single premiums, payable only when the contingent reversion comes into possession, are calculated on the Carlisle Table of Mortality, a 3 per-cent premium with a loading of 3s. and a nineteenth being employed in the numerator, and a 4 per-cent net premium in the denominator.

Single Premiums, payable as described, for a Contingent Insurance of £100.

Age of Reversioner	AGE OF LIFE TENANT				
	30	40	50	60	70
	£	£	£	£	£
20	84.5	55.1	35.0	19.0	10.4
30	123.5	78.1	47.4	25.2	13.9
40	...	120.0	69.2	34.1	18.1
50	116.9	53.1	25.0
60	111.9	53.7

A proposer entitled to a reversionary life interest might be offered an insurance under which an annual premium would be

payable during the remainder of his life after the death of the life tenant, the first payment falling due six months after he entered upon his reversionary life interest. This premium is correctly represented by $\frac{P_x(1+a_x)}{a_x-a_{xy}}$, which may be denoted by $a_{xy}P_x$. When office premiums are being calculated, the rate of interest in the annuities should be considerably higher than the rate on which the office annual premium, P_x , is based, in order to secure a sufficient profit from excess interest.

The following specimens of annual premiums, payable during the survivance of the reversioner after the death of the life tenant, are based on the whole-life premiums undernoted, with Carlisle $4\frac{1}{2}$ per-cent annuities in their conversion.

Annual Premiums, payable as described, for a Whole-Life Insurance of £100.

Age of Reversioner	Whole-Life Premium	AGE OF LIFE TENANT									
		30		40		50		60		70	
		£	s. d.	£	s. d.	£	s. d.	£	s. d.	£	s. d.
20	1 12 6	9 2 5	6 16 8	5 1 3	3 10 6	2 14 4					
30	2 0 0	12 18 5	9 11 1	6 18 1	4 12 7	3 9 8					
40	2 14 6	...	15 11 4	10 19 9	6 19 4	5 1 0					
50	4 0 0	20 11 2	12 1 10	8 5 8					

The advantages to the reversioner of the proposed arrangements of premiums can be most clearly illustrated by comparison of numerical results.

A proposer, aged 30, entitled to a contingent reversion of £1,000 on the death of a life tenant, aged 50, would, under existing methods, borrow a sum sufficient to commute the annual premiums, equal to $\text{£}1,000 \frac{P_{xy}(1+a_{xy})_4}{1-d_3}$, or $\text{£}178\cdot6$, assuming the office to be content to commute the premiums by a 4 per-cent annuity. In consideration of this advance he would give a charge upon his reversion and insurance of $\frac{\text{£}1,000 P_v(1+a_{xy})_4}{1-d_3(1+a_{xy})_3}$, or $\text{£}165\cdot8$, the reversion and insurance combining to make up an absolute reversion on the failure of the joint lives. When this charge is deducted there remains payable out of the reversion, if the proposer survives the life tenant, or out of the policy-moneys if he predecease, $\text{£}1,000 \left(1 - \frac{P_v(1+a_{xy})_4}{1-d_3(1+a_{xy})_3}\right)$, or $\text{£}534\cdot2$. If

annuities at $3\frac{1}{2}$ per-cent had been used throughout, the charge would have been $\text{£}1,000 \frac{P_{xy}^1(1+a_{xy})_{3\frac{1}{2}}}{1-d_5(1+a_{xy})_{3\frac{1}{2}}}$, or $\text{£}486\cdot6$, and the net result to the reversioner or his estate, $\text{£}1,000 \left\{ \frac{1-(P_{xy}^1+d_5)(1+a_{xy})_{3\frac{1}{2}}}{1-d_5(1+a_{xy})_{3\frac{1}{2}}} \right\}$ or $\text{£}513\cdot4$. The last expression, $\frac{1-(P_{xy}^1+d_5)(1+a_{xy})_{3\frac{1}{2}}}{1-d_5(1+a_{xy})_{3\frac{1}{2}}}$, represents the value of the contingent reversion divided by the value of the absolute reversion on the failure of the joint lives, and suggests that the net result to the reversioner might have been arrived at directly from the consideration that the office, in granting an insurance for a reversionary charge, was, in effect, converting the contingent reversion into an absolute reversion on the failure of the joint lives, of equal present value on the basis of calculation adopted.

By the new method the amount of insurance required is not $\text{£}1,000$ as above, but $\frac{\text{£}1,000}{1+\frac{P_{xy}^1}{P_{xy}^1}}$, or $\text{£}678\cdot4$, the single premium on

which, payable only when the reversion comes into possession, is

$\text{£}1,000 \left\{ \frac{\frac{P_{xy}^1}{P_{xy}^1}}{1+\frac{P_{xy}^1}{P_{xy}^1}} \right\}$, or $\text{£}321\cdot6$. This single premium is the only

deduction to be made from the reversion, and the net result to

the reversioner is, therefore, $\text{£}1,000 \left\{ 1 - \frac{\frac{P_{xy}^1}{P_{xy}^1}}{1+\frac{P_{xy}^1}{P_{xy}^1}} \right\}$, or $\frac{\text{£}1,000}{1+\frac{P_{xy}^1}{P_{xy}^1}}$,

equal to the amount of the insurance, $\text{£}678\cdot4$. Should the reversioner predecease the life tenant, the whole amount of the insurance, without deduction, becomes part of his estate. Later on it will appear that this result also might have been arrived at from the consideration that the office practically exchanges an absolute reversion on the failure of the joint lives for the contingent reversion in connection with which the insurance is effected, the amounts of the two reversions being in the inverse ratio of the present values.

If the proposer, aged 30, were entitled to a reversionary life interest of $\text{£}100$ per annum after a life tenant, aged 50, he would, under existing methods, borrow the present value of the premiums payable during the joint lives of himself and the life tenant.

The amount of insurance required to secure the value of his reversionary interest would be $\frac{£100}{P_x + d_5}$, or £1,179, and the value of the premiums during the joint lives, $£100 \frac{P_x(1 + a_{xy})_4}{P_x + d_5}$, or £366·6, again assuming the office to commute the premiums by a Carlisle 4 per-cent annuity. For the advance of this sum of £366·6 the reversioner would grant a reversionary charge of $£100 \frac{P_x(1 + a_{xy})_4}{P_x + d_5} \frac{1}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}}$, or £956, on the insurance at his death, interest to be paid at the rate of 5 per-cent on the charge during his survivance after the life tenant. The annual outlay for premiums and interest on the reversionary life interest coming into possession would be $£100 \left\{ \frac{P_x}{P_x + d_5} + \frac{d P_x(1 + a_{xy})_4}{P_x + d_5} \frac{1}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right\}$, or $£100 \frac{P_x}{P_x + d_5} \left(1 + \frac{d_5(1 + a_{xy})_4}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right)$, or £75·1, leaving, as a net result to the reversioner, an annual income of $£100 \left[1 - \frac{P_x}{P_x + d_5} \left(1 + \frac{d_5(1 + a_{xy})_4}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right) \right]$, which reduces to $\frac{£100 d_5}{P_x + d_5} \left(1 - \frac{P_x(1 + a_{xy})_4}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right)$, or £24·9, during his survivance after the life tenant, and an insurance of $£100 \left\{ \frac{1}{P_x + d_5} - \frac{\frac{P_x(1 + a_{xy})_4}{P_x + d_5}}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right\}$, or $\frac{£100}{P_x + d_5} \left(\frac{1 - d_5(1 + a_{xy})_{3\frac{1}{2}} - P_x(1 + a_{xy})_4}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right)$, or £523, to better his estate at his death.

With $3\frac{1}{2}$ per-cent annuities throughout, the net results to the reversioner would be an income of $\frac{£100 d_5}{P_x + d_5} \times \frac{1 - (P_x + d_5) \frac{1 + a_{xy}}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}}}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}}$, or £22·8, and an insurance of $\frac{£100}{P_x + d_5} \left(1 - \frac{(P_x + d_5) \frac{1 + a_{xy}}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}}}{1 - d_5(1 + a_{xy})_{3\frac{1}{2}}} \right)$, or £480, to fall into his estate.

Under the new methods the amount of insurance required is only $\frac{£100}{a_{xy} P_x + d_5}$, or £857, and the annual premium, payable when the reversionary life interest has come into possession, is $\frac{£100 a_{xy} P_x}{a_{xy} P_x + d_5}$, or £59·2. This premium is the only deduction

from the income, and the insurance is payable without deduction. The net results are, therefore, an income to the reversioner after the death of the life tenant of $\frac{\text{£}100 d_5}{a_{xy} P_x + d_5}$, or £40·8, and an insurance at death of $\frac{\text{£}100}{a_{xy} P_x + d_5}$, or £857.

Insurances under which the premiums are payable in either of the new ways proposed could only be granted when a charge upon the reversionary interest is given, and the office is satisfied that the payments are well secured. The new methods would be wholly inapplicable when the security offered, either on account of the investment of the funds in reversion, the powers to vary the investments, the character of the trustees, or from any other cause, was not such that the office might with confidence lend upon it if asked to do so.

This class of insurance would occupy a unique position at investigations. The risk under the insurance would run concurrently with the chance of receiving the single premiums secured upon the contingent reversion, and of entering upon the annual premiums secured upon the reversionary life interest, and it follows that the values may not always be positive. In the case of a contingent reversion when the reversioner and life tenant are of equal ages no values will attach on a net premium valuation, and a glance at the table of single premiums given above will show that in most of the cases likely to occur in practice a negative value will result. In the case of a reversionary life interest negative values will seldom result during the existence of both lives, but when the life tenant is dead, and the reversioner has not attained the age at which the premium for an insurance on his life is at least equal to the premium payable under the policy, the value will be negative. These negative values would, however, be legitimate, the payment of the premiums being absolutely secured, and a lapse being impossible. When a surrender is proposed the office will be in a position to settle upon what terms it will release its charge for the premiums on being relieved from further risk under the insurance, regard being paid to the health of the life tenant, and to the securing of whatever may be considered a fair profit in the circumstances. In most cases a payment would be required from the policyholder.

The effect of arranging the premiums in the ways indicated above is to enhance materially the value of contingent reversions and reversionary life interests, by a reduction of the outlay and of

the amounts of annuity and insurance required. The value of a contingent reversion now represented by $1 - (P_{xy}^1 + d)(1 + a_{xy})$ will be expressed by $\frac{1 - d(1 + a_{xy})}{1 + \frac{P_{xy}^1}{P_{xy}^1}}$, arrived at as follows:

For each $1 + \frac{P_{xy}^1}{P_{xy}^1}$ of the contingent reversion an insurance of 1 is effected, the single premium for which, as demonstrated on page 538, is $\frac{P_{xy}^1}{P_{xy}^1}$. With this insurance effected, the value of the contingent reversion to $1 + \frac{P_{xy}^1}{P_{xy}^1}$ is equal to the value of an absolute reversion of 1 on the failure of the joint lives, or $1 - d(1 + a_{xy})$, and the value of a contingent reversion to 1 must therefore be $\frac{1 - d(1 + a_{xy})}{1 + \frac{P_{xy}^1}{P_{xy}^1}}$.

The following table gives a comparison of the values by the existing and the proposed methods, the premiums being those mentioned or tabulated above, and the rates of interest in the numerator being 5 per-cent for d , and $3\frac{1}{2}$ per-cent for the annuity.

Values of a Contingent Reversion to £1,000.

Age of Reversioner	AGE OF LIFE TENANT					
	30	40	50	60	70	
	£	£	£	£	£	
20 {	Negative	94	218	400	568	By Jellicoe's method.
112		176	270	424	578	By proposed method.
30 {	Negative	69	197	382	552	By Jellicoe's method.
111		170	260	412	566	By proposed method.
40 {	...	33	162	356	534	By Jellicoe's method.
...		157	242	395	551	By proposed method.
50 {	105	307	505	By Jellicoe's method.
...		...	209	360	527	By proposed method.
60 {	215	414	By Jellicoe's method.
...		290	452	By proposed method.
30 {	...	163	250	396	542	By Sprague's method 1 6 7/10 for d and for
...		195	273	408	547	By proposed method 1 the annuity-value

In further illustration we may take the case of a reversioner aged 30 entitled to a contingent reversion of £1,000 on the death

becomes $a_{xy} P_x$, the value of the reversionary life interest is $\sqrt{v_3} \left\{ \frac{1 - d_3(1 + a_{xy})_{3\frac{1}{2}}}{a_{xy} P_x + d_3} \right\}$.

Similarly the formula $\sqrt{v_6} \left\{ \frac{1 - (P + d_6)(1 + a_{xy})_6}{P + d_3} \right\}$ will become $\sqrt{v_6} \left\{ \frac{1 - d_6(1 + a_{xy})_6}{a_{xy} P_x + d_3} \right\}$.

The next formula has to undergo some change in appearance

when the altered conditions are applied. $v_5 - \frac{1}{2} P$
 $P + d_3 - a_{xy} =$
 $\frac{1}{P + d_3} - (1 + a_{xy})_{3\frac{1}{2}} + \frac{\Lambda_{x5}}{2} - \frac{\Lambda_{xy3\frac{1}{2}}}{2}$ approximately. This, when the premium during the joint lives is nil, becomes $\frac{1 - d_3(1 + a_{xy})_{3\frac{1}{2}}}{a_{xy} P_x + d_3} + \frac{\Lambda_{x5}}{2} - \frac{\Lambda_{xy3\frac{1}{2}}}{2}$.

In the fourth formula, $(1 - \frac{1}{2} d_3) \left\{ \frac{1 - (P + d_6)(1 + a_{xy})_6}{P + d_3} \right\}$, P in the numerator again represents the premium payable during the joint lives, and the formula, when that premium is nil, becomes $(1 - \frac{1}{2} d_3) \left\{ \frac{1 - d_6(1 + a_{xy})_6}{a_{xy} P_x + d_3} \right\}$.

The following table gives the values by the first pair of formulas with the premiums already tabulated.

Values of a Reversionary Life Interest of £100 per annum.

Age of Reversioner	AGE OF LIFE TENANT					
	30	40	50	60	70	
	£	£	£	£	£	
20	Negative	40	224	513	787	By existing method.
	145	230	361	594	833	By proposed method.
30	Negative	15	180	452	714	By existing method.
	136	207	321	536	763	By proposed method.
40	...	Negative	93	339	588	By existing method.
	...	165	254	441	648	By proposed method.
50	Negative	194	415	By existing method.
	175	319	493	By proposed method.

The values by all the formulas are set out below in the order in which the formulas appear above for the case of a reversioner aged 30 and a life tenant aged 50.

By existing methods, £180, £275, £171, and £276.

By proposed methods, £321, £335, £313, and £337.

This case, on the first formula by the proposed methods, would work out as follows:

The amount paid to the vendor is . . . £321

The cost of an annuity of £39. 17s., to pay
interest at 5 per-cent on the outlay, is . . . 476

The total outlay is . . . £797

The amount of insurance effected is $\frac{£100}{a_{xy}P_x + d_5}$, or £857, and this, multiplied by $v\sqrt{v}$, equals the outlay, \sqrt{v} appearing as a result of the formula employed.

Various arrangements of premiums, dependent upon the condition that security for payment is obtained, might be made in connection with reversionary life interests with much less sweeping departure from existing practice. The first to occur to one is that of charging the premium for a survivorship insurance during the joint lives, and the whole-life premium at the age next birthday of the reversioner at the death of the life tenant for the remainder of the reversioner's life. This arrangement, it will be seen, is practically independent of the condition referred to, but is open to the serious objection that the premiums after the failure of the joint lives could not be fixed at the outset. To obviate this we must fall back upon the security for payment of premiums, and, with its help, fix a premium at the outset which will be equally profitable to the insuring office. The value of the equal premiums for the whole of life is $P_x(1 + a_x)$, and, deducting from this the value of the payments of P_{xy}^1 during the joint lives, we have $P_x(1 + a_x) - P_{xy}^1(1 + a_{xy})$, to be met by premiums payable during the survivorship of x after y . The survivorship annuity is $a_x - a_{xy}$, and the annual premium payable after the death of y is therefore $\frac{P_x(1 + a_x) - P_{xy}^1(1 + a_{xy})}{a_x - a_{xy}}$.

In the calculation of this premium annuity-values at a high rate of interest should again be adopted, in order to secure at least as much profit from interest as on an ordinary annual premium basis. A fair result would, I think, be obtained by the employment of Carlisle $4\frac{1}{2}$ per-cent annuities, with the same annual premiums for P_x and P_{xy}^1 as before. Taking, as an example, the case we have made use of throughout of a reversioner

aged 30, and a life tenant aged 50, we find that on the above basis the premium payable during the joint lives is £1. 8s. 10*d.* per-cent, and the premium payable after the death of the life tenant, £3. 7s. 5*d.* per-cent. Substituting these premiums in the formula $\sqrt{r} \left\{ \frac{1 - (P + d_5)(1 + a_{xy})^{31}}{P + d_5} \right\}$, the premium of £1. 8s. 10*d.* per-cent in the numerator, and the premium of £3. 7s. 5*d.* per-cent in the denominator, as we have already seen we are entitled to do, the value of a reversionary life interest of £100 per annum is found to be £236 against £180 by the existing method, and £321 by the method proposed in the preceding pages.

It does not require proof, because it is evident, that the smaller the premium charged during the joint lives the higher will be the value of the reversionary life interest, and it is my contention that though the departure from existing practice is greater the smaller we make that premium, and greatest when we eliminate it altogether, there is nothing unsound in the character of the departure, and nothing in the nature of the proposed methods to prevent their introduction into practice.

The insurance in connection with a life interest in possession is, in natural course, effected by an annual premium payable in advance, and, before describing the application of a deferred premium to this case, I may say that I do not attach to it much practical value, and have added it merely to include the third interest directly dependent upon life insurance with which Life Offices are in the habit of dealing.

The first n payments of the life interest might be applied wholly in payment of interest and in repayment of the advance or purchase price, and, to cover the balance outstanding in each year until the repayment was accomplished, a temporary decreasing insurance would be effected, the premiums under which would be payable only after the insurance had expired. The purchase price or advance being repayable in n years, it follows that its amount is $a_{\overline{n}|}$, and that the repayments in the first, second, and third, &c., years will be v^n , v^{n-1} , v^{n-2} , &c., respectively. Neglecting interest in the year of death, the insurance required will be $a_{\overline{n}|}$ at the outset, and will diminish by v^n at the end of the first year, v^{n-1} at the end of the second year, v^{n-2} at the end of the third year, and so on, until its extinction at the end of n years, when the whole purchase price or advance has been repaid. Interest on the amount outstanding for the portion of the year of death elapsed before payment of the policy-moneys must also be insured, half a

year's interest on the average with immediate payment of claim, or a year's interest if we assume payment to be made at the end of the year of death. On this latter assumption the annual premium, commencing with a payment one year after the expiry of the insurance, may be represented by

$$\frac{(1+i)\{v^n A_x + v^{n-1} A_x + \dots + v A_x\}}{a_x - a_x \underset{n}$$

$$\text{or } \frac{(1+i)\{v^n P_{x1} + v^{n-1} P_{x2} (1 + a_x) + \dots + v P_{xn} (1 + a_x)\}}{a_x - a_x \underset{n}$$

an insurance for one year only being required in respect of v^n , the repayment at the end of the first year; for two years only of v^{n-1} , the repayment at the end of the second year, and so on. The value of the life interest may be determined by equating the expression for the premium to unity and finding the value of n in the equation by trial, the remainder of the life interest after the expiry of n years being available in its entirety for the payment of the premiums. I have calculated the values for ages 30, 40, and 50 on the H^M Table, assuming $3\frac{1}{2}$ per-cent premiums with a loading of 15 per-cent, $4\frac{1}{2}$ per-cent annuities, and $4\frac{1}{2}$ per-cent as the return on the investment. The results are 15.687, 13.941, and 11.617 respectively.

The calculation is a laborious one, but a table of values at every age may readily be made up from accurate calculations at a few ages. The values given above are respectively 6.2 per-cent, 7.2 per-cent, and 8.2 per-cent in excess of the ordinary values on the basis of H^M $3\frac{1}{2}$ per-cent premiums with a loading of 15 per-cent, and it may fairly be assumed that for other ages the results would not be very dissimilar. In these examples n is not an integer, and if a decreasing temporary insurance were ever to be effected in connection with a life interest, it would be necessary, I think, to take the integral value of n next lower as the basis of the arrangement.

A very much simpler formula for the premium on the temporary decreasing insurance can be obtained, though, as it is not based on an office annual premium, it would not be easy to fix the loading that should be added with a view to securing profit from interest, unless in the first instance some examples had been calculated by the formula already given. Neglecting the interest to be insured in the year of death, the insurance required in the

first year is a_n , in the second a_{n-1} , in the third a_{n-2} , and so on, and the present value of the total insurance is therefore

$$a_n \frac{1}{x} \Lambda_x + a_{n-1} \frac{x^{l_x+1}}{l_x} \Lambda_x + a_{n-2} \frac{x^{l_x+2}}{l_x} \Lambda_x + \dots$$

$$+ a_1 \frac{x^{n-1} l_x + n - 1}{l_x} \Lambda_x + n - 1,$$

or, in commutation symbols,

$$\frac{a_n^- C_x + a_{n-1}^- C_{x+1} + a_{n-2}^- C_{x+2} + \dots + a_1^- C_{x+n-1}}{D_x}.$$

But $a_{\bar{n}!}$ is equal to $\frac{1-r^n}{i}$, and we can therefore break up the numerator into two parts, and obtain

$$\frac{(C_x + C_{x+1} + C_{x+2} + \dots + C_{x+n-1}) - (r^n C_x + r^{n-1} C_{x+1} + r^{n-2} C_{x+2} + \dots + r C_{x+n-1})}{i D_x}$$

which, if the same rate of interest is employed throughout, and for C_x in the second portion of the formula we write $e^x d_x$, can be converted into

$$\frac{\{(\mathbf{M}_x - \mathbf{M}_{x+n}) - v^{x+n} \sum^n d_x\}}{i \mathbf{D}_x},$$

The annual premium, beginning with a payment at the end of $n + 1$ years, will be represented on this basis by

$$\frac{\{(\mathbf{M}_x - \mathbf{M}_{x+n}) - t^{x+n} \sum^n d_x\}}{i \mathbf{N}_{x+n}}.$$

The values of the life interests at ages 30, 40 and 50 given above, are equal to the values of annuities-certain for 27·81, 22·43, and 16·81 years. Taking them at 27, 22 and 16 years respectively, the deferred premiums calculated accurately by the first formula are respectively ·882, ·917 and ·820. The net premiums by the formula last stated are respectively ·710, ·723 and ·626, which, with a loading of 30 per-cent, become respectively ·923, ·940 and ·814, so that it might be accepted that with double the loading (30 per-cent against 15 per-cent) that would be imposed on an ordinary whole-life premium, the results would be satisfactory so far as profit on the insurance is concerned. Had 5 per-cent been employed throughout in place of $4\frac{1}{2}$ per-cent, the net premiums by the last formula would have been ·777, ·781 and ·666 respectively, rates appreciably larger than the $4\frac{1}{2}$

per-cent results, and that in spite of the purchase price or advance, and consequently the insurance, being smaller. This is a result of the premiums being deferred until after the risk has been run, the interest on that account acting on a sum owing by the policyholder to the office, and not as in ordinary insurance contracts on a sum owing by the office to the policyholder.

DISCUSSION.

Mr. H. W. MANLY said that there was something novel and ingenious about the paper under discussion. It seemed that the object had been in some manner to increase the price to be offered for reversionary charges. The shortest way to do that would be to reduce the rate of interest in the usual formulas; but no matter in what way the formula was produced, if the price was increased the return must necessarily be less. The author endeavoured to arrive at this result in another way, so that the purchaser should still have the same return for his investment, but the loss, if any, should fall on the office that insured the risk. He divided the reversion into two parts, and assigned to them totally different values; consequently, although the formulas were extremely ingenious, they were not homogeneous. The probability would be that if an office was prepared to purchase one portion of the reversion at one price, it would also be prepared to purchase the other portion on the same terms. In that case they would have the very simple formula that the value of a contingent reversion to x at the death of y would be the single premium for the assurance of a sum on the death of y in the lifetime of x calculated at $\frac{1}{4}$ per-cent interest. Thus, if they called R the "reversion", Mr. Paterson divided R into two parts, M and N , so that the price given for N should be the single premium to insure the sum M in the event of x dying in the lifetime of y . He then said that if he could find an office that was willing to purchase N at the price of

$$N \cdot A_{xy}^{\frac{1}{4}} \text{ } \frac{1}{4} \text{ per-cent}$$

he would purchase the reversion to M and the insurance to M effected by the sum given for N at the price of

$$M[1 - d_5(1 + a_{xy}^{\frac{3}{4}})];$$

but it had not occurred to him that if an office were willing to purchase N at the price,

$$N \cdot A_{xy}^{\frac{1}{4}} \text{ } \frac{1}{4} \text{ per-cent,}$$

it would also be willing to purchase M at the price of

$$M \cdot A_{xy}^{\frac{1}{4}} \text{ } \frac{1}{4} \text{ per-cent.}$$

The value of the whole reversion to R would then be

$$R \cdot A^{\frac{1}{4}} \text{ } \frac{1}{4} \text{ per-cent.}$$

If contingent reversions were as numerous as ordinary policies of life assurance, the offices might be willing to purchase them at something like that price, but while the risks were, as a rule, very heavy and the numbers very small, the offices must make ample provision against adverse contingencies, so that he did not think that the author would find any office willing to purchase any portion of the reversion at the price he assumed in his formula. There was something, however, in the suggestion that an office able to take the whole risk of *M.* might say that in consideration of getting the whole transaction they would set up a policy on the terms suggested by the author. These transactions very often ran into large figures; and when re-insurances had to be effected he could not think that any office would be willing to grant a policy on the terms proposed.

MR. A. F. BURRIDGE said that passing to some of the difficulties in connection with Mr. Paterson's method, the gist of the argument was that by this means they could bring out superior values for a contingent reversion. He contrasted the author's value with other figures brought out by Jellicoe's formula. Mr. Sprague in his formula (vol. xiv, p. 121) commuted the discount and annuity on joint lives at 6 per-cent, and working out the example given by Mr. Paterson by Mr. Sprague's formula, the result would be found, under an ordinary policy, very close to Mr. Paterson's figures. But Mr. Paterson would say that if he used Mr. Sprague's formula in his kind of policy, his value also would rise still more. The difficulty was that with an office buying a contingent reversion of £1,000, the policy set up by this method was only £678. It, therefore, appeared that they did not cover the loss which might be made on the reversion; in other words, the premium account was starved at the expense of the profit on reversion account. It might be replied that that was a matter of no consequence outside the office; it was simply a matter between the office as an insurance and investment office, provided the office retained the whole of the assurance. These matters, however, often ran to very large figures. Supposing, for instance, the reversion was £80,000, and that the office retained £10,000 of the insurance and re-assured £70,000,—then one of three things would happen. Supposing the life tenant died early, the investing office would have undertaken to pay the re-assuring offices a much higher premium than was necessary, owing to the manner in which the premium was calculated. In the second possible case, supposing the joint lives endured for a longer period than the expectation showed them, the policy would be unprofitable, and to that extent the re-assuring offices, which had no share in any possible profit which might arise, would incur considerable loss from the very peculiar form of policy which had been set up. There was yet a third case—namely, where the reversioner predeceased the life tenant, and in that case policies, unprofitable in the second case, would be still more unprofitable, because there was no premium whatever payable. Therefore they had the offices who bore the burden of the major portion of the assurance standing the greatest possibility of a large loss, since life tenants like the annuitant were not, as a rule, people who died at an early date. In dealing with large transactions, it was necessary to observe perfect comity between the offices entering into them, which

could hardly exist if it was known or suspected that the purchasing office might make a profit, whereas, owing to this unusual form of policy the re-assuring offices stood a greater chance than they did of making a loss. He reminded the Institute of the practical difficulties which sometimes surrounded transactions with contingent reversions. In the case of a charge under a base fee, for instance, when the tenant for life died, it became necessary to produce the reversioner and to complete the security by obtaining his personal signature to the disentailing deed. It had been held that that could be done by power of attorney, but the balance of the best legal opinion was that the signature of the disentailing deed was a purely personal matter. The reversioner was not a person upon whom they could always lay their hand, and it might be that, having stripped himself of every interest in his reversion, he had left the country. He cited the instance of a reversioner who had executed a deed in favour of a certain office, and when the tenant for life died was found in prison. When the deed was presented for his signature he feigned inability to sign. Ultimately he went out of his mind. Happily for the office he was not a convict (in which case he would have been legally disqualified from signing), but was only a misdemeanant, and therefore the court instructed that the deed should be signed on his behalf. That was a practical instance of the difficulties that arose and of the necessity for preserving a good feeling between the offices, for all the time that these negotiations were going on, the policy was running off, and if the deed had not been signed within a year or eighteen months subsequent to the death, the security would have been in jeopardy. In another case, when the life tenant died, the reversioner, who had parted with all interest in his reversion, was literally a beggar in America. He got into the hands of a financier, who, knowing that his signature was of some value, insisted on certain terms before his signature could be obtained. This showed that these questions involved some of the most complicated and difficult operations with which they were concerned.

Mr. R. P. HARDY said that instead of recognizing that there had been a rise in the market value of reversions, all their formulas and endeavours were merely a silent recognition of that fact, and an attempt to cover it with a scientific justification. The subject now came before them under quite a novel aspect. It exhibited great ingenuity; but it seemed that it contained a fallacy, as shown by the following formula:

$$A_{xy}^1 + A_{xy}^1(1-P) = A_{xy}^1;$$

whence

$$P = \frac{A_{xy}^1}{A_{xy}^1}.$$

He could understand that the values came out higher, because they had left a large portion of the interest uncovered, and had made the seller a present of that which they had been imprudent enough to reject themselves. Under those circumstances while Mr. Paterson's formula might be applied to a certain number of limited cases, still for the majority of such transactions they must fall back upon the older and sounder methods.

Mr. G. KING said the object of the paper seemed to be to produce larger prices for reversions, not with the object of reducing the profit of the company, but by saving insurance. In that, however, the author did not succeed. He called Mr. Jellicoe's formula the existing formula, and showed that that involved, according to his views, larger insurances than those by the new method, but on analysis it really was not so. It could be shown that by reducing the existing formula, as Mr. Paterson called it, in one sense the policies need not be so large as the whole of the reversion. Mr. Paterson adduced the example of a man wishing to cover a contingent reversion which he possessed, so that the estate might not lose in the event of his dying before the life tenant. After all, the principles involved were identical with those where an office purchased a reversion, and it was easier to retain the primitive idea. If by A we represent the single premium for a contingent assurance, and by A^1 the value of a contingent reversion, and by C the charge to be given by a reversioner in order to secure a policy to cover the balance of his reversion; then $1-C$ is the amount of policy to be effected, for which the single premium is $A(1-C)$, and the charge in respect thereof is

$$\frac{A \cdot (1-C)}{A^1},$$

which is equal to C ; whence

$$C = \frac{A}{A + A^1}.$$

So far as the reversioner is concerned, the amount of policy to be effected is, therefore, $1-C$, and not the whole amount in reversion, and here the old formula agrees exactly with that of Mr. Paterson. The office might set up a policy in its books to cover the reversionary charge of C , but that had nothing to do with the reversioner. Under Mr. Paterson's methods exactly the same thing applied. The office had to pay a policy of the amount insured by Mr. Paterson's method, but if the claim ever became payable it was under a further liability for the premium which it would lose, and, as a matter of fact, the two together were exactly the amount in reversion. In the "existing formula" the amounts to be insured on behalf of the reversioner and on behalf of the company came to the same amount. The two formulas, so far as theory was concerned, were absolutely identical. The virtue of Mr. Paterson's method was in the loading, and the reason why he produced a larger value was simply that he loaded his premiums less. It was not an object to be aimed at to increase the prices of reversions simply by diminishing the loading on their premiums.

Mr. W. KING said the paper contemplated only certain "secured" contingent reversions, in which the author's plan could be carried out, and its scope would necessarily be limited. The purchaser of a contingent reversionary interest, in order to make it an investment, must secure by assurance the full amount of the interest he purchased, just as a reversionary company had to do. The paper proposed to assure only a portion of this amount, so that the purchaser would buy a part of the contingent reversion without any

assurance. At the present time, if a man wished to sell a contingent reversion, they could offer him a certain price, and beyond that they would have to pay a certain premium down. According to the proposal made by Mr. Paterson, there would be no occasion to pay the premium down, but instead a larger sum was to be secured, payable when the reversion came in. Supposing the reversioner did not die, then they got the contingent reversion without paying anything for premium, unless it was secured; and if it was to be secured the difference was nothing from the present system, except that the reversionary society kept the premium in its pocket with interest, and paid it at the end; whereas, under the present system, it simply paid the premium necessary to secure the reversion to start with. It, therefore, seemed to him that, where the premium was secured, it could make no difference except the slight difference that might be made between the rate of interest at which the reversionary company accumulated its funds and the rate at which the insurance company accumulated theirs.

Mr. S. G. WARNER said that the true way of looking at the suggested method of dealing with contingent reversions was that two purchases were made. The contingent reversion was purchased, and then the single premium for the necessary insurance, which otherwise would be taken by the office, was itself re-invested in the purchase of a further reversionary charge, and that investment charge was unprotected by insurance. Separate investments were thus made by the office, one at 5 per-cent and the other at 4 per-cent, and one essential point of controversy was, were they to enter into contingent reversionary transactions at a rate of interest of 4 per-cent? It was of no use to contend that this was only a question of taking a premium at the end instead of the beginning of the transaction. It was an *investment* of premiums, and the whole of their business lay in investing premiums. The premium was, according to the author's suggestion, re-invested in the purchase of a contingent reversionary charge at 4 per-cent. That was the reason why the better prices came out. The investment was split into two parts, one at 4 per-cent and uncovered by insurance, and the other at 5 per-cent, and it was thus that they were enabled to offer better terms than before. In the case of the reversionary life interest the same element occurred again, only in a more subtle form. There they were asked to buy a reversionary annuity of a surplus premium of such an amount as would enable them to forego present premiums, at $4\frac{1}{2}$ per-cent. But this rate affected the premium throughout the whole transaction, and in a reversionary life interest the premium was a very important element. In considering reversionary life interests they were dealing with very large figures, and a slight manipulation of the mathematical formulas would make a startling difference in the result. He failed to see the logic of Mr. Paterson's manipulation of Mr. Sprague's formulas. They must agree that the primary assumption involved therein was that the money employed in this transaction was improved at rates of interest not lower than 5 per-cent in possession and 6 per-cent in reversion. Here, however, there was a proposal to invest the premiums, whether in possession or reversion, at $4\frac{1}{2}$ per-cent. That seemed to vitiate the basis of Mr. Sprague's formulas,

and therefore he saw no logical significance in the figures brought out when Mr. Paterson stated that the new method gave £335 and £337, as against Mr. Sprague's £275 and £276. Even assuming the customary rates (*i.e.* commuting the premiums for life at $3\frac{1}{2}$ per-cent and taking the reversionary annuity at 5 per-cent), they would lose by the transaction. It would be thrown into the form least advantageous to the office; reducing the amount of policy (which presumably was a source of profit), reducing the amount of investment, giving away for no reason at all the benefit of selection, and yet increasing the outlay. The essential question was the rate of interest. Most of them had some idea of their difficulties in business, and the falling rate of interest was among the most serious. Here was a class of investments upon which they could get at present a fairly remunerative rate, and they looked to this to recompense them for reduction in other cases. Mr. Paterson went above the valuation rate, and appeared to think he did well. He (Mr. Warner) thought they should rather take their own mean rate realized, and try to get a little above that, because if they did not get above that on this class of investments they certainly would on no other.

Mr. A. D. BUMSTED said the whole discussion had been with regard to borrowers, or persons selling reversions, but in his paper Mr. Paterson referred to a reversioner who had no wish to anticipate in any way his reversion, and who might have an object in securing its value to his estate, and Mr. Paterson proposed to effect a policy which should secure the reversion, so that if the owner died before it fell in, his family might have the benefit. There was a way of doing it by insuring in the usual way, taking out an ordinary contingent policy. If the reversioner effected a policy under Mr. Paterson's scheme, he would have to prove his title to the reversion to the satisfaction of the insurance company; and to pay his own solicitor's fees and the fees of the office. He might then find that he had no title to the reversion at all. That would be sufficient to show that no person who wished to secure a reversion would be likely to do it in that way.

Mr. NIGHTINGALE stated that the chief object of the paper was the creation of a reversionary charge in respect of the premiums payable to cover a contingent interest and an examination of the effect of such charge on the present values. The result of an investigation showed that the favourable nature of Mr. Paterson's results, as compared with the ordinary values, was due to two causes:—1. The sacrifice of a portion of the premium loading when turning the values of the premiums into reversions. 2. Conversion of the premiums at a rate of interest below that actually realized on the whole security, as opposed to the formula rate used for discounting the premiums before conversion. He (Mr. Nightingale) said that after examination, he found the expression

$$\frac{P_{x,y}^1}{P_{x,y}^{-1}}$$

adopted by Mr. Paterson, should strictly be

$$\frac{P_{x,y}^1 + \phi}{P_{x,y}^{-1} - \phi}$$

where ϕ is the premium loading. The author of the paper omitted ϕ in the denominator. How far this was justifiable was an open question, but the omission was incompatible with Mr. Paterson's own assumption that the arrangement proposed must be at least as remunerative as the ordinary annual premiums. He objected to the introduction of a third rate of interest in the formula, thus still further obscuring the true rate yielded on the whole security. Further difficulties in the adoption of Mr. Paterson's suggestions were the necessity for contingent premiums at high rates of interest, the allowance for selection, and the sex of the life tenant.

Mr. COLENSO said that he would make an addition to Mr. King's demonstration of the theoretical consistency of Mr. Paterson's new formula for the value of a contingent reversion.

$$\left(1 + \frac{P^1_{x,y}}{P_{x,y}}\right)^{-1} [1 - (1 + a_{xy})d].$$

Taking the analogous formula for the value obtained on the assumption that the commuted single premium was made payable on the first death, this was easily shown to be

$$\left(1 - \frac{P^1_{xy}}{P_{xy}}\right) [1 - (1 + a_{xy})d].$$

If the elements of this formula were made homogeneous it at once reduced, like the other formulas, to

$$A_{xy} - A^1_{xy}.$$

For comparison, they could throw Jellicoe's formula into the shape

$$\left(1 - \frac{P^1_{xy}}{P'_{xy} + d' - d}\right) [1 - (1 + a_{xy})d],$$

where P^1_{xy} represented the pure premium of the annuity table, Carlisle or Government experience. It would have been interesting if Mr. Paterson had developed these comparisons between the existing formulas and his own, so as to indicate analytically the exact points of difference. The new method did not seem likely to have much influence on market values, unless, as Mr. Burridge suggested, it was pushed still further. But the paper was an interesting one from a theoretical point of view, and it was impossible to say that it might not sooner or later in some way or other affect practice.

The CHAIRMAN (Mr. G. S. Crisford), in moving a vote of thanks to Mr. Paterson, thought they were bound to come to the conclusion that there were points which would make it almost impossible to bring the author's plans into practice, except in very isolated cases, and there would be dangers in connection with the adoption of the principle laid down. Amongst others, there would be the frequent danger of putting a heavy loss upon the re-assuring companies, whereas the reversionary transaction might result in a large profit. He did not know whether any office had been tested as to whether it would grant insurances of this kind, because unless offices were willing to transact re-insurances and wait an indefinite time for the payment of the

premiums a company would be quite unable to carry out these large transactions in the way that Mr. Paterson had foreshadowed. Payment of the premium in advance by the office effecting the reassurances, while having to depend for repayment thereof upon Mr. Paterson's suggested method, would completely upset the basis upon which the method itself was founded.

MR. PATERSON, in responding, said that in matters of this kind, when one departed from the ordinary accepted rules, criticism must be expected. A great deal had been said regarding the rate of interest. The rate of interest involved in the conversion of premiums was not material. He had no objection to its being made at 5 per-cent. The effect was not to materially diminish the value of the contingent reversion or reversionary life-interest. The increased values obtained by his methods resulted from a saving in outlay and a consequent reduction in the amount of annuity and insurance required. Mr. Manly had said that the result of his formula was to make the return to an office purchasing less, and he had indicated that the reason of the return being less was in some way connected with the loading on the premiums, and that part of the total outlay was uncovered by insurance. As to the question about a saving being effected by a reduction of loading, that was a misapprehension. The office annual premium was the basis of the premiums proposed to be charged and such mortality tables or equivalent ages in a mortality table as would be favourable to the office were intended to be used in their conversion. The outlay was entirely covered by insurance, the fact being that a smaller insurance was required because the outlay was less. Mr. Hardy had stated that a fallacy existed in the formula, but it had been demonstrated by Mr. King that on a net basis the formula was reducible to A_{xy}^1 , as every sound formula whether old or new must be. The departure from existing practice was not one of loading, rate of interest, or insufficient insurance, but rested solely upon the principle laid down in the first part of the paper, that if there was adequate security for the payment of premiums, it did not matter whether the premiums were paid at the beginning of the year or the end, or at some later date, provided the total value of the premiums received be the same.

ACTUARIAL NOTES.

I.—On a Formula for calculating Policy-values.

It may seem superfluous to add another to the numerous formulas for obtaining policy-values, which are duly set out on pages 314 to 323 of the *Text-Book* (Part II), but the formula now to be mentioned has been found of service under special circumstances, which may frequently arise. Let it be required to value in one total the policies on a particular page of the class-lists, and the only process at present available is that of

valuing the sums assured and the premiums separately. For valuation returns to the Board of Trade the separate valuing is necessary, but for many other purposes it is not necessary, and can be abolished with advantage.

Using x for the valuation age, S for sums assured and bonuses, and P for premiums to be valued, the formula is as follows:

$$V_x = \left(P + \frac{P}{i} + S \right) A_x - \left(P + \frac{P}{i} \right).$$

The following is the proof:

$$A_x = 1 - d(1 + a_x)$$

$$\therefore (1 + a_x) = (1 - A_x) \frac{1+i}{i}$$

$$P(1 + a_x) = P(1 - A_x) \frac{1+i}{i}$$

$$V_x = SA_x - P(1 + a_x) = SA_x - P(1 - A_x) \frac{1+i}{i}$$

$$= \left(S + P \frac{1+i}{i} \right) A_x - P \frac{1+i}{i}$$

$$= \left(S + P + \frac{P}{i} \right) A_x - \left(P + \frac{P}{i} \right).$$

As an illustration, required the value, at age 40, by H^M 3 per cent, of policies for £1,700, with bonuses of £30.05, subject to annual premiums amounting to £40.714 in all.

$$\begin{array}{rcl}
 40.714 & = & P \\
 1357.133 & = & P \cdot i^{-1} = \frac{100P}{3} \\
 1730.050 & = & S \\
 \hline
 3127.897 & & \\
 795074 & = & A_{40} \text{ (inverted)} \\
 \hline
 125116 & & \\
 21895 & & \\
 156 & & \\
 28 & & \\
 2 & & \\
 \hline
 1471.97 & & \\
 1397.85 & = & P + P i^{-1} \text{ (from above)} \\
 \hline
 74.12 & = & V_x \\
 \hline \hline
 \end{array}$$

The particular figures used are the values, taken together, of four policies surrendered in the same year, the question having arisen in the course of my investigation into the "Progress of Profit" (see page 523 of my paper in the present number of the *Journal*). There is room for use of the formula in all enquiries into expected mortality strain, involving, as they do, the calculation of a number of policy-values.

The formula is equally applicable to endowment assurances, using the proper value of $\Lambda_{x:n}$, but not to whole-life policies and endowment assurances taken together, because the values of Λ would not be the same.

II.—On the Transformation of Values from H^M to $H^{M.5}$ Mortality.

The process described in the previous note will enable us to pass readily from H^M to $H^{M.5}$ policy-values on the same data. There will be no change in the rate of interest nor in the premiums valued. The multiplication can be made by the difference of the Λ_x values, and the final subtraction will disappear in the difference of the results.

Thus, to transform the above-mentioned example into $H^{M.5}$:

$$\begin{array}{r}
 3127.9 \\
 181600 = \Lambda_{40}(H^{M.5} - H^M) \\
 \hline
 1877 \\
 125 \\
 25 \\
 \hline
 20.27 = V_x(H^{M.5} - H^M) \\
 74.12 = V_x(H^M) \\
 \hline
 94.39 = V_x(H^{M.5}) \\
 \hline
 \end{array}$$

This will help to remove a considerable difficulty in the way of an investigation into sources of profit, when a combined valuation is employed by a company.

It may be convenient to print the following table.

Difference between $H^{M(5)}$ and H^M Values of A_x at 3 per cent.

Age	Difference of A_x	Age	Difference of A_x	Age	Difference of A_x	Age	Difference of A_x
10	·017084	32	·009023	51	·003844	76	·000967
11	·018325	33	·008698	55	·003572	77	·000796
12	·019357	34	·008494	56	·003327	78	·000791
13	·020212	35	·008297	57	·003119	79	·000694
14	·020869	36	·007935	58	·002907	80	·000530
15	·021364	37	·007566	59	·002748	81	·000469
16	·021810	38	·007192	60	·002635	82	·000472
17	·022184	39	·006792	61	·002486	83	·000452
18	·022503	40	·006481	62	·002370	84	·000582
19	·022951	41	·006220	63	·002243	85	·000730
20	·023140	42	·006052	64	·002104	86	·000919
21	·022693	43	·005878	65	·001888	87	·001180
22	·021649	44	·005770	66	·001708	88	·001395
23	·020259	45	·005676	67	·001511	89	·001404
24	·018507	46	·005563	68	·001366	90	·001234
25	·016556	47	·005469	69	·001187	91	·001468
26	·014745	48	·005394	70	·001115	92	·001612
27	·013196	49	·005247	71	·001080	93	·001875
28	·011821	50	·005032	72	·001083	94	·001418
29	·010721	51	·004770	73	·001078	95	·001812
30	·009907	52	·004444	74	·001086	96	·005195
31	·009402	53	·004155	75	·001031	97	·029126

THOMAS J. SEARLE.

THE INSTITUTE OF ACTUARIES.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

Examiners—F. E. COLENSO, G. MARKS, G. F. HARDY,
and S. DAY.

First Paper.

1. A person (C) buys an estate, and some time afterwards disposes of it to another (D) for £5,400. At this price it yields D $2\frac{3}{4}$ per-cent. C previously obtained 3 per-cent for his investment, but his income from the estate, owing to less careful management, was £36 less than D's. What did C originally pay for the estate?

2. Extract the square root of $7 + \sqrt{13}$, and the cube root of $10 + \sqrt{108}$.

3. Solve the equations:

$$(i) \quad \sqrt{x^2 + 2x - 1} + \sqrt{x^2 + x + 1} = \sqrt{2} + \sqrt{3}.$$

$$(ii) \quad \frac{y}{x^2 - 3} = -\frac{x}{y^2 - 3} = \frac{7}{x^3 - y^3}.$$

4. Find the sum of the infinite series:

$$x + 4x^2 + 9x^3 + 16x^4 + \&c.$$

and of the series:

$$\frac{1}{8.18} + \frac{1}{10.21} + \frac{1}{12.24} + \frac{1}{14.27} + \&c. \text{ ad inf.}$$

5. A bag contains 9 counters marked 1 to 9. How many different numbers, each of nine digits, can be formed from five counters taken from the bag, and four other counters each marked 0?

6. A body is composed partly of copper and partly of tin. If the copper had been tin, and the tin copper, the weight of the body would have been greater by 9 per-cent than what it actually is. The weights of equal volumes of copper and tin are as 8.96 to 7.29. Find how much of the body is copper and how much tin.

7. Between what negative whole numbers do the logarithms of .02, .00197, and .00002576 to the base 9 lie?

Given that

$$\log_9 279936 = 7$$

find x and y from the equations:

$$\begin{aligned}\sqrt{x+y} &= 2, \\ (x+y)3^x &= 279936.\end{aligned}$$

8. Show that the annuity for n years which £1 will produce, less the annuity which will accumulate to £1 in n years, is equal to the interest on £1 for 1 year.

9. Two masons, one of whom, (A), commenced work a day and a half later than the other, (B), built a wall in $5\frac{1}{2}$ days. A could have built it alone in 3 days less than B. In how many days could the latter, working alone, have built it?

10. Prove that if $x=a$, $y=\beta$ be one pair of values satisfying in positive integers the equation

$$ax - by = c,$$

all possible values are given by the formulæ

$$x = a + bt, \quad y = \beta + at.$$

Find all the positive integral solutions of

$$4x + 5y = 39.$$

11. An observed event must have arisen from some one only of 3 causes, A, B, C, the antecedent probabilities of which are respectively $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, the probability that the observed event would follow in each case being 1, $\frac{1}{2}$ and $\frac{1}{3}$ respectively. What is the probability of the observed event having arisen either from B or C?

12. Equal straight lines in a circle are equally distant from the centre.

Find the locus of the middle points of equal chords of a circle.

Second Paper.

1. What is the object of a Waste Book? If a lease worth £100 per annum is purchased for £1,264, the investment yielding 5 per cent interest, what Ledger entries will be required during the first year?

2. Find the value of

$$\frac{1\frac{2}{3}\frac{9}{5}}{\frac{2}{6}\frac{7}{6}} \text{ of } \frac{\frac{2}{9} + \frac{3}{4} - \frac{4}{15}}{\frac{3}{34} - \frac{1}{24} + \frac{5}{84}}$$

3. Given P and Q, the p th and q th terms, find the series—

(1) in arithmetical progression,

(2) in geometrical progression,

(3) in harmonic progression.

4. Solve the equations :

$$(i) \begin{cases} x+y=a \\ x^3+y^3=b \end{cases}$$

$$(ii) \quad x^2 - y\sqrt{xy} = 336$$

$$y^2 - x\sqrt{xy} = 112$$

5. Assuming the Binomial theorem to be true when the exponent is a positive integer, show that it is true when the exponent is a positive fraction.

If N be a large number and a the integer next greater than its fourth root, show that

$$N^{\frac{1}{4}} = a \frac{3a^4 + 5N}{5a^4 + 3N} \text{ approximately.}$$

6. The worth of a diamond varies as the square of its weight, and the square of the value of a ruby varies as the cube of its weight. A diamond weighing a carats is worth m times a ruby of b carats, and both together are worth £ c . Find the value of a diamond, and of a ruby, each weighing n carats.

7. Find four equidifferent numbers such that their product shall be 384, and the sum of their squares 120.

8. Write down the expansion of a^x in ascending powers of x and

deduce therefrom the expansion of $\log_e (1+y)$ in ascending powers of y .

Show that the logarithm to the base e of the product of

$$(1+ax)^{\frac{1-ax}{2a}} \quad \text{and} \quad (1-ax)^{\frac{1+ax}{2a}} \\ = \frac{ax^2}{1 \cdot 2} + \frac{a^3x^4}{3 \cdot 4} + \frac{a^5x^6}{5 \cdot 6} + \text{ad inf.}$$

9. A capital of k is invested at compound interest at the rate of i per unit per annum. The accumulating fund is charged at the end of each year with a contribution to expenses of management $= \frac{i}{4}k$.

Show that the fund will amount by the end of n years to

$$\frac{3(1+i)^n + 1}{4}k.$$

10. A and B throw alternately with three dice. A undertakes to throw 9 before B throws 11. What is the chance that A wins the game, assuming that he has the first throw?

11. In a box are A counters marked with the number a , B marked with the number b , and C marked with the number c . A counter is drawn r times and each time replaced. What is the probability that the sum of the numbers drawn

(α) is equal to n ?

(β) is greater than n ?

12. Describe a parallelogram that shall be equal to a given triangle and have one of its angles equal to a given rectilineal angle.

Construct an isosceles triangle equal in area to any given triangle.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE, OR TO THE CLASS OF FELLOW (PART II).

Examiners—H. J. ROTHERY, ERNEST WOODS, R. CROSS,
and A. W. SUNDERLAND.

First Paper.

1. A lease was granted at a rental of £ s for a term of m years, of which x years have expired, the full annual value of the property being £($s+t$) per annum. Find—

(a) What sum should be paid down for the extension of the lease at the present rental for n years beyond the original term? What does this become if n is infinite?

(b) What sum should be paid down to purchase the freehold?

2. Each successive payment of an annuity-certain payable in advance is to accumulate until the first payment has doubled itself. To what sum will the whole annuity then amount? Prove the result by general reasoning.

3. Give a very brief description of the principal books and instruments available for the use of the actuary in arithmetical calculations.

4. Prove that—

$$A_{xy}^1 = \frac{1}{2} \left(1 - d(1 + a_{xy}) + v p_y(1 + a_{x, y+1}) - v p_x(1 + a_{x+1, y}) \right)$$

In using this formula to calculate an office rate how many places of decimals would you retain in the values of the annuities? In using the formula $A_{xy} = 1 - d(1 + a_{xy})$ to calculate an office rate, how many places of decimals would you retain in the value of the annuity?

Translate the formulas into Commutation symbols (De Morgan's form).

5. Interpret the following symbols—

$${}_n \overline{a}_{xyz}^2, {}_n a_{(xy)}, a_{xyz} \overline{a}_{xyz}, {}_1 A_{xyz}, {}_n V_{xm}.$$

6. Find the annual premiums for assurances for a term of n years on two lives aged x and y respectively—

(a) On the joint lives.

(b) On the life of the survivor.

If n is small how would you obtain a rough approximation to the office premiums from a table of office premiums for short term assurances on single lives?

7. Find the single and annual premiums for an assurance upon a life aged x , the sum assured increasing in amount at compound interest. What would be the policy-value after t years?

8. A whole life policy at an annual premium payable throughout life was effected eight years ago on a life then aged 25 next birthday, and a reversionary bonus B has been declared thereon. It is now desired to change the policy into an endowment assurance, payable on attaining age 60 or at previous death, and to commute all future premiums by 15 annual payments. Find a formula for the new annual premium.

9. Discuss Galloway's method of obtaining the numbers exposed to risk, and compare it with that adopted in compiling the Institute of Actuaries' Mortality Tables.

10. Explain in detail how you would calculate tables of temporary and deferred life annuities, and how you would verify them.

11. In what important legal respects do life assurance contracts differ from fire and marine insurances.

12. A loan of £25,000 was made 10 years ago to a Local Board and was repayable by 60 equal half-yearly payments, including principal and interest, the latter at the nominal rate of $4\frac{1}{2}$ per-cent per annum, but convertible half-yearly. Find—

(a) The equal half-yearly payment.

(b) The sum now payable to redeem the loan.

(A table of logarithms may be used for this question.)

Second Paper.

13. Explain the meaning of the term “Force of Mortality.”

Taking as the algebraic definition of the “Force of Mortality” the value to which the expression

$$-\frac{1}{l_x} \cdot \frac{l_{x+n} - l_x}{n}$$

approaches when n is made very small, prove that the force of mortality is the same for all ages if the law of mortality is expressed by the equation $l_x = l_0 a^x$.

On the same hypothesis as to the law of mortality, find an expression for the expectation of life.

14. A debt is to be discharged by n equal annual payments to include (a) principal, (b) interest and (c) the premium for an assurance to provide for the cessation of the annual payments and the extinction of the debt, if the debtor aged x die before the expiration of the n years. What should be the amount of the annual payment?

15. Find the value of an assurance payable at the death of (x), if he die before (y) or within t years after. If t is small, what means would you adopt to obtain an approximate value?

16. What is the meaning of ${}_n a_{y:x}$? Show how to calculate its value if interest is assumed at the rate i during the lifetime of (y), and at the rate j thereafter.

17. Show that

$${}_{m+n}V_x = 1 - (1 - {}_mV_x)(1 - {}_nV_{x+m})$$

and give a verbal interpretation of the formula

$${}_nV_x = A_{x+n} \left(1 - \frac{P_x}{P_{x+n}} \right).$$

18. Find the annual premium for an endowment assurance for n years upon a life (x), where the sum payable on survivorship is

double that payable if death occur before the attainment of the endowment age. Find also the policy-value and the amount of the paid-up policy when the assurance has been t years in force.

Would you expect to find the rate of premium increase rapidly with the age at entry? Give reasons for your answer.

19. If a population consists of l_x persons aged x , l_{x+1} aged $x+1$ and so on to the end of life, find—

- (a) the average single premium to provide an assurance of £1 at the end of the year of death of each person (irrespective of age).
- (b) the average value of an annuity of £1 payable during the lifetime of each person (irrespective of age).

20. How would you determine the gain or loss from mortality under the annuity contracts of an assurance office? Obtain a formula and explain how you would use it in practice.

21. How would you construct a mortality table from the results of two quinquennial censuses and the birth and death returns for the intermediate years?

22. Explain briefly the methods of construction and the uses of Orchard's tables.

23. State the provisions of the Gambling Act 1744 and of the Policies of Assurance Act 1867.

24. A bond for £100, bearing interest at 6 per-cent per annum, payable half-yearly, and redeemable at par at the end of 40 years from the date of issue, was issued at par 30 years ago, the present market value being £115.

- (a) Find the rate of interest yielded to a purchaser now buying at the market price.
- (b) Find the rate of interest obtained by the original holder when his profit on sale is taken into account.
- (c) If it were proposed to convert the bond into one for £125 bearing $3\frac{1}{2}$ per-cent interest, redeemable at par in 30 years, what gain or loss would there be to the holder of the bond on conversion?

(A table of logarithms may be used for this question.)

25. Discuss the merits of the following securities as investments for a life office, when trade is declining and the rate of interest on sound securities falling.

- (1) Railway ordinary stocks,
- (2) Railway perpetual debenture stocks,
- (3) Perpetual debenture stocks of trading companies.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III).*Examiners*—T. G. F. BROWNE, G. H. RYAN, and W. J. H. WHITTALL.*First Paper.*

1. Give the principal formulas which have been used or suggested for ascertaining the number "exposed to risk" in an investigation of the mortality experience of assured lives.

Which method would you prefer, and on what grounds?

2. State what you know of the mortality table designated $H^{M^{10-4}}$, and of the purposes for which it has been employed.

Discuss the question whether the $H^{M^{50}}$ table should be retained among the tables based upon any new mortality experience of British offices.

3. Explain Woolhouse's Method of Graduation, and give your opinion of its merits as an instrument for adjusting mortality tables.

4. Describe fully the system of graduation explained in the following paragraph, and its advantages:—

"A graduation of the numbers living by Makeham's formula being first made, the figures thus adjusted were made the base line, and the deviations of the original figures from the formula were taken as the numbers for adjustment by the graphic method."

5. What arguments can be advanced in favour of and against the Net-Premium and Gross-Premium methods of valuation respectively?

Consider the cases of—

(1) A prosperous office.

(2) An office whose solvency is in question.

6. Explain the "Re-insurance" method of valuation. Show mathematically under what circumstances the reserve obtained by it is greater than, equal to, or less than, the net-premium reserve.

In what modified form was this method employed until quite recently?

7. Mention the four methods of dividing profits most generally adopted by British offices.

Discuss the relative merits and demerits of each.

8. On what basis would you compute the rates for a new life office which has resolved to adopt the system of distribution of profits known as the Compound Reversionary Bonus plan?

9. What principles should guide a life office in converting its cash surplus into reversionary bonuses, and in subsequently commuting the reversionary bonuses for cash payments?

10. How would you trace to its several sources the profit realized by a life office in a valuation period?

11. How should rated-up policies be treated in a valuation—

- (1) When the extra risk is met by an addition to the premium?
- (2) When the sum assured is made subject to a contingent debt?

Theoretically, should any distinction be made in the amount of reversionary bonus allocated to policies under the respective conditions?

12. A corporate body owning house property subject to running leases, offers to substitute for every lease, having not less than 40 years unexpired, a new lease for 999 years, subject to an increased ground rent.

State your opinion of the desirability of this conversion on general grounds, and indicate how you would compute the new ground rent.

Second Paper.

13. Give a summary of the powers conferred on a Mortgagee by the Conveyancing Act, 1881, unless a contrary intention is expressed in the mortgage deed.

14. Discuss shortly the advantages and disadvantages of an existing Company constituted by Deed of Settlement registering itself as a limited company under the Joint Stock Companies Acts.

After being fully registered, what steps would the Company have to take to effect an alteration in—

- (a) The memorandum of association?
- (b) The articles of association?

15. What do you understand by a “base fee”?

Under what conditions and by what means can a tenant-in-tail practically borrow on the fee-simple in reversion of his estates?

16. A, tenant for life, has full power of appointment over the trust funds at his death, and appoints a sum to B, who desires to borrow on his reversionary interest. What precautions should be taken, and what enquiries made by a lender?

Do appointments thus made rank *pari passu*, or in priority of date?

17. What are the chief requirements in a standard of exchange?

Discuss shortly the effect of the adoption of a double standard on—

- (1) Joint-stock banks.
- (2) The cotton industry.
- (3) Foreign exchanges.

18. A country is involved in a great war, and requires to borrow large sums. On what terms would you recommend the money to be raised?

State the objection to the method pursued by Great Britain at the end of the last century.

19. What are the principal conditions which govern the rate of interest on permanent investments? Apply your answer to form an opinion as to the future of the rate of interest in this country.

Is there any connection between the market rate of discount and the rate of interest on mortgages of land?

20. Give an outline of the weekly account of the Bank of England, and explain the items appearing in it. What is the Bank rate, and what causes its fluctuations?

21. Give an expression for the reversionary annuity to be charged in respect of an advance of £1, to be secured on the borrower's life interest in an estate after the death of another person, the borrower to have an option to redeem within n years. Explain verbally the borrower's position before the expiry of the n years.

22. A company has agreed to lend £40,000 on a freehold agricultural estate, subject to valuation. Draft a letter to the borrower, setting out the terms and conditions on which the loan would be entertained.

Assuming the conditions to be fully complied with by the borrower, could the latter compel the company to carry out the transaction?

23. A company is asked to purchase a reversion to £10,000 Trust funds, contingent on A surviving B. Draft a schedule of questions for obtaining the necessary information to enable the suitability of the transaction to be judged and a price to be quoted.

24. The probability of an event happening at a single trial being p , in n trials it will happen on the average np times. Write down the expression for the average magnitude of the deviations, irrespective of sign, from this mean result, and give an instance of its application to mortality investigations.

The Candidate having handed in his answers to the foregoing questions, will be supplied with any book or books of tables that he may require in the solution of the following question.

25. A, a widow aged 40, is entitled for life to the income of the following fund—

£5,000 Consols.

£5,000 Victoria 4 per-cent Stock.

£5,000 on mortgage, at $4\frac{1}{2}$ per-cent.

B, her son aged 21, is entitled to a maintenance allowance of £100 per annum as a first charge on the estate, and, further, is absolutely entitled to the reversion. A and B agree to divide the fund. Apportion it into the respective shares.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1892-93.

First Ordinary Meeting, 28 November 1892.

The first ordinary meeting of the session 1892-93 was held at the Hall of the Institute, on the 28th day of November 1892.

The President (MR. AUG. HENDRIKS) in the Chair.

The President delivered an inaugural address.

Mr. Newbatt proposed, and Mr. Crisford seconded, a vote of thanks to Mr. Hendriks for his address, which was carried unanimously.

In replying, the President announced that Mr. Cooke, late Honorary Secretary, had resigned, and that Mr. Hy. Cockburn had been appointed in his place.

Second Ordinary Meeting, 19 December 1892.

The President (MR. A. HENDRIKS) in the Chair.

Mr. George King read a paper entitled "Family Annuities."

The following gentlemen took part in the discussion:—Messrs. R. P. Hardy, Whittall, Frankland, Fox, Watson, Nash, Manly, and Dr. Hunter, M.P. (a visitor).

Third Ordinary Meeting, 30 January 1893.

The President (MR. A. HENDRIKS) in the Chair.

Mr. Kyd's and Mr. Tarn's Prize Essays on "Leasehold Enfranchisement" were read in abstract. Mr. Tarn read the abstract of his own paper; that of Mr. Kyd, owing to the author's unavoidable absence, was read by Mr. Cockburn, Hon. Sec.

The following gentlemen took part in the discussion:—Messrs. Browne, R. P. Hardy, McKenzie, Dowsett (a visitor), Smyth, Bailey, Higham, Manly, and the President.

Fourth Ordinary Meeting, 27 February 1893.

The President (MR. A. HENDRIKS) in the Chair.

A paper "On the Progress of Profit in a Life Assurance Fund" was read by the author, Mr. T. J. Searle.

The following gentlemen took part in the discussion:—Messrs. Ryan, Wyatt, Higham, Schooling, J. A. Robertson, and the President.

Fifth Ordinary Meeting, 27 March 1893.

MR. CRISFORD (Vice-President) in the Chair.

A paper "On Contingent Reversions, Reversionary Life Interests, and Life Interests in Possession" was read by the author, Mr. W. B. Paterson.

The following gentlemen took part in the discussion:—Messrs. Manly, Burridge, R. P. Hardy, George King, W. King, Warner, Bumsted, Nightingale, and Colenso.

Sixth Ordinary Meeting, 24 April 1893.

The President (MR. A. HENDRIKS) in the Chair.

A paper entitled "A Discussion of some Points of Life Assurance Administration in respect of which Divergence of Practice exists: a Plea for Uniformity", was read by the author, Mr. H. C. Thiselton.

The following gentlemen took part in the discussion:—Messrs. Bailey, S. Day, Weeks (a visitor), George King, and Newbatt.

The Forty-Fifth Annual General Meeting, 3 June 1893.

The President (MR. A. HENDRIKS) in the Chair.

The proceedings at the Annual General Meeting will be found on page 576.

REPORT. 1892-93.

"The Council have pleasure in reporting to the members upon the progress of the Institute during the session of 1892-93, the forty-fifth year of its existence.

"The increase in the number of members was 29, as compared with 25 in the preceding year. Since the date of the Charter the number of members has been as follows:

1884-85	.	.	434	1888-89	.	.	563
1885-86	.	.	441	1889-90	.	.	601
1886-87	.	.	484	1890-91	.	.	620
1887-88	.	.	521	1891-92	.	.	645
			1892-93	.	.		674

"The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1893.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1892 .	2	171	201	260	11	645
ii. Withdrawals by						
(1) Death	4	2	44
(2) Resignation	3	12	...	
(3) Default in pay- ment of Sub- scriptions	1	1	21	...	
iii. Additions to Member- ship	2	166	195	227	11	601
(1) By Order of Council	72	...	73
(2) By Re-instatement	1	...	
iv. Transfers	2	166	195	300	11	674
(1) By Examination:						
<i>from Associates</i>	4
<i>to Fellows</i>	4
(2) By Examination:	2	170	191	300	11	674
<i>from Students</i>	1
<i>to Fellows</i>	1
(3) By Examination:	2	171	191	299	11	674
<i>from Students</i>	5
<i>to Associates</i>	5
v. Number of Members in each Class on 31 March 1893 .	2	171	196	294	11	674

"The Council have, with great regret, to report the loss by death during the year of four Fellows, namely, Mr. C. J. Bunyon, Mr. A. G. Finlaison, Mr. C. G. Laing, and Mr. C. McCabe.

"Mr. Bunyon's connection with the Institute dated from 1849, and his well-known writings on Insurance Law, and on various actuarial subjects, form works of acknowledged usefulness.

"Mr. Finlaison, also well known as a writer, and as the Actuary to the Commissioners for the Reduction of the National Debt, was an original member of the Institute. Both of these gentlemen had served the office of Vice-President. Mr. Laing, also, had served on the Council.

"The accounts for the year are satisfactory, the total amount of funds now being £4,552. 16s. 3d., showing an increase in the year of £199. 10s. 9d.

"The income and expenditure account and balance sheet are given herewith.

"The annual subscriptions, together with fines for re-instatements, admission and other fees, amounted to £1,310. 8s., a slight increase over those of the previous year. The total income was £1,727. 1s. 10d.

"The total expenditure for the year was £1,527. 11s. 1d., showing an increase over that of the previous year of £222. 8s. 7d., accounted for by the honorarium to the late Assistant Secretary on his retirement, and the payment of prizes from the Brown Prize Fund.

"The stock in hand of the Institute publications at date is as follows:

No. of Copies	Description of Work
383	<i>Text-Book</i> , Part I.
595	" " II.
131	Mortality Experience Tables.
21	Mortality Experience.
673	Logarithm Cards.
410	Messenger Prize Essay (Friendly Societies).
518	Index to 10 Vols.
115	" to 20 "
9,463	Parts of <i>Journal</i> .

"The following papers were submitted at the sessional meetings of the Institute, namely:

"28 *November* 1892—'An Inaugural Address by the President, Mr. Aug. Hendriks.'

"19 *December* 1892—'On Family Annuities'—by Mr. George King.

"30 *January* 1893—'Abstracts of Messrs. Kyd and Tarn's Prize Essays on "Leasehold Enfranchisement."' "

"27 *February* 1893—'On the Progress of Profit in a Life Assurance Fund'—by Mr. T. J. Searle.

"27 *March* 1893—'Contingent Reversions, Reversionary Life Interests, and Life Interests in Possession'—by Mr. W. B. Paterson.

"24 *April* 1893—'A Discussion of some Points in Life Assurance Administration in respect of which Divergence of Practice exists:—A Plea for Uniformity'—by Mr. H. C. Thiselton.

"An Intermediate Examination in Part I was held on 28 October 1892, when 13 candidates presented themselves, of whom five passed, as follows:

In Class I None.

In Class II W. C. Sharman.

In Class III { J. M. Allen.
P. H. Merfield.
W. A. Sim.
J. Watson.

"As already intimated, the Intermediate Examination in October will not be continued.

"For the Examinations held in the United Kingdom on 21 and 22 April last, 107 candidates presented themselves, namely:

50 for Part I, of whom 19 were successful.

10 .. II. .. 14 ..

17 .. III, .. 6 ..

"The following are the successful candidates, the names in each Class being arranged alphabetically:

PART I.

Class I:

None.

Class II:

(H. M. Davies,
 H. P. Mitchell,
 R. S. B. Savery,
 C. T. Weeden.

Class III:

(F. Appleton.
 A. R. Barrand.
 E. Cumberland.
 N. M. Johannessen.
 J. C. Macmillan.
 F. Marchbank.
 W. A. Martin.
 A. J. Mascall.
 N. Miller.
 O. Parisot.
 J. E. Shimmell.
 H. Slade.
 W. R. Strong.
 A. J. Welman.
 L. A. Wintle.

PART II.

Class I:

A. D. Besant.

Class II:

(J. N. Lewis.
 D. A. Munro.
 H. A. Thompson.
 J. Watson

Class III:

(J. Burn.
 R. E. Gardiner.
 W. C. Laing.
 J. S. Lidbury.
 B. C. Morgan.
 R. Poston.
 A. Taylor.
 W. Warthington.
 R. Y. M. Wright.

PART III.

Class I:

A. W. Watson.

Class II:

W. Hutton.

Class III:

(A. S. Harris.
 J. W. Milner.
 E. C. Thomas.
 A. C. Thorne.

" In the Colonies the Examination entries numbered 16, as under:

For Part I, 34.

" II, 12.

" The results of the Colonial Examinations* will be duly announced.

" The Council of the Institute, believing that the time has arrived when a fresh investigation into the mortality among assured lives and annuity nominees may usefully be entered into, recently appointed a Committee to take the necessary preliminary steps. The Committee, desiring to co-operate in the matter with the Faculty of Actuaries, placed themselves in communication with that body, and were glad to learn that a similar movement was in contemplation in Scotland, and that the Institute may rely on the co-operation of the Faculty in carrying out the work proposed.

" The Council have received and approved an application from Messieurs Bégault and Le Jeune, of Antwerp, and M. Mahillon, of Brussels, for permission to publish in French, at their own cost, the *Institute Text-Book*, parts I and II. The translated edition is now in the hands of the printers, and will, it is hoped, be published before the end of this year.

" In consequence of the retirement through ill-health of Mr. T. H. Cooke from the office of Joint Honorary Secretary, and of the election in his place of Mr. H. Cockburn, the latter gentleman resigned the post of Joint Honorary Librarian, and the Council appointed thereto Mr. Frederick Schooling, who is now acting as Joint Honorary Librarian with Mr. Whittall.

" Under the supervision of the Library Committee and the Honorary Librarians a new catalogue of the Library has been for some time in preparation, and will, it is expected, be in the printer's hands ere long. As a good many additions have been made to the Library of late years, the issue of a new catalogue will, it is hoped, prove of considerable use and interest.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Saturday, 3 June, the President, Mr. Augustus Hendriks, in the chair.

The Report of the Council (given on p. 571) having been read,

The PRESIDENT (Mr. A. Hendriks), in proposing the adoption of the Report, said the Report dealt shortly, but very effectively, with the business and prospects of this Institute. It might be divided into two parts—first, the statistics and accounts, and, secondly, the work of the Institute during the session. Under both headings the Institute might be fairly congratulated upon its position, and upon the progress it had made during the session. Alluding first to the increase in the number of members, that increase had not been by any leaps and bounds, but by the healthy augmentation which a society of this nature had a right to expect, that is, by the gradual accretion of younger members. Each year the increase had proceeded at almost the same ratio. Between the years 1884 and 1893 they had had an accession of no fewer than 240 members. Commencing with 434, they had now attained to 674. Coming next to that subject which was always of painful interest at those annual meetings. There were very few years in which they did not have to record the removal by death of some most valued members. He felt sure that the names mentioned in the Report would be remembered by all with feelings of affection and regard. They all did good work, each in his way, for the Institute. As regards Mr. Bunyon, his name would long live, not only in the Institute but with the public, and he would be remembered in present and future generations by his works on life and fire assurance. They also had to deplore the death of Mr. Finlaison, who did good work for this country as Government actuary, and was also a good friend of this

* The results of the Colonial Examinations are given on page 580.

Institute. Mr. Laing and Mr. McCabe, although one of them was not so actively engaged in the service of the Institute, still were members whom they lost with regret. They had also to add the name of a gentleman of whose death he (the President) heard only yesterday, and with a great deal of sorrow. He alluded to Mr. Joseph Mills. He commenced his work in the Company of which he (the President) had the honour to be the actuary, and he could bear testimony, during his younger years, to his zeal and capacity, and to that straightforwardness of character which qualified him for the higher duties which he assumed subsequently. The regret of those who had more knowledge of him in his later years would, he felt sure, be quite commensurate with that of those who were acquainted with his worth when he commenced his career. The accounts for the year were stated to be satisfactory, and must be held to be so, as they showed an increase in funds as in past years. The annual subscriptions came in with regularity, and the total income amounted to no less than £1,727 a year. The total expenditure had been kept within due bounds. The only special item was that for payment of the prizes from the Brown Prize Fund, and for the departure of that sum out of the funds into the pockets of the writers of those essays he, for one, felt very thankful. The objects of the Brown Fund had been worthily carried out in the prizes awarded to the competing gentlemen. Coming to what might be said to be the most important work of the session, that is, the papers which had been read; there were six papers in all, and certainly five out of the six had undoubtedly preserved a high character and merit. The five papers might be said to be equally divided between practical questions and scientific questions. When he said divided, he ought, perhaps, to explain. In these days practical papers dealt also with mathematics and with scientific subjects, and papers dealing with the science of our profession frequently dealt also with the practical part of our business. The fact is the papers, although they might be qualified as practical or as scientific, contained each of them information upon both subjects. As regards the sixth paper, which was the first in the list, and to which he referred with some diffidence, he would only say this, that if it succeeded in its object, that also was a satisfactory paper. The examinations had been proceeded with, and with very excellent result. A large number of candidates presented themselves for examination, and a satisfactory proportion succeeded in passing. Whilst congratulating those whose names appeared in the list, they should also congratulate those whose names were absent therefrom; because it required a considerable amount of attention to read up for the examinations; and it should be consolatory to remember that some of the most honoured men of the Institute had been through the same process of temporary disappointment, to be rewarded ultimately by achieving that success which all men merit who devote themselves with ability to the study of the science. Then followed the announcement that "the Council of the Institute, believing that the time has come when a fresh investigation into the mortality among assured lives and annuity nominees may usefully be entered into", had appointed a committee. That committee had been unceasing in its labours. The question was a very large one, and had to be dealt with in a very comprehensive way; and if they had not gone further than the preliminaries, it meant that the preliminaries were, perhaps, of as much importance, and in many ways of greater importance, than the actual work to be performed, because to settle the principles upon which it would be conducted, and to settle the form of card, and the information they would require, was perhaps the most important part of the arduous duty which the Institute had undertaken. It was a matter of very great pleasure to find that the other body of actuaries, the Faculty of Actuaries, had most cheerfully associated themselves with the Institute. The *Text-Book* had been translated by a Belgian gentleman, Monsieur Bégault, with the co-operation of two of his confrères, Messieurs Le Jeune and Mahillon: and they had had not only

the satisfaction of seeing that work as translated, and of going through it, but they had also had the satisfaction of seeing those three gentlemen in this country, and had welcomed them here in the name of the Institute. The last paragraph dealt with the Library Committee. The committee were indefatigable in their labours, and they had now the duty of preparing a new catalogue for the library. That information would be received with a great deal of gratitude by those who used the books of the library, because a good catalogue was almost indispensable for a proper appreciation of any library. He concluded by expressing the greatest satisfaction at the present condition of the Institute, both as regards its internal position, and its relations to the public. It continued to do excellent work, both to the companies of which they were actuaries, and also to that larger community who trust to insurance companies their savings for the future provisions of their families.

MR. T. G. C. BROWNE, as the senior Vice-President present, in seconding the motion, said the meeting would be impressed with the fact that the Institute had enjoyed a year of great prosperity, and there was every indication that in the future that prosperity would be maintained. Referring particularly to the important paragraph which mentioned that it was in contemplation that another investigation should be made into the mortality experience of the life assurance companies of the country, he was sure that after an interval of 30 years they would all feel that the result of that investigation must be of very great use to the offices. It was a subject of great congratulation that in that investigation, and the arduous work connected with it, they were to have the co-operation of the Faculty of Actuaries.

The resolution was unanimously adopted.

The ballot was then taken, and at its close the Scrutineers, Messrs. CROSS and JUSTICAN, reported that the President, Vice-President, Council, and Officers, as given in the balloting list recommended by the Council for election, had been unanimously elected.

President.

AUGUSTUS HENDRIKS.

Vice-Presidents.

THOMAS G. C. BROWNE.
RALPH PRICE HARDY.

CHARLES D. HIGHAM.
GEORGE HUMPHREYS.

Council.

ARTHUR HUTCHESON BAILEY.
THOMAS G. C. BROWNE.
*ARTHUR FRANCIS BURRIDGE.
HENRY COCKBURN.
ERNEST COLQUHOUN.
GEORGE STEPHEN CRISFORD.
STANLEY DAY.
DAVID DEUCHAR.
ALEX. J. FINLISON, C.B.
*GEORGE FRANCIS HARDY.
RALPH P. HARDY.
AUGUSTUS HENDRIKS.
CHARLES DANIEL HIGHAM.
GEORGE HUMPHREYS, M.A.
GEORGE KING.

*ALEX. GEORGE MACKENZIE.
HENRY WILLIAM MANLY.
BENJAMIN NEWBATT.
HOWARD JAMES ROTHERY.
GERALD H. RYAN.
*FREDERICK SCHOOLING.
JAMES SORLEY.
THOMAS BOND SPRAGUE, M.A.
A. W. SUNDERLAND, M.A.
WILLIAM SUTTON, M.A.
*JOHN BELL TENNANT.
GEORGE TODD, M.A.
*ROBERT CHARLES TUCKER.
WILLIAM J. H. WHITTALL.
FRANK BERTRAND WYATT.

* New Members of Council.

Treasurer.

HENRY WILLIAM MANLY.

Honorary Secretaries.

HENRY COCKBURN.

| GEORGE KING.

Mr. SEARLE proposed that Messrs. MAKEHAM, MILLER, and MOORE be and hereby are elected Auditors for the ensuing year.

Mr. DAVIES seconded the resolution, which was unanimously agreed to.

Mr. ARCHIBALD DAY then proposed a vote of thanks to the President, the Vice-Presidents, the Council, and Officers, for their services during the past year. He said it was somewhat singular that he had never until that day been qualified to move this vote. He had hitherto been interested in it as the recipient of a share of the thanks that it gave, having been a member of Council for the long period of 35 years. The Report had been adopted, and that Report contained the record of the work of the Council, and as they had approved the Report, they had sufficient justification now for thanking the Council for what they had done. With regard to the President, the members would all agree that the performance of his duty had fully equalled the expectations that they all formed when they elected him to that office. He had worked very hard, and was starting the Institute now upon the new mortality experience, a tremendous business, which some of his predecessors had thought about, but had passed over for the next coming President. It had fallen to Mr. Hendriks' lot to undertake it, and he was quite certain that he would bring it to a very successful issue. After referring in felicitous terms to the labours of the Vice-Presidents, the Treasurer, and the Honorary Secretaries, he said they must also include in the vote the Librarians, and he was sure that what the President had said about their services would be echoed by all who sat around him to-day. He also mentioned the Examiners as being worthy of special recognition.

Mr. BURRIDGE seconded the vote of thanks proposed by Mr. DAY. The record of the work of the Council and other Officers during the past year had been to some extent disclosed by the Report. But that work had been supplemented to an enormous extent by meetings in committee, and in other less conspicuous ways, to which all the Officers gave the most earnest attention and no small portion of their spare time. One of the most important branches of this vote was the question of examination. The number of the candidates was increasing to an extraordinary extent. During the official year just passed, in October 13 candidates presented themselves, and in April no less than 107 aspirants appeared. The task of examining these gentlemen in the three different parts devolved upon four examiners for Part I, four examiners for Part II, and three examiners for Part III. It was a very important and onerous branch of their work to gauge the papers with accuracy and to do justice to the answers sent in by all these candidates. They had also added to the ordinary examinations at home examinations in the Colonies. The result of those examinations, in which 46 candidates were examined, had not yet been disclosed, but he thought that a most satisfactory departure on the part of this Institute in its educational department.

The motion was unanimously agreed to.

Mr. C. D. HIGHAM, on behalf of the President, Vice-Presidents, Council, Officers, and Examiners, returned thanks for the vote. The work the Council had to do was very great, though he should not venture to say so were not his own share so small, and the proposer and seconder had referred to the special vocations of various officers, only omitting one, the Editor of the *Journal* (Mr. H. J. Rothery), who had a difficult duty to perform.

Mr. TUCKER proposed a vote of thanks to the Auditors for their services during the past year.

Mr. SCHOOLING seconded the vote of thanks.

The resolution was unanimously adopted.

The PRESIDENT briefly replied to the personal compliments of which he had been the recipient, and announced that the meeting was adjourned to 27 November next.

COLONIAL EXAMINATIONS.

Examinations were held on 21 and 22 April 1893, at Cape Town, Melbourne, Montreal, Sydney, Toronto, and Wellington, with the following results:

PART I.

Thirty-four Candidates sent in their names, of whom twenty-four presented themselves, and eleven passed as follows:

Class I:

Henderson, R. (Montreal).

Class II:

Boddy, H. M. (Toronto).

Elliott, C. A. (Sydney).

Galwey, C. E. (Wellington).

Johnston, F. H. (Montreal).

McMinn, W. J. R. (Montreal).

Class III:

Adams, C. F. (Wellington).

Bradshaw, T. (Toronto).

Hollingworth, A. C. (Sydney).

Paradice, W. H. (Sydney).

Thodey, R. (Melbourne).

PART II.

Twelve Candidates sent in their names, and presented themselves, of whom three passed as follows:

Class I:

None.

Class II:

Blackadar, A. K. (Montreal).

Class III:

Muter, Percy (Wellington).

Day, W. R. (Sydney).

London, 13 July 1893.

HENRY COCKBURN, }
GEORGE KING, } *Hon. Secretaries.*

ERRATA.

1. Mr. E. L. STABLER contributes the following list of errata in his paper "On Mr. Makeham's Theory of Inverse Probabilities" (*J.I.A.*, xxx, 239):

Page 240, 3rd and 4th lines of 3rd paragraph. omit 12 words, commencing with "of" and ending with "mean value."

„ 240, 4th line of 4th paragraph, instead of "form" write "from."

„ 242, 2nd column, 5th line, instead of $\times \frac{2}{3} \times \left(\frac{1}{3}\right)^3$ read $4 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^3$.

„ 243, the last factor of the 1st term of the 2nd line of the formula should be s^n instead of s^m .

2. Mr. A. W. TARN sends the following corrections to be made in his Prize Essay:

Page 411, line 12, for "15th" read "13th."

„ 412, note (1), for "given" read "is given."

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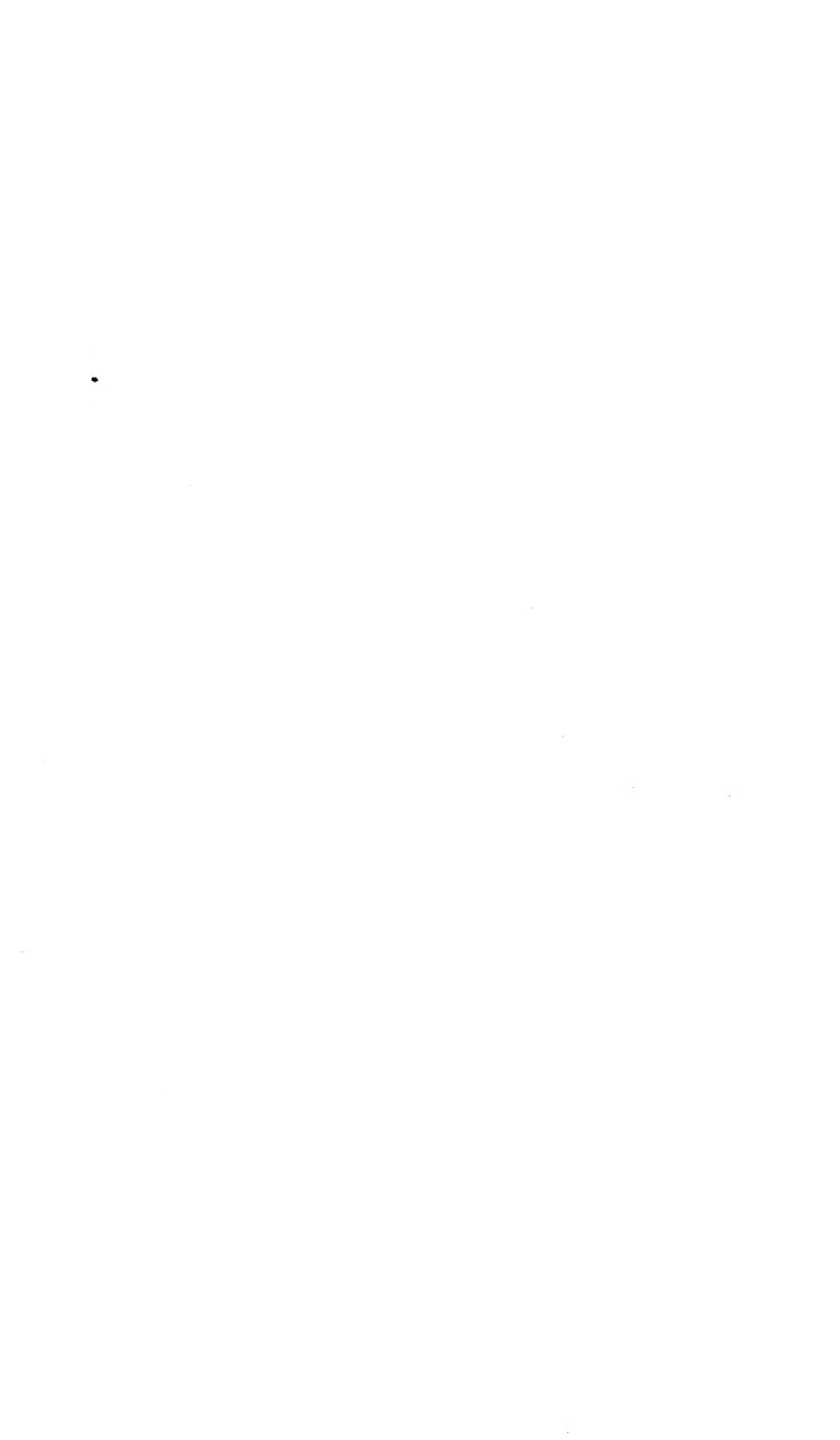
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